EECS2200 Electric Circuits

Chapter 3

Simple Resistive Circuits

Topics Covered

- Resistance in series and parallel
- Voltage and current dividers
- Measuring voltage, current, and resistance
- Delta-to-Wye (Δ to Y)

Objectives of Chapter 3

- Know how to identify components connected in series and in parallel
- Be able to find the equivalent resistance of series-connected and parallel-connected resistors
- Be able to recognize circuits where voltage division occurs and know how to use voltage division to analyze such circuits
- Be able to recognize circuits where current division occurs and know how to use current division to analyze such circuits

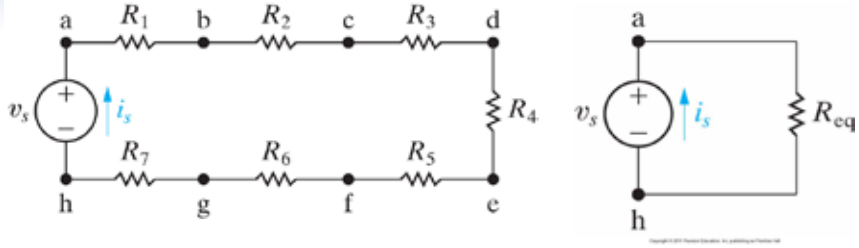
EECS2200 Electric Circuits

Resistors in series & parallel

Resistors in Series

- Connected end-to-end
- Have the same current (because of KCL)
- Equivalent resistance:

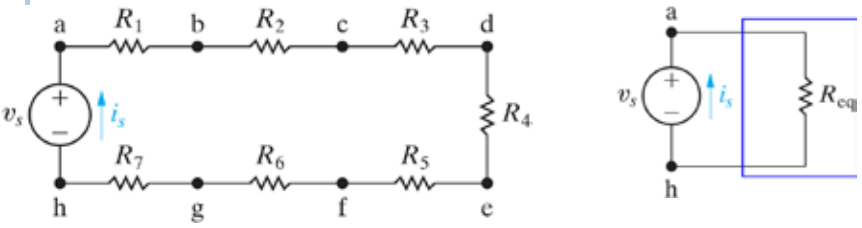
$$R_{eq} = R_1 + R_2 + \cdots + R_7$$



Equivalent Resistance

$$\text{KVL}: -v_s + R_1 i_s + R_2 i_s + \cdots + R_7 i_s = 0 \quad \Rightarrow \quad v_s = (R_1 + R_2 + \cdots + R_7) i_s$$

$$\text{KVL}: -v_s + R_{eq} i_s = 0 \quad \Rightarrow \quad v_s = (R_{eq}) i_s$$

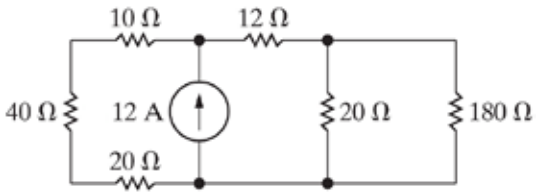


Resistors in series add: $R_{eq} = (R_1 + R_2 + \cdots + R_7)$

Activity 1

(1) Which of the resistors in the circuit below are connected in series?

(2) Find the equivalent resistance of the series-connected resistors.

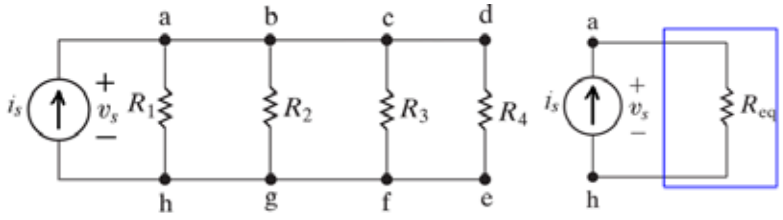


Resistors in Parallel

- Connected at both ends
- Have the same voltage (because of KVL)

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{R_4}$$

(use conductance)



