Parallel Algorithm for Minimum Spanning Tree

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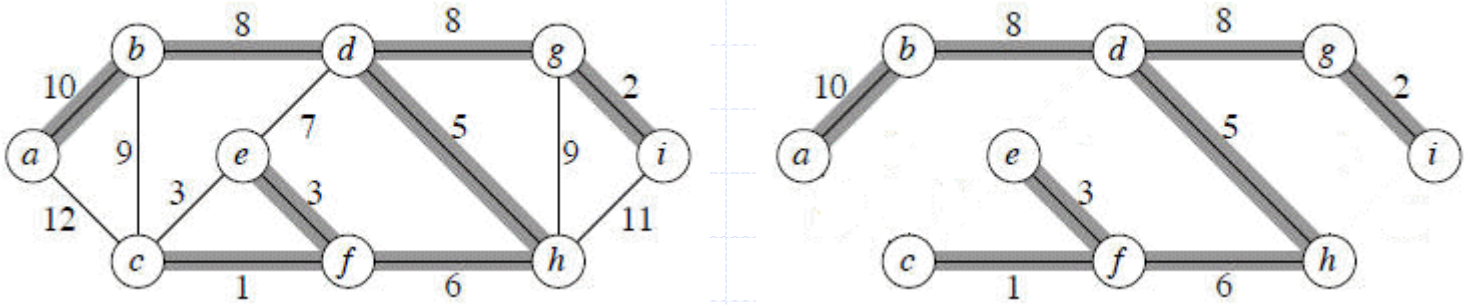
Outline

- Brief introduction.
- Sequential algorithm.
- Concurrent Prim's algorithm.
- Some related problem.

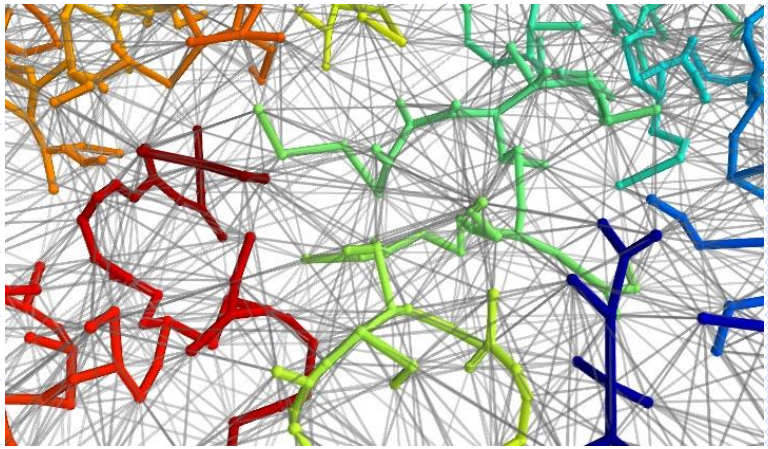
Introduction

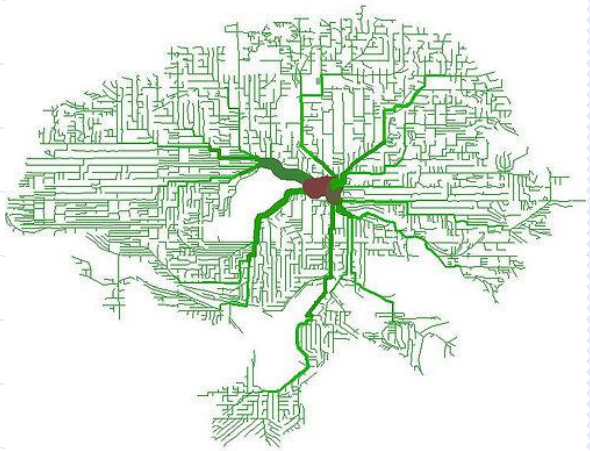
- Minimum Spanning Tree(MST) is one of the well known classical graph problems.



