P 6.13 [a] $i = C\frac{dv}{dt} = (5 \times 10^{-6})[500t(-2500)e^{-2500t} + 500e^{-2500t}]$

$$= 2.5 \times 10^{-3}e^{-2500t}(1 - 2500t)\ \text{A}$$

[b] $v(100\,\mu) = 500(100 \times 10^{-6})e^{-0.25} = 38.94\,\text{mV}$

$$i(100\,\mu) = (2.5 \times 10^{-3})e^{-0.25}(1 - 0.25) = 1.46\,\text{mA}$$

$$p(100\,\mu) = vi = (38.94 \times 10^{-3})(1.46 \times 10^{-3}) = 56.86\,\mu\text{W}$$

[c] $p > 0$, so the capacitor is absorbing power.

[d] $v(100\,\mu) = 38.94\,\text{mV}$

$$w = \frac{1}{2}Cv^2 = \frac{1}{2}(5 \times 10^{-6})(38.94 \times 10^{-3})^2 = 3.79\,\text{nJ}$$

[e] The energy is maximum when the voltage is maximum:

$$\frac{dv}{dt} = 0 \text{ when } (1 - 2500t) = 0 \text{ or } t = 0.4\,\text{ms}$$

$$v_{\text{max}} = 500(0.4 \times 10^{-3})^2e^{-1} = 73.58\,\text{mV}$$

$$p_{\text{max}} = \frac{1}{2}Cv_{\text{max}}^2 = 13.53\,\text{nJ}$$

P 6.14 [a]

$$\begin{aligned} v &= 0 && t < 0 \\ v &= 10t\ \text{A} && 0 \le t \le 2\,\text{s} \\ v &= 40 - 10t\ \text{A} && 2 \le t \le 6\,\text{s} \\ v &= 10t - 80\ \text{A} && 6 \le t \le 8\,\text{s} \\ v &= 0 && 8\,\text{s} < t \end{aligned}$$

[b] $i = C\frac{dv}{dt}$

$$\begin{aligned} i &= 0 && t < 0 \\ i &= 2\,\text{mA} && 0 < t < 2\,\text{s} \\ i &= -2\,\text{mA} && 2 < t < 6\,\text{s} \\ i &= 2\,\text{mA} && 6 < t < 8\,\text{s} \\ i &= 0 && 8\,\text{s} < t \end{aligned}$$

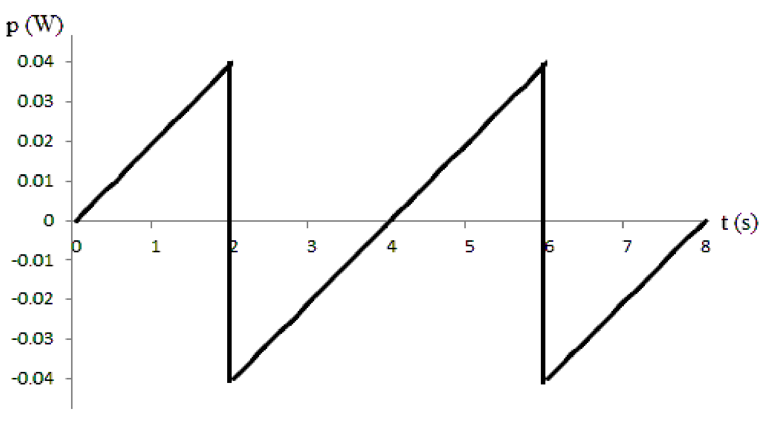
$p = vi$

$$
\begin{aligned}
p &= 0 && t < 0 \\
p &= (10t)(0.002) = 0.02t\,\text{W} && 0 < t < 2\,\text{s} \\
p &= (40 - 10t)(-0.002) = 0.02t - 0.08\,\text{W} && 2 < t < 6\,\text{s} \\
p &= (10t - 80)(0.002) = 0.02t - 0.16\,\text{W} && 6 < t < 8\,\text{s} \\
p &= 0 && 8\,\text{s} < t
\end{aligned}
$$

