P 8.1 **[a]** $i_{\rm R}(0) = \dfrac{25}{125} = 200\,\text{mA}$

$$i_{\rm L}(0) = -300\,\text{mA}$$

$$i_{\rm C}(0) = -i_{\rm L}(0) - i_{\rm R}(0) = 300 - 200 = 100\,\text{mA}$$

[b] $\alpha = \dfrac{1}{2RC} = \dfrac{1}{2(125)(5\times 10^{-6})} = 800$

$$\omega_o = \sqrt{\frac{1}{LC}} = \sqrt{\frac{1}{(200\times 10^{-3})(8\times 10^{-6})}} = 1000$$

$\alpha^2 < \omega_0^2$ The response is underdamped

$$\omega_d = \sqrt{\omega_0^2 - \alpha^2} = \sqrt{1000^2 - 800^2} = 600$$

$$v = B_1 e^{-800t}\cos 600t + B_2 e^{-800t}\sin 600t$$

$$v(0) = B_1 = 25$$

$$\frac{dv}{dt}(0) = -\alpha B_1 + \omega_d B_2 = \frac{1}{C} i_{\rm C}(0)$$

So, $\quad -800(25) + 600B_2 = \dfrac{1}{5\times 10^{-6}}(0.1) = 20{,}000$

$\therefore \quad B_2 = \dfrac{20{,}000 + 800(25)}{600} = 66.67$

$$v = 25e^{-800t}\cos 600t + 66.67e^{-800t}\sin 600t\ \text{V}, \qquad t \geq 0$$

[c] $i_C = C\frac{dv}{dt}$

$$= 5 \times 10^{-6}[20{,}000e^{-800t}\cos 600t - 68{,}333.33e^{-800t}\sin 600t]$$

$$= 100e^{-800t}\cos 600t - 341.67e^{-800t}\sin 600t \,\text{mA}$$

$$i_R = \frac{v}{R} = 200e^{-800t}\cos 600t + 533.36e^{-800t}\sin 600t \,\text{mA}$$

$$i_L = -i_C - i_R = -300e^{-800t}\cos 600t - 191.7e^{-800t}\sin 600t \,\text{mA}, \quad t \geq 0$$

P 8.2 $\frac{1}{2RC} = \frac{1}{2(100)(5 \times 10^{-6})} = 1000$

$$\omega_0 = \sqrt{\frac{1}{LC}} = \sqrt{\frac{1}{(200 \times 10^{-3})(5 \times 10^{-6})}} = 1000$$

$\alpha^2 = \omega_0^2$ So the response is critically damped

$$v(t) = D_1te^{-1000t} + D_2e^{-1000t}$$

$$v(0^+) = 25\,\text{V} = D_2$$

$$\frac{dv(0^+)}{dt} = D_1 - \alpha D_2 = \frac{1}{C}\left(-I_0 - \frac{V_0}{R}\right)$$

So, $D_1 - 1000(25) = \frac{1}{5 \times 10^{-6}}\left(0.3 - \frac{25}{100}\right)$

$\therefore$ $D_1 = 35{,}000$

$$v(t) = 35{,}000e^{-1000t} + 25e^{-1000t}\,\text{V}, \qquad t \geq 0$$

P 8.3 $\alpha = \frac{1}{2RC} = \frac{1}{2(80)(5 \times 10^{-6})} = 1250$

$$\omega_0 = \sqrt{\frac{1}{LC}} = \sqrt{\frac{1}{(200 \times 10^{-3})(5 \times 10^{-6})}} = 1000$$

$\alpha^2 > \omega_0^2$ So the response is overdamped

$$s_{1,2} = -1250 \pm \sqrt{1250^2 - 1000^2} = -500, -2000$$

$$v(t) = A_1e^{-500t} + A_2e^{-2000t}$$

$$v(0) = A_1 + A_2 = 25$$

$$\frac{dv}{dt}(0) = -500A_1 - 2000A_2 = \frac{1}{C}\left(-I_0 - \frac{V_0}{R}\right) = \frac{1}{5 \times 10^{-6}}\left(0.3 - \frac{25}{80}\right) = -2500$$

