P 6.1 [a] $v = L\frac{di}{dt}$

$$= (150 \times 10^{-6})(25)[e^{-500t} - 500te^{-500t}] = 3.75e^{-500t}(1 - 500t)\,\text{mV}$$

[b] $i(5\,\text{ms}) = 25(0.005)(e^{-2.5}) = 10.26\,\text{mA}$

$$v(5\,\text{ms}) = 0.00375(e^{-2.5})(1 - 2.5) = -461.73\,\mu\text{V}$$

$$p(5\,\text{ms}) = vi = (10.26 \times 10^{-3})(-461.73 \times 10^{-6}) = -4.74\,\mu\text{W}$$

[c] delivering $4.74\,\mu\text{W}$

[d] $i(5\,\text{ms}) = 10.26\,\text{mA}$ (from part [b])

$$w = \frac{1}{2}Li^2 = \frac{1}{2}(150 \times 10^{-6})(0.01026)^2 = 7.9\,\text{nJ}$$

[e] The energy is a maximum where the current is a maximum:

$$\frac{di_L}{dt} = 0 \quad \text{when} \quad 1 - 500t = 0 \quad \text{or} \quad t = 2\,\text{ms}$$

$$i_{\text{max}} = 25(0.002)e^{-1} = 18.39\,\text{mA}$$

$$w_{\text{max}} = \frac{1}{2}(150 \times 10^{-6})(0.01839)^2 = 25.38\,\text{nJ}$$

P 6.2 [a]

$$\begin{aligned} i &= 0 && t < 0 \\ i &= 4t\ \text{A} && 0 \le t \le 25\,\text{ms} \\ i &= 0.2 - 4t\ \text{A} && 25 \le t \le 50\,\text{ms} \\ i &= 0 && 50\,\text{ms} < t \end{aligned}$$

[b] $v = L\frac{di}{dt} = 500 \times 10^{-3}(4) = 2\,\text{V} \qquad 0 \le t \le 25\,\text{ms}$

$$v = 500 \times 10^{-3}(-4) = -2\,\text{V} \qquad 25 \le t \le 50\,\text{ms}$$

$$\begin{aligned} v &= 0 && t < 0 \\ v &= 2\,\text{V} && 0 < t < 25\,\text{ms} \\ v &= -2\,\text{V} && 25 < t < 50\,\text{ms} \\ v &= 0 && 50\,\text{ms} < t \end{aligned}$$

$$p = vi$$

$$
\begin{aligned}
p &= 0 && t < 0 \\
p &= (4t)(2) = 8t\,\mathrm{W} && 0 < t < 25\,\mathrm{ms} \\
p &= (0.2 - 4t)(-2) = 8t - 0.4\,\mathrm{W} && 25 < t < 50\,\mathrm{ms} \\
p &= 0 && 50\,\mathrm{ms} < t
\end{aligned}
$$

$$
\begin{aligned}
w &= 0 && t < 0 \\
w &= \int_0^t (8x)\,dx = 8\frac{x^2}{2}\,\bigg|_0^t = 4t^2\,\mathrm{J} && 0 < t < 25\,\mathrm{ms} \\
w &= \int_{0.025}^t (8x - 0.4)\,dx + 2.5 \times 10^{-3} \\
&= 4x^2 - 0.4x\,\bigg|_{0.025}^t + 2.5 \times 10^{-3} \\
&= 4t^2 - 0.4t + 10 \times 10^{-3}\,\mathrm{J} && 25 < t < 50\,\mathrm{ms} \\
w &= 0 && 10\,\mathrm{ms} < t
\end{aligned}
$$

P 6.8 $0 \le t \le 100\,\mathrm{ms}:$

$$i_L = \frac{10^3}{50}\int_0^t 2e^{-100x}\,dx + 0.1 = 40\frac{e^{-100x}}{-100}\Big|_0^t + 0.1$$

