Unsigned Binary Integers

- Given an n-bit number

$$x = x_{n-1}2^{n-1} + x_{n-2}2^{n-2} + \cdots + x_1 2^1 + x_0 2^0$$

- Range: 0 to $+2^n - 1$
- Example
  - $0000\ 0000\ 0000\ 0000\ 0000\ 0000\ 0000\ 1011_2$
    $= 0 + \ldots + 1 \times 2^3 + 0 \times 2^2 + 1 \times 2^1 + 1 \times 2^0$
    $= 0 + \ldots + 8 + 0 + 2 + 1 = 11_{10}$
- Using 32 bits
  - 0 to +4,294,967,295

2s-Complement Signed Integers

- Given an n-bit number

$$x = -x_{n-1}2^{n-1} + x_{n-2}2^{n-2} + \cdots + x_1 2^1 + x_0 2^0$$

- Range: -2^{n-1} to $+2^{n-1} - 1$
- Example
  - $1111\ 1111\ 1111\ 1111\ 1111\ 1111\ 1111\ 1100_2$
    $= -1\times2^{31} + 1\times2^{30} + \ldots + 1\times2^2 + 0\times2^1 + 0\times2^0$
    $= -2{,}147{,}483{,}648 + 2{,}147{,}483{,}644 = -4_{10}$
- Using 32 bits
  - –2,147,483,648 to +2,147,483,647

2s-Complement Signed Integers

- Bit 31 is sign bit
  - 1 for negative numbers
  - 0 for non-negative numbers
- $-(-2^{n-1})$ can't be represented
- Non-negative numbers have the same unsigned and 2s-complement representation
- Some specific numbers
  - 0: 0000 0000 … 0000
  - –1: 1111 1111 … 1111
  - Most-negative: 1000 0000 … 0000
  - Most-positive: 0111 1111 … 1111

Signed Negation

- Complement and add 1
  - Complement means 1 → 0, 0 → 1

$$x + \overline{x} = 1111\ldots111_2 = -1$$

$$\overline{x} + 1 = -x$$

- Example: negate +2
  - $+2 = 0000\ 0000 \ldots 0010_2$
  - $-2 = 1111\ 1111 \ldots 1101_2 + 1$
    $= 1111\ 1111 \ldots 1110_2$

2’s Complement

complement all the bits

0101
and add a 1

0110 (6)

	2’sc binary	decimal
$-2^3 =$	1000	-8
$-(2^3 - 1) =$	1001	-7
	1010	-6
	1011	-5
	1100	-4
	1101	-3
	1110	-2
	1111	-1
	0000	0
	0001	1
	0010	2
	0011	3
	0100	4
	0101	5
	0110	6
$2^3 - 1 =$	0111	7

Sign Extension

- Representing a number using more bits
  - Preserve the numeric value
- In MIPS instruction set
  - `addi`: extend immediate value
  - `lb`, `lh`: extend loaded byte/halfword
  - `beq`, `bne`: extend the displacement
- Replicate the sign bit to the left
  - c.f. unsigned values: extend with 0s
- Examples: 8-bit to 16-bit
  - +2: 0000 0010 => 0000 0000 0000 0010
  - –2: 1111 1110 => 1111 1111 1111 1110

Representing Instructions

- Instructions are encoded in binary
  - Called machine code
- MIPS instructions
  - Encoded as 32-bit instruction words
  - Small number of formats encoding operation code (opcode), register numbers, …
  - Regularity!
- Register numbers
  - $t0 – $t7 are reg's 8 – 15
  - $t8 – $t9 are reg's 24 – 25
  - $s0 – $s7 are reg's 16 – 23

MIPS R-format Instructions

op	rs	rt	rd	shamt	funct
6 bits	5 bits	5 bits	5 bits	5 bits	6 bits

- Instruction fields
  - op: operation code (opcode)
  - rs: first source register number
  - rt: second source register number
  - rd: destination register number
  - shamt: shift amount (00000 for now)
  - funct: function code (extends opcode, selects the specific variant of the operation specified in the opcode field)

R-format Example

op	rs	rt	rd	shamt	funct
6 bits	5 bits	5 bits	5 bits	5 bits	6 bits

```
add $t0, $s1, $s2
```

special	$s1	$s2	$t0	0	add
0	17	18	8	0	32_{ten}
000000	10001	10010	01000	00000	100000