Solving, $\qquad A_1 = 31.67, \qquad A_2 = -6.67$

$$v(t) = 31.67e^{-500t} - 6.67e^{-2000t}\,\text{V}, \qquad t \geq 0$$

P 8.5 **[a]** $-\alpha + \sqrt{\alpha^2 - \omega_o^2} = -1000$

$$-\alpha - \sqrt{\alpha^2 - \omega_o^2} = -4000$$

Adding the above equations, $\quad -2\alpha = -5000$

$$\alpha = 2500\,\text{rad/s}$$

$$-2500 \pm \sqrt{2500^2 - \omega_0^2} = -1000 \qquad \text{so} \qquad \sqrt{2500^2 - \omega_0^2} = 1500$$

$$\therefore \qquad -\omega_0^2 = 1500^2 - 2500^2 \qquad \text{thus} \qquad \omega_0 = 2000$$

$$\omega_0^2 = \frac{1}{LC} = \frac{1}{(0.01)C} = 2000^2 \qquad \text{so} \qquad C = \frac{1}{(0.01)2000^2} = 25\,\mu\text{F}$$

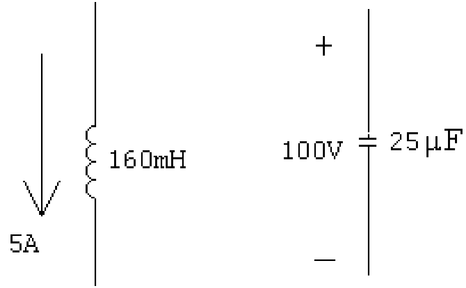
$$\alpha = \frac{1}{2RC} = \frac{1}{2R(25 \times 10^{-6})} = 2500 \qquad \text{so} \qquad R = \frac{1}{2(25 \times 10^{-6})(2500)} = 8\,\Omega$$

[b] $i_{\mathrm{R}} = \dfrac{v(t)}{R} = 5e^{-1000t} - 11.25e^{-4000t}\,\mathrm{A}, \qquad t \geq 0^{+}$

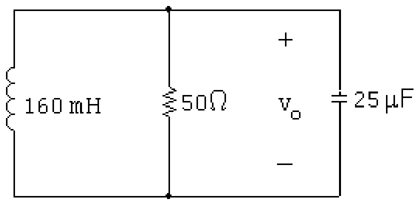
$$i_{\mathrm{C}} = C\frac{dv(t)}{dt} = 9e^{-4000t} - e^{-1000t}\,\mathrm{A}, \qquad t \geq 0^{+}$$

$$i_{\mathrm{L}} = -(i_{\mathrm{R}} + i_{\mathrm{C}}) = 2.25e^{-4000t} - 4e^{-1000t}\,\mathrm{A}, \qquad t \geq 0$$

P 8.11 $t < 0: \qquad V_o = 100\,\text{V}, \qquad I_o = 5\,\text{A}$



$t > 0$:



$$\alpha = \frac{1}{2RC} = \frac{1}{2(50)(25 \times 10^{-6})} = 400 \text{ rad/s}$$

$$\omega_o = \sqrt{\frac{1}{LC}} = \sqrt{\frac{1}{(160 \times 10^{-3})(25 \times 10^{-6})}} = 500$$

$\alpha^2 < \omega_0^2 \qquad$ Response is underdamped

$$\omega_d = \sqrt{500^2 - 400^2} = 300$$

$$\therefore \quad v_o = B_1 e^{-400t}\cos 300t + B_2 e^{-400t}\sin 300t$$

$$v_o(0) = B_1 = 100$$

$$\frac{dv_o}{dt}(0) = -\alpha B_1 + \omega_d B_2 = \frac{1}{C}\left(-I_0 - \frac{V_0}{R}\right)$$

$$\therefore \qquad -(400)(100) + 300B_2 = \frac{1}{25\times 10^{-6}}\left(-5 - \frac{100}{50}\right) \qquad \text{so} \qquad B_2 = -800$$

$$\therefore \quad v_o = 100e^{-400t}\cos 300t - 800e^{-400t}\sin 300t\,\mathrm{V}, \qquad t \geq 0$$