Applications

- Minimum Spanning Tree has many applications in VSLI layout and routing, wireless communication and various other fields.





Sequential Algorithms

- The first serial algorithm for MST was given by a Czech scientist Borůvka, 1926.

Borůvka(1899-1995)

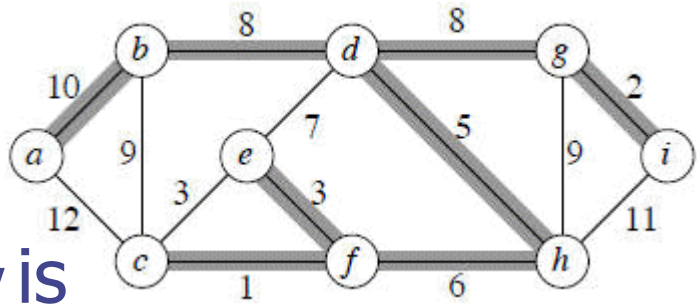
Prim(1921-now)

- Other two commonly used Greedy algorithms are Kruskal's algorithm (1956) and Prim's algorithm(1957).

Kruskal(1928-2010)

Prim's Algorithm

- Initialize: $\boldsymbol{V}_{mst} = \{x\}, x$ is an arbitrary node from $\boldsymbol{V}$, $\boldsymbol{E}_{mst} = \{\emptyset\}$.
- Repeat until $\boldsymbol{V}_{mst} == \boldsymbol{V}$
  - Choose an edge (u,v) with minimal weight such that u is in $\boldsymbol{V}_{mst}$ and v is not.
  - Add v to $\boldsymbol{V}_{mst}$ and (u,v) to the $\boldsymbol{E}_{mst}$.

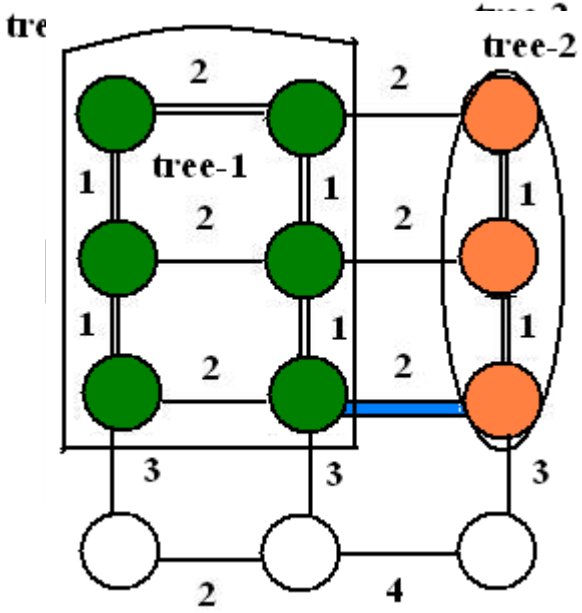


Concurrent Prim's Algorithm

- Key ideas:
  - Each thread grows/colors their own single trees in parallel.
  - When collision occurs between two threads i and j(i<j), thread j merges with i. i keeps growing and j chooses another node to grow a new tree.

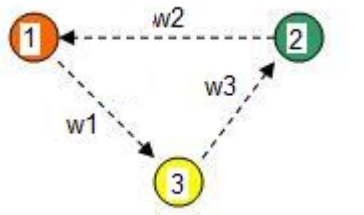
Concurrent Prim's Algorithm

- Things remains the same for growing a single tree in an individual thread.
- For the merge part, we call the MergeTree(i,j) method to do that.



Concurrent Prim's Algorithm

- Lemma 1.1 No cycles are formed during MergeTree operation.



W1<W2;
W2<W3; W3<W1; → W2<W1
Contradiction!

- Lemma 1.2 The edges added by the this this algorithm belong to MST.