$00000010001100100100000000100000_2 = 02324020_{16}$

Hexadecimal

- Base 16
  - Compact representation of bit strings
  - 4 bits per hex digit

0	0000	4	0100	8	1000	c	1100
1	0001	5	0101	9	1001	d	1101
2	0010	6	0110	a	1010	e	1110
3	0011	7	0111	b	1011	f	1111

- Example: eca8 6420
  - 1110 1100 1010 1000 0110 0100 0010 0000

MIPS I-format Instructions

op	rs	rt	constant or address
6 bits	5 bits	5 bits	16 bits

- Immediate arithmetic and load/store instructions
  - rt: destination -- rs source register number
  - Constant: -2^{15} to $+2^{15}-1$
  - Address: offset added to base address in rs
- ***Design Principle 4:* Good design demands good compromises**
  - Different formats complicate decoding, but allow 32-bit instructions uniformly
  - Keep formats as similar as possible

MIPS I-format Instructions

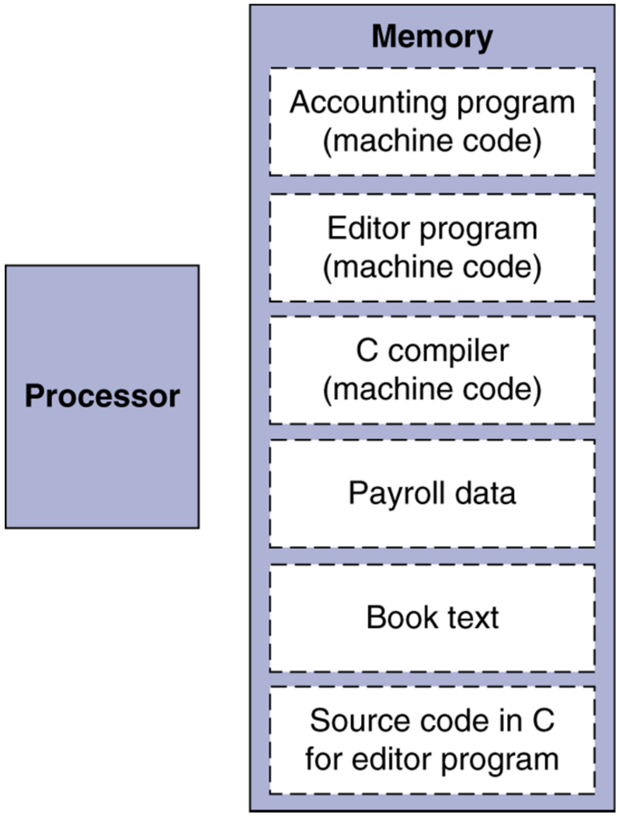
op	rs	rt	constant or address
6 bits	5 bits	5 bits	16 bits

`addi $t0, $s1, 10`

addi	$t0	$s1	constant
8	8	17	10
001000	10000	10001	0000000000001010

Stored Program Computers

The BIG Picture



- Instructions represented in binary, just like data
- Instructions and data stored in memory
- Programs can operate on programs
  - e.g., compilers, linkers, …
- Binary compatibility allows compiled programs to work on different computers
  - Standardized ISAs

Logical Operations

- Instructions for bitwise manipulation

Operation	C	Java	MIPS
Shift left	<<	<<	`sll`
Shift right	>>	>>>	`srl`
Bitwise AND	&	&	`and, andi`
Bitwise OR	|	|	`or, ori`
Bitwise NOT	~	~	`nor`

- Useful for extracting and inserting groups of bits in a word

Shift Operations

op	rs	rt	rd	shamt	funct
6 bits	5 bits	5 bits	5 bits	5 bits	6 bits

- shamt: how many positions to shift
- Shift left logical
  - Shift left and fill with 0 bits
  - `sll` by *i* bits multiplies by 2^i
- Shift right logical
  - Shift right and fill with 0 bits
  - `srl` by *i* bits divides by 2^i (unsigned only)

AND Operations

- Useful to mask bits in a word
  - Select some bits, clear others to 0

```
and $t0, $t1, $t2
```

$t2	0000 0000 0000 0000 0000 1101 1100 0000
$t1	0000 0000 0000 0000 0011 1100 0000 0000
$t0	0000 0000 0000 0000 0000 1100 0000 0000

OR Operations

- Useful to include bits in a word
  - Set some bits to 1, leave others unchanged

```
or $t0, $t1, $t2
```