$w = \int p\,dx$

$$
\begin{aligned}
w &= 0 && t < 0 \\
w &= \int_0^t (0.02x)\,dx = 0.01x^2 \Big|_0^t = 0.01t^2\,\text{J} && 0 < t < 2\,\text{s} \\
w &= \int_2^t (0.02x - 0.08)\,dx + 0.04 \\
&= (0.01x^2 - 0.08x) \Big|_2^t + 0.04 \\
&= 0.01t^2 - 0.08t + 0.16\,\text{J} && 2 < t < 6\,\text{s} \\
w &= \int_6^t (0.02x - 0.16)\,dx + 0.04 \\
&= (0.01x^2 - 0.16x) \Big|_6^t + 0.04 \\
&= 0.01t^2 - 0.16t + 0.64\,\text{J} && 6 < t < 8\,\text{s} \\
w &= 0 && 8\,\text{s} < t
\end{aligned}
$$

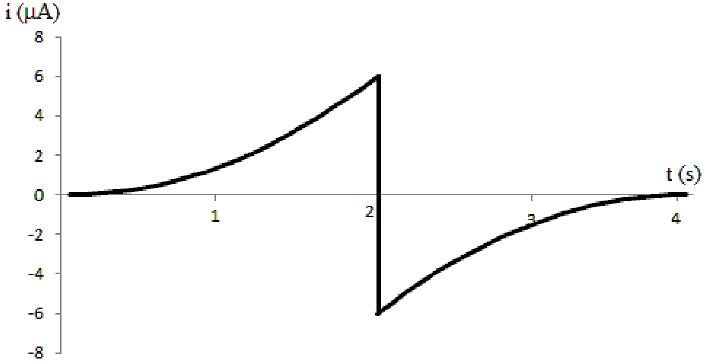
[**c**]



P 6.16 $i_C = C(dv/dt)$

$$0 < t < 2\,\text{s}: \qquad i_C = 100 \times 10^{-9}(15)t^2 = 1.5 \times 10^{-6}t^2\ \text{A}$$

$$2 < t < 4\,\text{s}: \qquad i_C = 100 \times 10^{-9}(-15)(t-4)^2 = -1.5 \times 10^{-6}(t-4)^2\ \text{A}$$



i (μA)
8
6
4
2
0
-2
-4
-6
-8
t (s)
1
2
3
4

P 6.19 **[a]**

$$v = \frac{1}{0.5\times 10^{-6}}\int_0^{500\times 10^{-6}} 50\times 10^{-3}e^{-2000t}\,dt - 20$$

$$= 100\times 10^3 \frac{e^{-2000t}}{-2000}\Big|_0^{500\times 10^{-6}} - 20$$

$$= 50(1-e^{-1}) - 20 = 11.61\,\text{V}$$

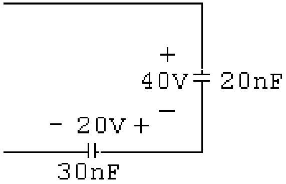
$$w = \tfrac{1}{2}Cv^2 = \tfrac{1}{2}(0.5)(10^{-6})(11.61)^2 = 33.7\,\mu\text{J}$$

[b] $v(\infty) = 50 - 20 = 30\text{V}$

$$w(\infty) = \frac{1}{2}(0.5\times 10^{-6})(30)^2 = 225\,\mu\text{J}$$

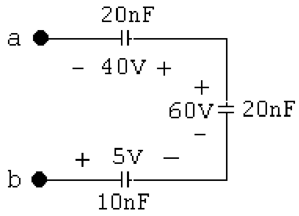
P 6.27 **[a]** $\frac{1}{C_1} = \frac{1}{48} + \frac{1}{24} = \frac{1}{16}; \qquad C_1 = 16\,\text{nF}$

$$C_2 = 4 + 16 = 20\,\text{nF}$$



$$\frac{1}{C_3} = \frac{1}{20} + \frac{1}{30} = \frac{1}{12}; \qquad C_3 = 12\,\text{nF}$$