P 8.27 $\omega_o = \sqrt{\frac{1}{LC}} = \sqrt{\frac{1}{(25\times 10^{-3})(62.5\times 10^{-6})}} = 800 \text{ rad/s}$

$$\alpha = \frac{1}{2RC} = \frac{1}{2(12.5)(62.5\times 10^{-6})} = 640 \text{ rad/s} \qquad \therefore \quad \text{underdamped}$$

$$\omega_d = \sqrt{800^2 - 640^2} = 480$$

$$I_f = 2\,\text{A}$$

$$i_L = 2 + B_1' e^{-640t}\cos 480t + B_2' e^{-640t}\sin 480t$$

$$i_L(0) = 2 + B_1' = 1 \qquad \text{so} \qquad B_1' = -1$$

$$\frac{di_L}{dt}(0) = -\alpha B_1' + \omega_d B_2' = \frac{V_0}{L}$$

$$\therefore \qquad -640(-1) + 480B_2' = \frac{50}{25\times 10^{-3}} \qquad \text{so} \qquad B_2' = 2.83$$

$$i_L(t) = 2 - e^{-640t}\cos 480t + 2.83e^{-640t}\sin 480t\,\text{A}, \quad t \geq 0$$

P 8.31 $\alpha = \dfrac{1}{2RC} = \dfrac{1}{2(20)(31.25 \times 10^{-6})} = 800 \text{ rad/s}$

$$\omega_o = \sqrt{\frac{1}{LC}} = \sqrt{\frac{1}{(50 \times 10^{-3})(31.25 \times 10^{-6})}} = 800 \text{ rad/s}$$

$\alpha^2 = \omega_0^2$ Critically damped

$$V_0 = v_C(0) = 60\,\text{V}; \qquad I_0 = i_o(0) = 0; \qquad I_f = i_o(\infty) = \frac{60}{20} = 3\,\text{A}$$

$$i_o = 3 + D_1' te^{-800t} + D_2' e^{-800t}$$

$$i_o(0) = 3 + D_2' = 0 \qquad \text{so} \qquad D_2' = -3$$

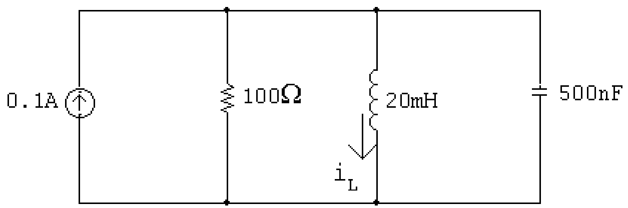
$$\frac{di_o}{dt}(0) = D_1' - \alpha D_2' = \frac{V_0}{L} \qquad \text{so} \qquad D_1' - 800(-3) = \frac{60}{50 \times 10^{-3}}$$

Solving, $D_1' = -1200$

$$i_o(t) = 3 - 1200te^{-800t} - 3e^{-800t}\,\text{A}, \qquad t \geq 0$$

P 8.37 $t < 0: \qquad i_{\rm L}(0^-) = \dfrac{36}{300} = 0.12\,\text{A}; \qquad v_{\rm C}(0^-) = 0\,\text{V}$

The circuit reduces to:



$$i_{\rm L}(\infty) = 0.1\,\text{A}$$

$$\omega_o = \sqrt{\frac{1}{LC}} = \sqrt{\frac{1}{(20\times10^{-3})(500\times10^{-9})}} = 10{,}000 \text{ rad/s}$$

$$\alpha = \frac{1}{2RC} = \frac{1}{(100)(500\times10^{-9})} = 10{,}000 \text{ rad/s}$$

Critically damped:

$$i_L = 0.1 + D_1'te^{-10{,}000t} + D_2'e^{-10{,}000}$$

$$i_L(0) = 0.1 + D_2' = 0.12 \qquad \text{so} \qquad D_2' = 0.02$$

$$\frac{di_L}{dt}(0) = D_1' - \alpha D_2' = \frac{V_0}{L} \qquad \text{so} \qquad D_1' - (10{,}000)(0.02) = 0$$