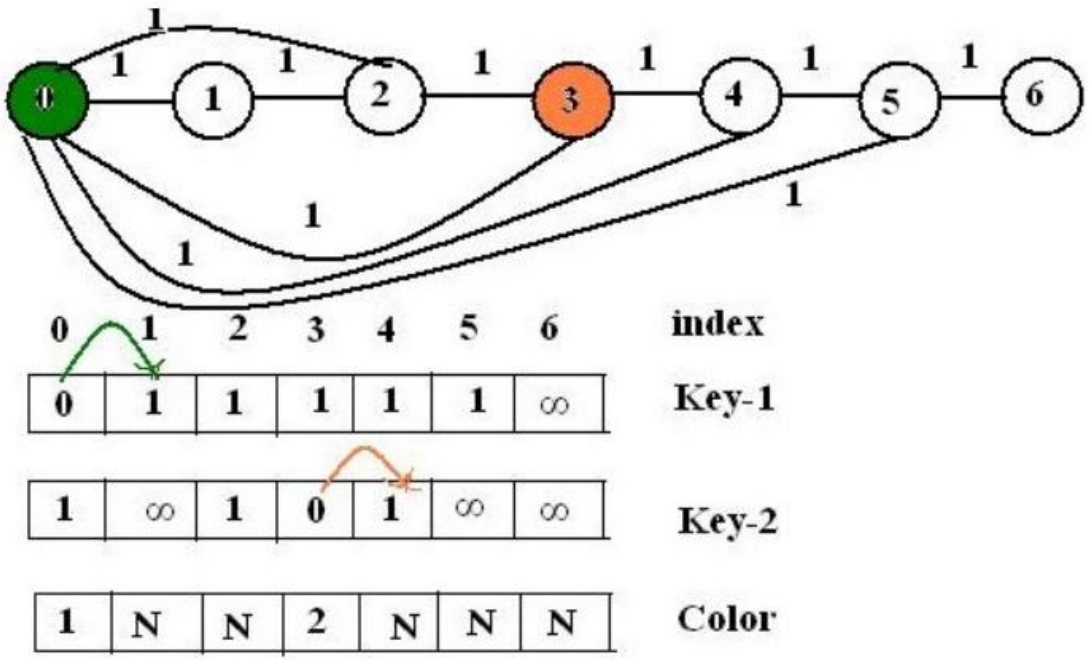
Load Balancing scheme

- Base Problem Size
  - A threshold value for the number of uncolored nodes. If # of nodes fall below that threshold, we terminate the thread j instead of let it pick a new random node to grow a new tree.

The writer has an empirical value equal to $\frac{|V|}{p}$, if there are p threads created.

Heuristics-1

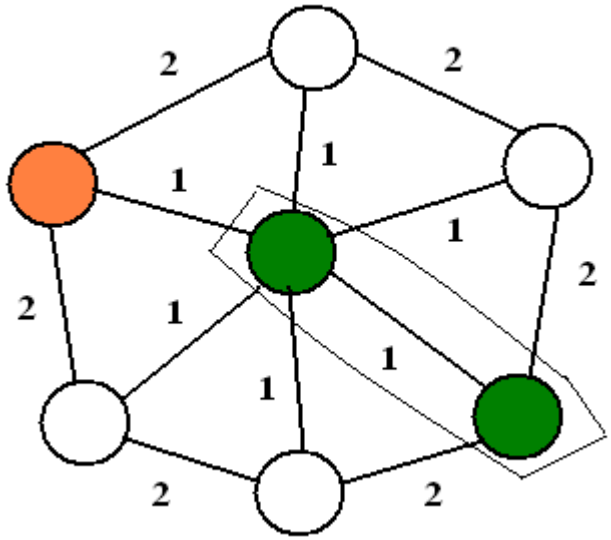
- Warp-around find-min



Heuristics-2

- Threshold nodes
  - Kill the underperformed thread

If the size of tree generated by some thread consistently smaller than N_{th} for k times, we killed that thread.



Experiments and results