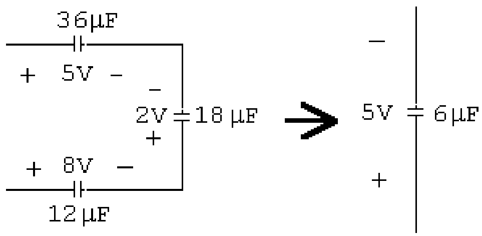
$$C_4 = 12 + 8 = 20\,\text{nF}$$

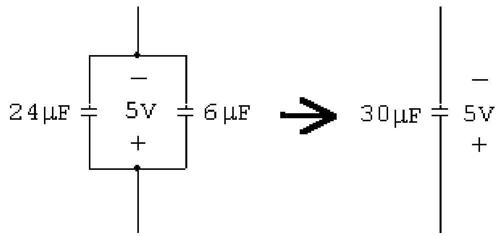


$$\frac{1}{C_5} = \frac{1}{20} + \frac{1}{20} + \frac{1}{10} = \frac{1}{5}; \qquad C_5 = 5\,\text{nF}$$

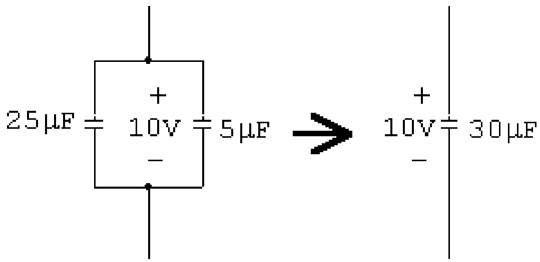
Equivalent capacitance is 5 nF with an initial voltage drop of +15 V.

[b] $\frac{1}{36} + \frac{1}{18} + \frac{1}{12} = \frac{1}{6} \quad \therefore \quad C_{\text{eq}} = 6\,\mu\text{F} \qquad\qquad 24 + 6 = 30\,\mu\text{F}$

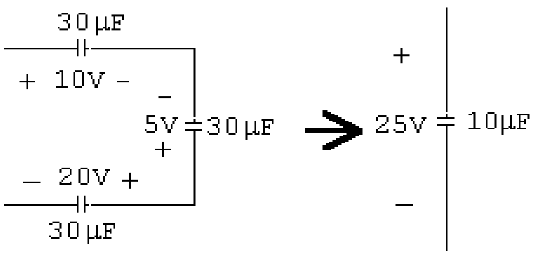




$25 + 5 = 30\,\mu\text{F}$

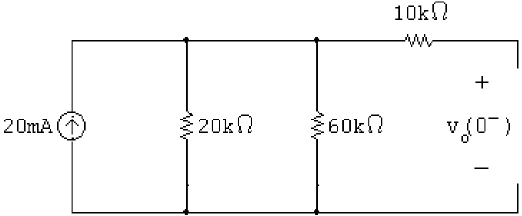


$$\frac{1}{30} + \frac{1}{30} + \frac{1}{30} = \frac{3}{30} \quad \therefore \quad C_{\text{eq}} = 10\,\mu\text{F}$$



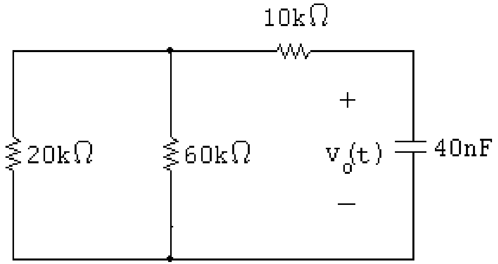
Equivalent capacitance is 10 μF with an initial voltage drop of +25 V.