Solving, $\quad D_1' = 200$

$$i_L(t) = 0.1 + 200te^{-10{,}000t} + 0.02e^{-10{,}000t}\,\text{A}, \qquad t \geq 0$$

P 8.41 **[a]** $-\alpha + \sqrt{\alpha^2 - \omega_0^2} = -4000; \qquad -\alpha - \sqrt{\alpha^2 - \omega_0^2} = -16{,}000$

$$\therefore \quad \alpha = 10{,}000 \text{ rad/s}, \qquad \omega_0^2 = 64 \times 10^6$$

$$\alpha = \frac{R}{2L} = 10{,}000; \qquad R = 20{,}000L$$

$$\omega_o^2 = \frac{1}{LC} = 64 \times 10^6; \qquad L = \frac{10^9}{64 \times 10^6(31.25)} = 0.5\,\mathrm{H}$$

$$R = 10{,}000\ \Omega$$

[**b**] $i(0) = 0$

$$L\frac{di(0)}{dt} = v_c(0); \qquad \frac{1}{2}(31.25)\times 10^{-9}v_c^2(0) = 9\times 10^{-6}$$

$$\therefore \quad v_c^2(0) = 576; \qquad v_c(0) = 24\,\text{V}$$

$$\frac{di(0)}{dt} = \frac{24}{0.5} = 48\,\text{A/s}$$

[**c**] $i(t) = A_1e^{-4000t} + A_2e^{-16{,}000t}$

$$i(0) = A_1 + A_2 = 0$$

$$\frac{di(0)}{dt} = -4000A_1 - 16{,}000A_2 = 48$$

Solving,

$$\therefore \quad A_1 = 4\,\text{mA}; \qquad A_2 = -4\,\text{mA}$$

$$i(t) = 4e^{-4000t} - 4e^{-16{,}000t}\,\text{mA}, \qquad t \geq 0$$

[**d**] $\dfrac{di(t)}{dt} = -16e^{-4000t} + 64e^{-16{,}000t}$

$$\frac{di}{dt} = 0 \text{ when } 64e^{-16{,}000t} = 16e^{-4000t}$$

$$\text{or } e^{12{,}000t} = 4$$

$$\therefore \quad t = \frac{\ln 4}{12{,}000} = 115.52\,\mu\text{s}$$

[**e**] $i_{\text{max}} = 4e^{-0.4621} - 4e^{-1.8484} = 1.89\,\text{mA}$

[**f**] $v_L(t) = 0.5\dfrac{di}{dt} = [-8e^{-1000t} + 32e^{-4000t}]\,\text{V}, \quad t \geq 0^+$

P 8.45 **[a]** $t < 0$:

$$i_o = \frac{100}{50} = 2\,\mathrm{A}; \qquad v_o = -4(100) = -400\,\mathrm{V}$$

$t > 0$:

$$\alpha = \frac{R}{2L} = \frac{500}{2(0.4)} = 625 \text{ rad/s}$$

$$\omega_o = \sqrt{\frac{1}{LC}} = \sqrt{\frac{1}{(0.4)(10 \times 10^{-6})}} = 500 \text{ rad/s}$$

$$\alpha^2 < \omega_o^2 \quad \therefore \quad \text{overdamped}$$

$$s_{1,2} = -625 \pm \sqrt{625^2 - 500^2} = -250, -1000 \text{ rad/s}$$

$$i_o = A_1 e^{-250t} + A_2 e^{-1000t}$$

$$i_o(0) = A_1 + A_2 = 2$$

$$\frac{di_o}{dt}(0) = -250A_1 - 1000A_2 = \frac{1}{L}(-V_0 - RI_0) = -1500$$

$$\text{Solving,} \qquad A_1 = \frac{2}{3}; \qquad A_2 = \frac{4}{3}$$

$$\therefore \qquad i_o(t) = \frac{2}{3}e^{-250t} + \frac{4}{3}e^{-1000t}\,\mathrm{A}, \quad t \geq 0$$