\$t2	0000 0000 0000 0000 0000 1101 1100 0000
\$t1	0000 0000 0000 0000 0011 1100 0000 0000
\$t0	0000 0000 0000 0000 0011 1101 1100 0000

NOT Operations

- Useful to invert bits in a word
  - Change 0 to 1, and 1 to 0
- MIPS has NOR 3-operand instruction
  - a NOR b == NOT (a OR b)

```
nor $t0, $t1, $zero
```

Register 0: always read as zero

$zero	0000 0000 0000 0000 0000 0000 0000 0000
$t1	0000 0000 0000 0000 0011 1100 0000 0000
$t0	1111 1111 1111 1111 1100 0011 1111 1111

Conditional Operations

- Branch to a labeled instruction if a condition is true
  - Otherwise, continue sequentially
- `beq rs, rt, L1`
  - if (rs == rt) branch to instruction labeled L1;
- `bne rs, rt, L1`
  - if (rs != rt) branch to instruction labeled L1;
- `j L1`
  - unconditional jump to instruction labeled L1

Conditional Operations

- `beq $s0, $s1, L1`
- `bne $s0, $s1, L1`
- Instruction format

How to specify L1

op	rs	rt	constant or address
6 bits	5 bits	5 bits	16 bits

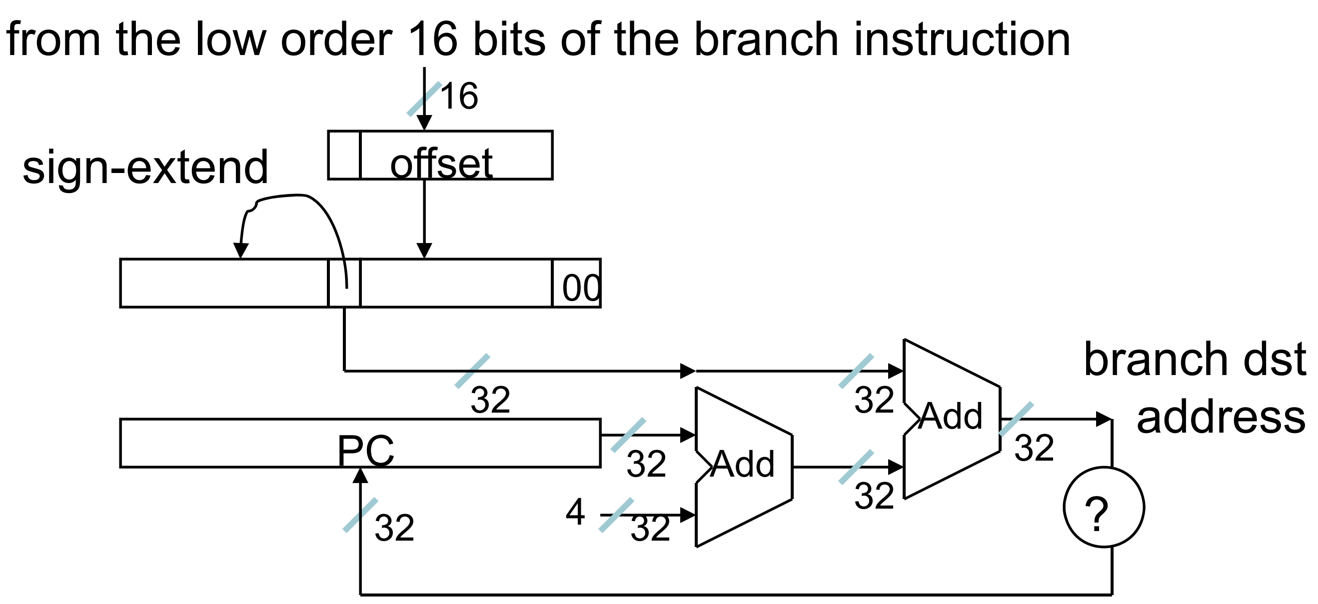
5	16	17	L1

4	16	17	L1

Specifying Branch Destination

- We could specify the memory location, but that will require 32 bits ???
- Can use a base register, the base register is PC
- Limits jumps to $-2^{15} \rightarrow 2^{15} - 1$
- In reality, 00 is appended to the immediate thus instructions (words not bytes)

Branch destination



Jump instruction

- J Label #go to label

op	26-bit address
6 bits	

2	xxxx..xx

- Again, concatenating 00 increase the effective number to 28 + the left-most 4 bits of the PC (added to the PC)

Jump instruction

from the low order 26 bits of the jump instruction