P 7.22 For $t < 0$:



$$V_o = (20{,}000 \| 60{,}000)(20 \times 10^{-3}) = 300\,\text{V}$$

For $t \geq 0$:



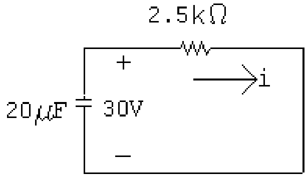
$$R_{\text{eq}} = 10{,}000 + (20{,}000 \| 60{,}000) = 25\,\text{k}\Omega$$

$$\tau = R_{\text{eq}}C = (25{,}000)(40 \times 10^{-9}) = 1\,\text{ms}$$

$$v(t) = V_o e^{-t/\tau} = 300e^{-1000t}\,\text{V} \qquad t \geq 0$$

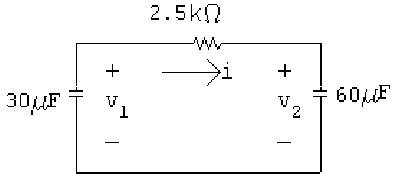
P 7.25 **[a]** $v_1(0^-) = v_1(0^+) = (0.006)(5000) = 30\,\text{V}$ $\qquad v_2(0^+) = 0$

$$C_{\text{eq}} = (30)(40)/90 = 20\,\mu\text{F}$$



$$\tau = (2.5 \times 10^3)(20 \times 10^{-6}) = 50\text{ms}; \qquad \frac{1}{\tau} = 20$$

$$i = \frac{30}{2500}e^{-20t} = 12e^{-20t}\,\text{mA}, \qquad t \geq 0^+$$



$$v_1 = \frac{-1}{30^{-6}}\int_0^t 12 \times 10^{-3}e^{-20x}\,dx + 30 = 20e^{-20t} + 10\,\text{V}, \qquad t \geq 0$$

$$v_2 = \frac{1}{60 \times 10^{-6}} \int_0^t 12 \times 10^{-3} e^{-20x}\, dx + 0 = -10e^{-20t} + 10\,\text{V}, \qquad t \geq 0$$

[b] $w(0) = \frac{1}{2}(30 \times 10^{-6})(30)^2 = 13.5\,\text{mJ}$

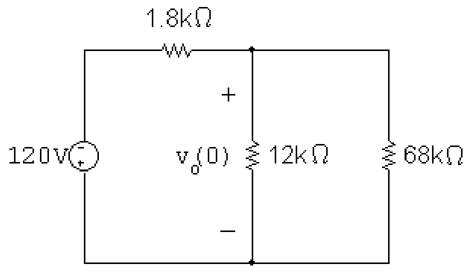
[c] $w_{\text{trapped}} = \frac{1}{2}(30 \times 10^{-6})(10)^2 + \frac{1}{2}(60 \times 10^{-6})(10)^2 = 4.5\,\text{mJ}.$

The energy dissipated by the 2.5 kΩ resistor is equal to the energy dissipated by the two capacitors; it is easier to calculate the energy dissipated by the capacitors:

$$w_{\text{diss}} = \frac{1}{2}(20 \times 10^{-6})(30)^2 = 9\,\text{mJ}.$$

Check: $w_{\text{trapped}} + w_{\text{diss}} = 4.5 + 9 = 13.5\,\text{mJ}; \qquad w(0) = 13.5\,\text{mJ}.$

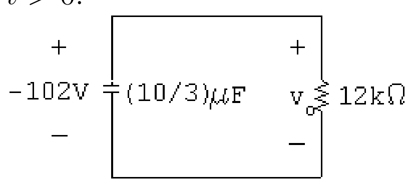
P 7.26 [a] $t < 0$:



$$R_{\text{eq}} = 12\,\text{k} \| 8\,\text{k} = 10.2\,\text{k}\Omega$$

$$v_o(0) = \frac{10{,}200}{10{,}200 + 1800}(-120) = -102\,\text{V}$$

$t > 0$:



$$\tau = [(10/3) \times 10^{-6})(12{,}000) = 40\,\text{ms}; \qquad \frac{1}{\tau} = 25$$

$$v_o = -102e^{-25t}\,\text{V}, \quad t \geq 0$$

$$p = \frac{v_o^2}{12{,}000} = 867 \times 10^{-3} e^{-50t}\,\text{W}$$

$$w_{\text{diss}} = \int_0^{12 \times 10^{-3}} 867 \times 10^{-3} e^{-50t}\, dt$$