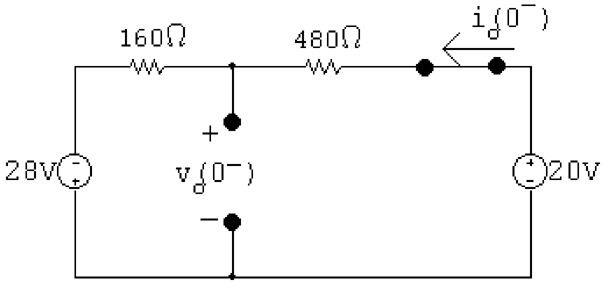
$$\textbf{[b]}\quad v_o(t) = \frac{1}{10\times 10^{-6}}\int_0^t i_o(x)\,dx - 400$$

$$= 10^5\left(\int_0^t \frac{2}{3}e^{-250x}\,dx + \int_0^t \frac{4}{3}e^{-1000x}\,dx\right) - 400$$

$$= 10^5\left(\frac{(2/3)e^{-250x}}{-250}\Bigg|_0^t + \frac{(4/3)e^{-1000x}}{-1000}\Bigg|_0^t\right) - 400$$

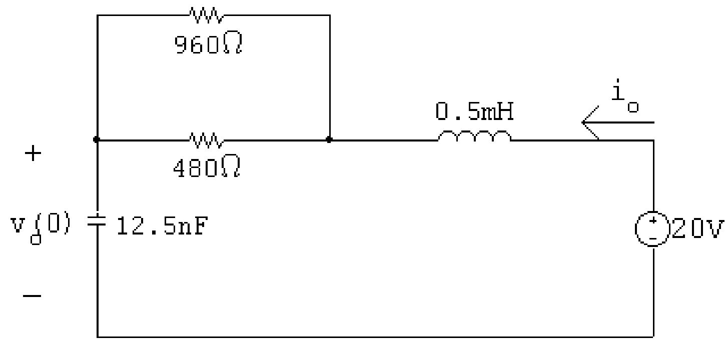
$$= -266.67e^{-250t} - 133.33e^{-1000t}\ \text{V},\quad t \geq 0$$

P 8.52 $t < 0$:



$$i_o(0^-) = \frac{20 + 28}{160 + 480} = 75\,\mathrm{mA}$$

$$v_o(0^-) = 20 - 480(0.075) = -16\,\mathrm{V}$$



As $t \to \infty$, $V_f = 20$ V.

$$R_{\text{eq}} = 960 \| 480 = 320\,\Omega$$

$$\alpha = \frac{R_{\text{eq}}}{2L} = \frac{320}{2(0.5 \times 10^{-3})} = 320{,}000 \text{ rad/s}$$

$$\omega_o = \sqrt{\frac{1}{LC}} = \sqrt{\frac{1}{(0.5 \times 10^{-3})(12.5 \times 10^{-9})}} = 400{,}000\,\text{rad/s}$$

$$\alpha^2 < \omega_0^2: \qquad \text{underdamped}$$

$$\omega_d = \sqrt{400{,}000^2 - 320{,}000^2} = 240{,}000 \text{ rad/s}$$

$$v_o = 20 + B_1' e^{-320{,}000t} \cos 240{,}000t + B_2' e^{-320{,}000t} \sin 240{,}000t$$

$$v_o(0) = 20 + B_1' = -16 \qquad \text{so} \qquad B_1' = -36\,\text{V}$$

$$\frac{dv_o}{dt}(0) = -\alpha B_1' + \omega_d B_2' = \frac{I_0}{C} \qquad \text{so} \qquad -320{,}000(-36) + 240{,}000B_2' = \frac{75 \times 10^{-3}}{12.5 \times 10^{-9}}$$

$$\text{solving,} \qquad B_2' = -23$$

$$\therefore \qquad v_o(t) = 20 - 36e^{-320{,}000t} \cos 240{,}000t - 23e^{-320{,}000t} \sin 240{,}000t\,\text{V} \quad t \geq 0$$