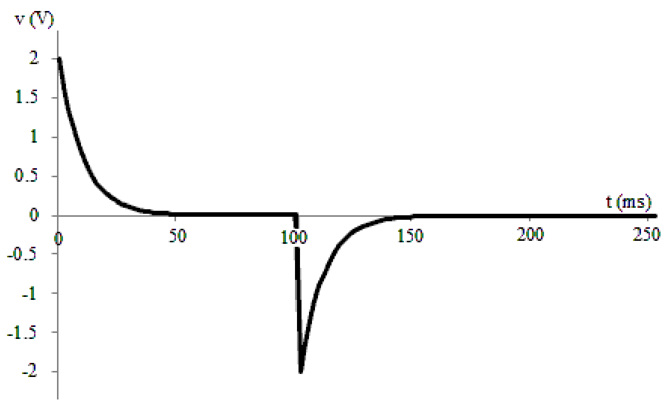
$$= -0.4e^{-100t} + 0.5\,\mathrm{A}$$

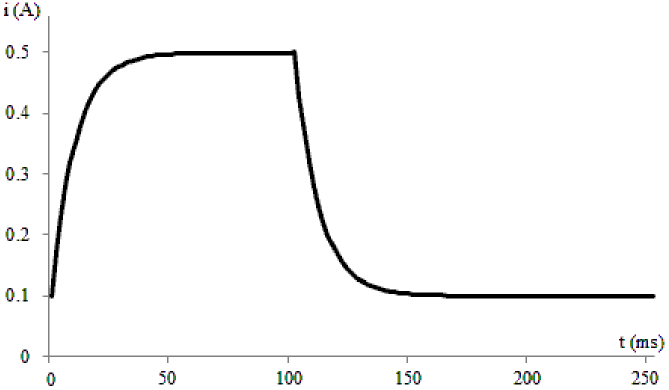
$$i_L(0.1) = -0.4e^{-10} + 0.5 = 0.5\,\mathrm{A}$$

$t \ge 100\,\mathrm{ms}:$

$$i_L = \frac{10^3}{50}\int_{0.1}^t -2e^{-100(x-0.1)}\,dx + 0.5 = -40\frac{e^{-100(x-0.1)}}{-100}\Big|_{0.1}^t + 0.5$$

$$= 0.4e^{-100(t-0.1)} + 0.1\,\mathrm{A}$$




i (A)
0.5
0.4
0.3
0.2
0.1
0
t (ms)
0
50
100
150
200
250

P 6.22 **[a]** $15\|30 = 10\,\text{mH}$

$10 + 10 = 20\,\text{mH}$

$20\|20 = 10\,\text{mH}$

$12\|24 = 8\,\text{mH}$

$10 + 8 = 18\,\text{mH}$

$18\|9 = 6\,\text{mH}$

$L_{\text{ab}} = 6 + 8 = 14\,\text{mH}$

[b] $12 + 18 = 30\,\mu\text{H}$

$30\|20 = 12\,\mu\text{H}$

$12 + 38 = 50\,\mu\text{H}$

$30\|75\|50 = 15\,\mu\text{H}$

$15 + 15 = 30\,\mu\text{H}$

$30\|60 = 20\,\mu\text{H}$

$L_{\text{ab}} = 20 + 25 = 45\,\mu\text{H}$

P 6.41 **[a]** $v_{ab} = L_1 \frac{d(i_1 - i_2)}{dt} + M\frac{di_2}{dt}$

$$0 = L_1 \frac{d(i_2 - i_1)}{dt} - M\frac{di_2}{dt} + M\frac{d(i_1 - i_2)}{dt} + L_2\frac{di_2}{dt}$$

Collecting coefficients of $[di_1/dt]$ and $[di_2/dt]$, the two mesh-current equations become

$$v_{ab} = L_1\frac{di_1}{dt} + (M - L_1)\frac{di_2}{dt}$$

and

$$0 = (M - L_1)\frac{di_1}{dt} + (L_1 + L_2 - 2M)\frac{di_2}{dt}$$

Solving for $[di_1/dt]$ gives

$$\frac{di_1}{dt} = \frac{L_1 + L_2 - 2M}{L_1L_2 - M^2}v_{ab}$$

from which we have

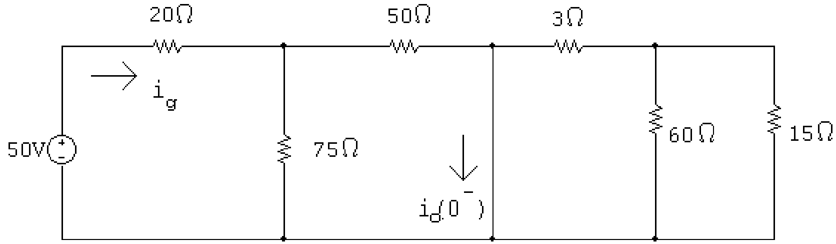
$$v_{ab} = \left(\frac{L_1L_2 - M^2}{L_1 + L_2 - 2M}\right)\left(\frac{di_1}{dt}\right)$$