$$= 17.34 \times 10^{-3}(1 - e^{-50(12 \times 10^{-3})}) = 7824\,\mu\text{J}$$

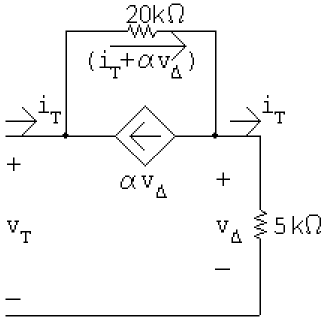
[b] $w(0) = \left(\frac{1}{2}\right)\left(\frac{10}{3}\right)(102)^2 \times 10^{-6} = 17.34\,\text{mJ}$

$$0.75w(0) = 13\,\text{mJ}$$

$$\int_0^{t_o} 867 \times 10^{-3} e^{-50x}\,dx = 13 \times 10^{-3}$$

$$\therefore \quad 1 - e^{-50t_o} = 0.75; \qquad e^{50t_o} = 4; \quad \text{so} \quad t_o = 27.73\,\text{ms}$$

P 7.31 **[a]**



$$v_T = 20 \times 10^3(i_T + \alpha v_\Delta) + 5 \times 10^3 i_T$$

$$v_\Delta = 5 \times 10^3 i_T$$

$$v_T = 25 \times 10^3 i_T + 20 \times 10^3 \alpha(5 \times 10^3 i_T)$$

$$R_{\text{Th}} = 25{,}000 + 100 \times 10^6 \alpha$$

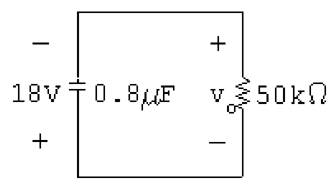
$$\tau = R_{\text{Th}} C = 40 \times 10^{-3} = R_{\text{Th}}(0.8 \times 10^{-6})$$

$$R_{\text{Th}} = 50\,\text{k}\Omega = 25{,}000 + 100 \times 10^6 \alpha$$

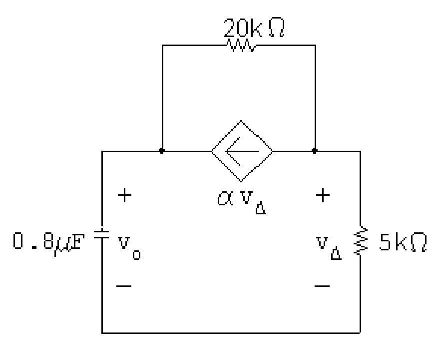
$$\alpha = \frac{25{,}000}{100 \times 10^6} = 2.5 \times 10^{-4}\,\text{A/V}$$

[b] $v_o(0) = (-5 \times 10^{-3})(3600) = -18\,\text{V} \qquad t < 0$

$t > 0$:



$$v_o = -18e^{-25t}\,\text{V}, \quad t \geq 0$$

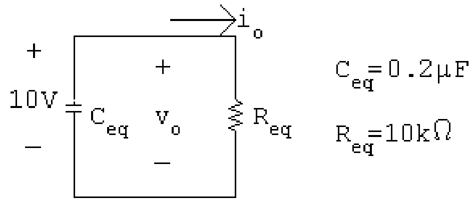


$$\frac{v_\Delta}{5000} + \frac{v_\Delta - v_o}{20{,}000} + 2.5 \times 10^{-4} v_\Delta = 0$$

$$4v_\Delta + v_\Delta - v_o + 5v_\Delta = 0$$

$$\therefore \quad v_\Delta = \frac{v_o}{10} = -1.8e^{-25t}\,\text{V}, \quad t \geq 0^+$$

P 7.34 **[a]** The equivalent circuit for $t > 0$:



$$\tau = 2\,\text{ms}; \qquad 1/\tau = 500$$

$$v_o = 10e^{-500t}\,\text{V}, \qquad t \geq 0$$

$$i_o = e^{-500t}\,\text{mA}, \qquad t \geq 0^+$$

$$i_{24\text{k}\Omega} = e^{-500t}\left(\frac{16}{40}\right) = 0.4e^{-500t}\,\text{mA}, \qquad t \geq 0^+$$

$$p_{24\text{k}\Omega} = (0.16 \times 10^{-6}e^{-1000t})(24{,}000) = 3.84e^{-1000t}\,\text{mW}$$

$$w_{24\text{k}\Omega} = \int_0^\infty 3.84 \times 10^{-3}e^{-1000t}\,dt = -3.84 \times 10^{-6}(0-1) = 3.84\,\mu\text{J}$$

$$w(0) = \frac{1}{2}(0.25 \times 10^{-6})(40)^2 + \frac{1}{2}(1 \times 10^{-6})(50)^2 = 1.45\,\text{mJ}$$

$$\%\text{ diss}\,(24\,\text{k}\Omega) = \frac{3.84 \times 10^{-6}}{1.45 \times 10^{-3}} \times 100 = 0.26\%$$

[b] $p_{400\Omega} = 400(1 \times 10^{-3}e^{-500t})^2 = 0.4 \times 10^{-3}e^{-1000t}$

$$w_{400\Omega} = \int_0^\infty p_{400}\,dt = 0.40\,\mu\text{J}$$

$$\%\text{ diss}\,(400\,\Omega) = \frac{0.4 \times 10^{-6}}{1.45 \times 10^{-3}} \times 100 = 0.03\%$$

$$i_{16k\Omega} = e^{-500t}\left(\frac{24}{40}\right) = 0.6e^{-500t}\,\text{mA}, \quad t \geq 0^+$$

$$p_{16k\Omega} = (0.6 \times 10^{-3}e^{-500t})^2(16{,}000) = 5.76 \times 10^{-3}e^{-1000t}\,\text{W}$$

$$w_{16k\Omega} = \int_0^\infty 5.76 \times 10^{-3}e^{-1000t}\,dt = 5.76\,\mu\text{J}$$

$$\%\text{ diss}\,(16\,\text{k}\Omega) = 0.4\%$$

[c] $\sum w_{\text{diss}} = 3.84 + 5.76 + 0.4 = 10\,\mu\text{J}$

$$w_{\text{trapped}} = w(0) - \sum w_{\text{diss}} = 1.45 \times 10^{-3} - 10 \times 10^{-6} = 1.44\,\text{mJ}$$

$$\%\ \text{trapped} = \frac{1.44}{1.45} \times 100 = 99.31\%$$

Check: $0.26 + 0.03 + 0.4 + 99.31 = 100\%$

P 7.53 [a] Use voltage division to find the initial value of the voltage:

$$v_c(0^+) = v_{9k} = \frac{9\,\text{k}}{9\,\text{k} + 3\,\text{k}}(120) = 90\,\text{V}$$

[b] Use Ohm's law to find the final value of voltage:

$$v_c(\infty) = v_{40k} = -(1.5 \times 10^{-3})(40 \times 10^3) = -60\,\text{V}$$

[c] Find the Thévenin equivalent with respect to the terminals of the capacitor:

$$V_{\text{Th}} = -60\,\text{V}, \qquad R_{\text{Th}} = 10\,\text{k} + 40\,\text{k} = 50\,\text{k}\Omega$$

$$\tau = R_{\text{Th}}C = 1\,\text{ms} \; = 1000\,\mu\text{s}$$

[d] $v_c = v_c(\infty) + [v_c(0^+) - v_c(\infty)]e^{-t/\tau}$

$$= -60 + (90 + 60)e^{-1000t} = -60 + 150e^{-1000t}\,\text{V}, \quad t \geq 0$$

We want $v_c = -60 + 150e^{-1000t} = 0$:

Therefore $t = \dfrac{\ln(150/60)}{1000} = 916.3\,\mu\text{s}$

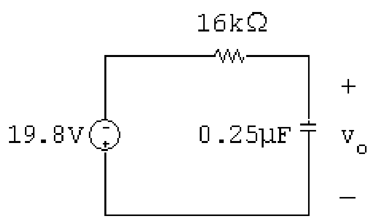
P 7.60 For $t > 0$

$$V_{\rm Th} = (-25)(16{,}000)i_{\rm b} = -400 \times 10^3 i_{\rm b}$$

$$i_{\rm b} = \frac{33{,}000}{80{,}000}(120 \times 10^{-6}) = 49.5\,\mu\text{A}$$

$$V_{\rm Th} = -400 \times 10^3(49.5 \times 10^{-6}) = -19.8\,\text{V}$$

$$R_{\rm Th} = 16\,\text{k}\Omega$$



$$v_o(\infty) = -19.8\,\text{V}; \qquad v_o(0^+) = 0$$

$$\tau = (16,000)(0.25 \times 10^{-6}) = 4\,\text{ms}; \qquad 1/\tau = 250$$

$$v_o = -19.8 + 19.8e^{-250t}\,\text{V}, \qquad t \geq 0$$

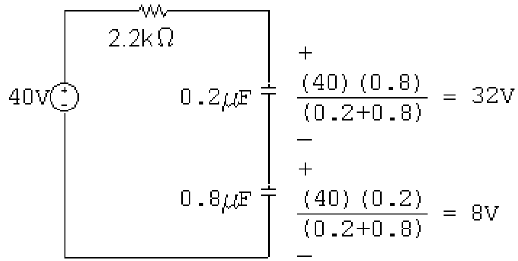
$$w(t) = \frac{1}{2}(0.25 \times 10^{-6})v_o^2 = w(\infty)(1 - e^{-250t})^2\,\text{J}$$

$$(1 - e^{-250t})^2 = \frac{0.36w(\infty)}{w(\infty)} = 0.36$$

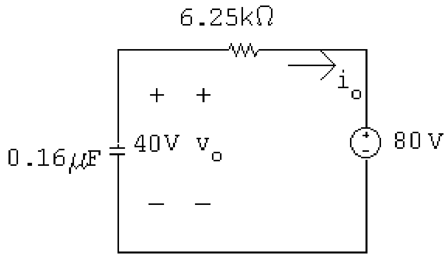
$$1 - e^{-250t} = 0.6$$

$$e^{-250t} = 0.4 \quad \therefore \qquad t = 3.67\,\text{ms}$$

P 7.65 **[a]** $t < 0$



$t > 0$



$$v_o(0^-) = v_o(0^+) = 40\,\text{V}$$

$$v_o(\infty) = 80\,\text{V}$$

$$\tau = (0.16 \times 10^{-6})(6.25 \times 10^3) = 1\,\text{ms}; \qquad 1/\tau = 1000$$

$$v_o = 80 - 40e^{-1000t}\,\text{V}, \qquad t \geq 0$$

[b] $i_o = -C\dfrac{dv_o}{dt} = -0.16 \times 10^{-6}[40{,}000e^{-1000t}]$

$$= -6.4e^{-1000t}\,\text{mA}; \qquad t \geq 0^+$$

[c] $v_1 = \dfrac{-1}{0.2 \times 10^{-6}} \displaystyle\int_0^t -6.4 \times 10^{-3} e^{-1000x}\,dx + 32$

$$= 64 - 32e^{-1000t}\,\text{V}, \qquad t \geq 0$$

[d] $$v_2 = \frac{-1}{0.8 \times 10^{-6}} \int_0^t -6.4 \times 10^{-3} e^{-1000x}\, dx + 8$$

$$= 16 - 8e^{-1000t}\,\text{V}, \qquad t \geq 0$$

[e] $$w_{\text{trapped}} = \frac{1}{2}(0.2 \times 10^{-6})(64)^2 + \frac{1}{2}(0.8 \times 10^{-6})(16)^2 = 512\,\mu\text{J}.$$