$$\therefore \quad L_{ab} = \frac{L_1L_2 - M^2}{L_1 + L_2 - 2M}$$

[b] If the magnetic polarity of coil 2 is reversed, the sign of M reverses, therefore

$$L_{ab} = \frac{L_1L_2 - M^2}{L_1 + L_2 + 2M}$$

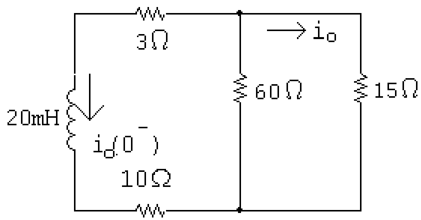
P 7.2 [a] For $t < 0$



$$i_g = \frac{50}{20 + (75\|50)} = \frac{50}{50} = 1\,\text{A}$$

$$i_o(0^-) = \frac{50}{75+50}(1) = 0.4\,\text{A} = i_o(0^+)$$

For $t > 0$



$$i_o(t) = i_o(0^+)e^{-t/\tau}\,\text{A}, \qquad t \geq 0$$

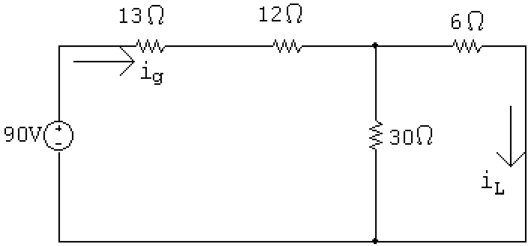
$$\tau = \frac{L}{R} = \frac{0.02}{3 + 60\|15} = 1.33\,\text{ms}; \qquad \frac{1}{\tau} = 750$$

$$i_o(t) = 0.4e^{-750t}\,\text{A}, \qquad t \geq 0$$

[b] $$v_\text{L} = L\frac{di_o}{dt} = 0.02(-750)(0.4e^{-750t}) = -6e^{-750t}\,\text{V}$$

$$v_o = \frac{60\|15}{3 + 60\|15}v_\text{L} = \frac{12}{15}(-6e^{-750t}) = -4.8e^{-750t}\,\text{V} \qquad t \geq 0^+$$

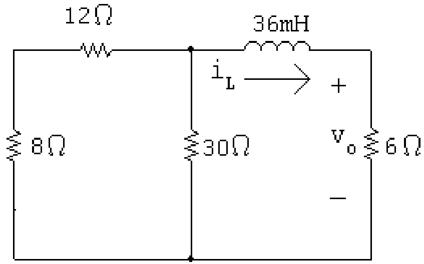
P 7.4 $t < 0$:



$$i_g = \frac{90}{13 + 12 + 6\|30} = 3\,\text{A}$$

$$i_L(0^-) = \frac{30}{36}(3) = 2.5\,\text{A}$$

$t > 0$:



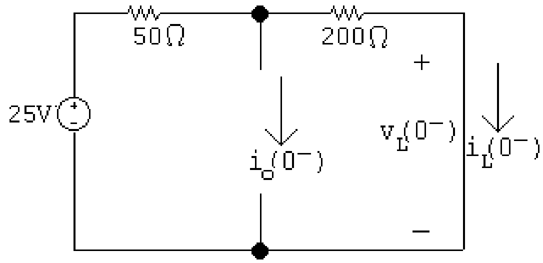
$$R_e = 6 + 30\|(8 + 12) = 6 + 12 = 18\,\Omega$$

$$\tau = \frac{L}{R_e} = \frac{36 \times 10^{-3}}{18} = 2\,\text{ms}; \qquad \frac{1}{\tau} = 500$$

$$\therefore \quad i_L = 2.5e^{-500t}\,\text{A}$$

$$v_o = 6i_o = 15e^{-500t}\,\text{V}, \quad t \geq 0^+$$

P 7.9 [a] For $t = 0^-$ the circuit is:

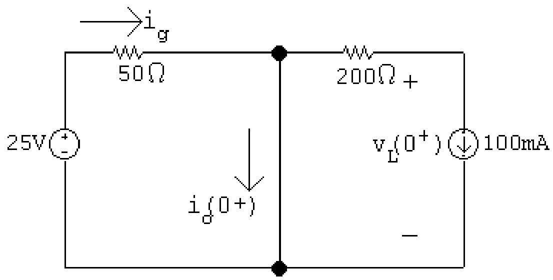


$$i_o(0^-) = 0 \quad \text{since the switch is open}$$

$$i_L(0^-) = \frac{25}{250} = 0.1 = 100\,\text{mA}$$

$$v_L(0^-) = 0 \quad \text{since the inductor behaves like a short circuit}$$

[b] For $t = 0^+$ the circuit is:



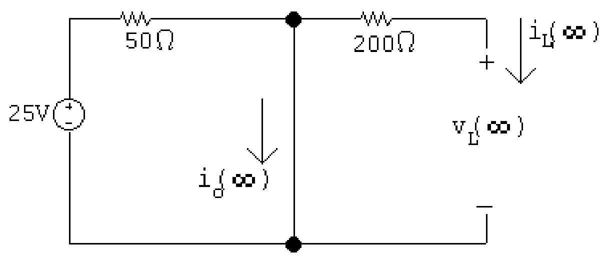
$$i_L(0^+) = i_L(0^-) = 100\,\text{mA}$$

$$i_g = \frac{25}{50} = 0.5 = 500\,\text{mA}$$

$$i_o(0^+) = i_g - i_L(0^+) = 500 - 100 = 400\,\text{mA}$$

$$200 i_L(0^+) + v_L(0^+) = 0 \qquad \therefore \qquad v_L(0^+) = -200 i_L(0^+) = -20\,\text{V}$$

[c] As $t \to \infty$ the circuit is:



$$i_L(\infty) = 0; \qquad v_L(\infty) = 0$$

$$i_o(\infty) = \frac{25}{50} = 500\,\text{mA}$$

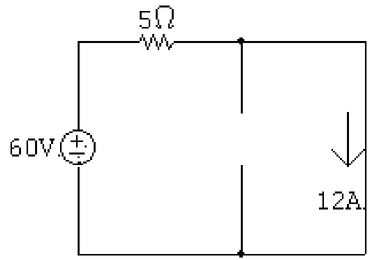
[d] $\tau = \frac{L}{R} = \frac{0.05}{200} = 0.25\,\text{ms}$

$$i_\text{L}(t) = 0 + (0.1 - 0)e^{-4000t} = 0.1e^{-4000t}\,\text{A}$$

[e] $i_o(t) = i_\text{g} - i_\text{L} = 0.5 - 0.1e^{-4000t}\,\text{A}$

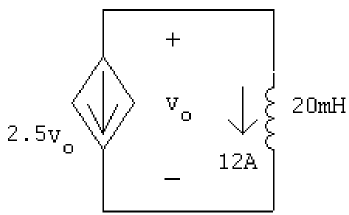
[f] $v_\text{L}(t) = L\frac{di_\text{L}}{dt} = 0.05(-4000)(0.1)e^{-4000t} = -20e^{-4000t}\,\text{V}$

P 7.14 $t < 0$

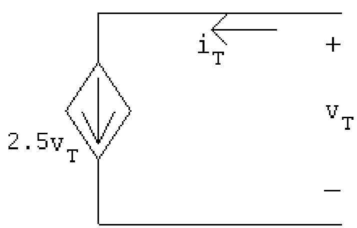


$$i_L(0^-) = i_L(0^+) = 12\,\text{A}$$

$t > 0$

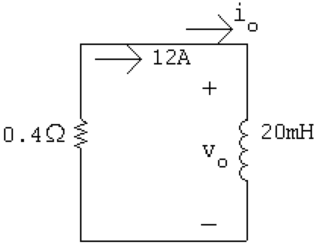


Find Thévenin resistance seen by inductor:



$$i_T = 2.5v_T; \qquad \frac{v_T}{i_T} = R_{\text{Th}} = \frac{1}{2.5} = 0.4\,\Omega$$

$$\tau = \frac{L}{R} = \frac{20 \times 10^{-3}}{0.4} = 50\,\text{ms}; \qquad 1/\tau = 20$$

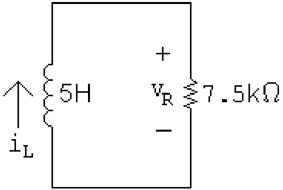


$$i_o = 12e^{-20t}\,\text{A}, \qquad t \geq 0$$

$$v_o = L\frac{di_o}{dt} = (20 \times 10^{-3})(-240e^{-20t}) = -4.8e^{-20t}\,\text{V}, \quad t \geq 0^+$$

P 7.17 **[a]** $t > 0$:

$$L_{\text{eq}} = 1.25 + \frac{60}{16} = 5\,\text{H}$$



$$i_L(t) = i_L(0)e^{-t/\tau}\,\text{mA}; \qquad i_L(0) = 2\,\text{A}; \qquad \frac{1}{\tau} = \frac{R}{L} = \frac{7500}{5} = 1500$$

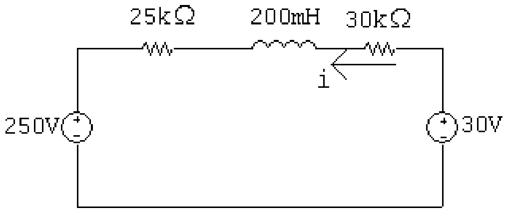
$$i_L(t) = 2e^{-1500t}\,\text{A}, \qquad t \geq 0$$

$$v_R(t) = Ri_L(t) = (7500)(2e^{-1500t}) = 15{,}000e^{-1500t}\,\text{V}, \qquad t \geq 0^+$$

$$v_o = -3.75\frac{di_L}{dt} = 11{,}250e^{-1500t}\,\text{V}, \qquad t \geq 0^+$$

[b] $i_o = \dfrac{-1}{6}\displaystyle\int_0^t 11{,}250e^{-1500x}\,dx + 0 = 1.25e^{-1500t} - 1.25\,\text{A}$

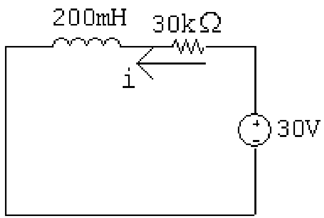
P 7.35 [a] For $t < 0$, calculate the Thévenin equivalent for the circuit to the left and right of the 200 mH inductor. We get



$$i(0^-) = \frac{30 - 250}{25\,\text{k} + 30\,\text{k}} = -4\,\text{mA}$$

$$i(0^-) = i(0^+) = -4\,\text{mA}$$

[b] For $t > 0$, the circuit reduces to



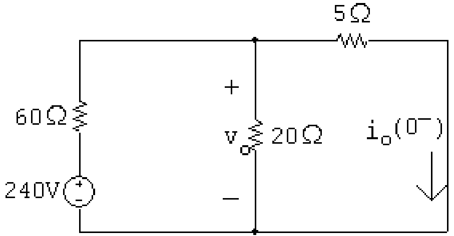
Therefore $\quad i(\infty) = 30/30{,}000 = 1\,\text{mA}$

[c] $\tau = \dfrac{L}{R} = \dfrac{200 \times 10^{-3}}{30{,}000} = 6.67\,\mu\text{s}$

[d] $i(t) = i(\infty) + [i(0^+) - i(\infty)]e^{-t/\tau}$

$$= 0.001 + [-0.004 - 0.001]e^{-150{,}000t} = 1 - 5e^{-150{,}000t}\,\text{mA}, \qquad t \geq 0$$

P 7.37 **[a]** $t < 0$



KVL equation at the top node:

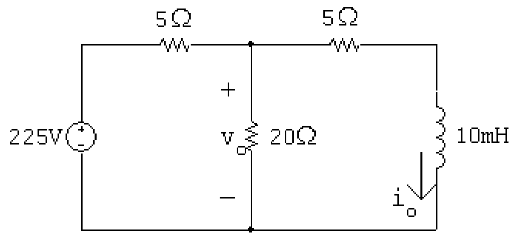
$$\frac{v_o - 240}{60} + \frac{v_o}{20} + \frac{v_o}{5} = 0$$

Multiply by 60 and solve:

$$240 = (3 + 1 + 12)v_o; \qquad v_o = 15\,\text{V}$$

$$\therefore \quad i_o(0^-) = \frac{v_o}{5} = 15/5 = 3\,\text{A}$$

$t > 0$



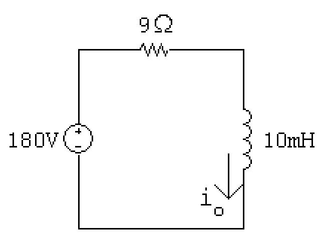
Use voltage division to find the Thévenin voltage:

$$V_{\text{Th}} = v_o = \frac{20}{20+5}(225) = 180\,\text{V}$$

Remove the voltage source and make series and parallel combinations of resistors to find the equivalent resistance:

$$R_{\text{Th}} = 5 + 20\|5 = 5 + 4 = 9\,\Omega$$

The simplified circuit is:



$$\tau = \frac{L}{R} = \frac{10 \times 10^{-3}}{9} = 1.11\,\text{ms}; \qquad \frac{1}{\tau} = 900$$

$$i_o(\infty) = \frac{190}{9} = 20\,\text{A}$$

$$\therefore \quad i_o = i_o(\infty) + [i_o(0^+) - i_o(\infty)]e^{-t/\tau}$$

$$= 20 + (3 - 20)e^{-900t} = 20 - 17e^{-900t}\,\text{A}, \qquad t \geq 0$$

[b]

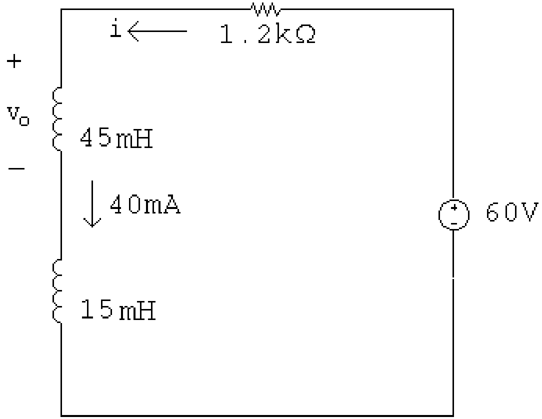
$$v_o = 5i_o + L\frac{di_o}{dt}$$

$$= 5(20 - 17e^{-900t}) + 0.01(-900)(17e^{-900t})$$

$$= 100 - 85e^{-900t} + 153e^{-900t}$$

$$v_o = 100 + 68e^{-900t}\,\text{V}, \qquad t \geq 0^+$$

P 7.46 For $t < 0$, $\quad i_{45\text{mH}}(0) = 80\,\text{V}/2000\,\Omega = 40\,\text{mA}$
For $t > 0$, after making a Thévenin equivalent of the circuit to the right of the inductors we have



$$i = \frac{V_s}{R} + \left(I_o - \frac{V_s}{R}\right) e^{-t/\tau}$$

$$\frac{1}{\tau} = \frac{R}{L} = \frac{1200}{60 \times 10^{-3}} = 20{,}000$$

$$I_o = 40\,\text{mA}; \qquad I_f = \frac{V_s}{R} = \frac{60}{1200} = 50\,\text{mA}$$

$$i = 0.05 + (0.04 - 0.05)e^{-20{,}000t} = 50 - 10e^{-20{,}000t}\,\text{mA}, \qquad t \geq 0$$

$$v_o = 0.045\frac{di}{dt} = 0.045(-0.01)(-20{,}000e^{-20{,}000t}) = 9e^{-20{,}000t}\,\text{V}, \qquad t \geq 0^+$$