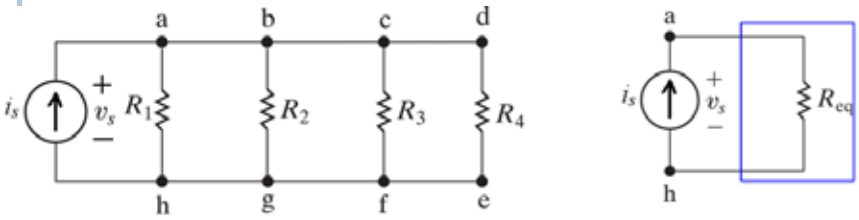
Equivalent Resistance

$\text{KCL}: -i_s + v_s/R_1 + v_s/R_2 + \cdots + v_s/R_4 = 0$

$\Rightarrow \quad i_s = (1/R_1 + 1/R_2 + \cdots + 1/R_4)v_s$

$\text{KCL}: -i_s + v_s/R_{eq} = 0$

$\Rightarrow \quad i_s = v_s/(R_{eq})$

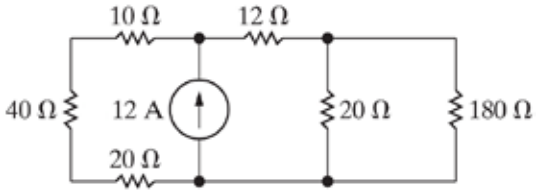


Resistors in parallel: $1/R_{eq} = (1/R_1 + 1/R_2 + \cdots + 1/R_4)$

Activity 2

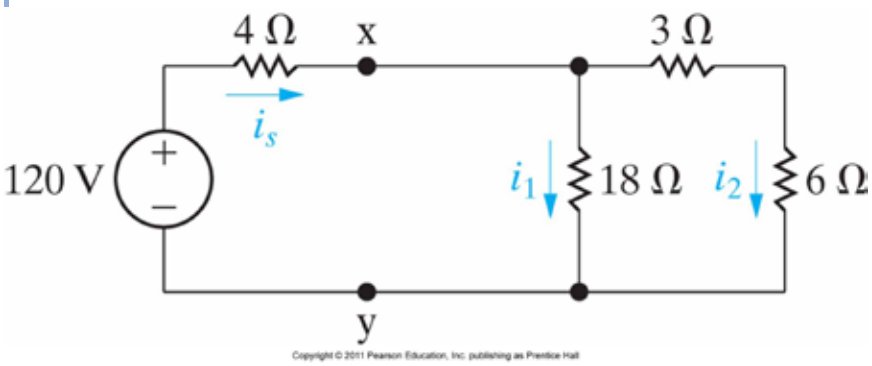
(1) Which of the resistors in the circuit below are connected in parallel?

(2) Find the equivalent resistance of the parallel-connected resistors.



Activity 3

Find i_s, i_1, and i_2



EECS2200 Electric Circuits

Voltage and Current Divider

Voltage Divider

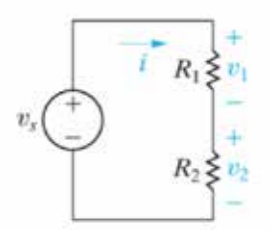
- Get more than one voltage level from a single voltage supply
- What is v_1 and v_2 relative to v_s?

$$v_s = iR_1 + iR_2$$

$$i = \frac{v_s}{R_1 + R_2}$$

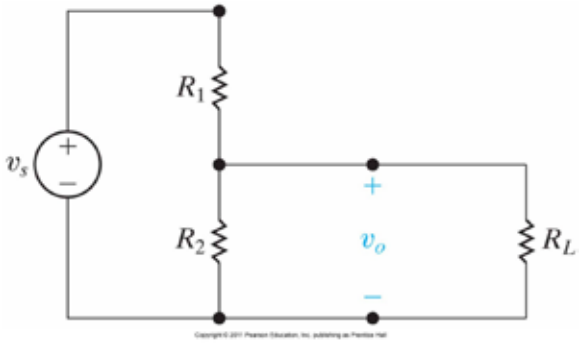
$$v_1 = iR_1 = \frac{v_s}{R_1 + R_2} R_1 = \frac{R_1}{R_1 + R_2} v_s$$

$$v_2 = iR_2 = \frac{v_s}{R_1 + R_2} R_2 = \frac{R_2}{R_1 + R_2} v_s$$



The effect of loading

- A load on any circuit consists of one or more circuit elements that draw power from the circuit.
- R_L is a load in below circuit.

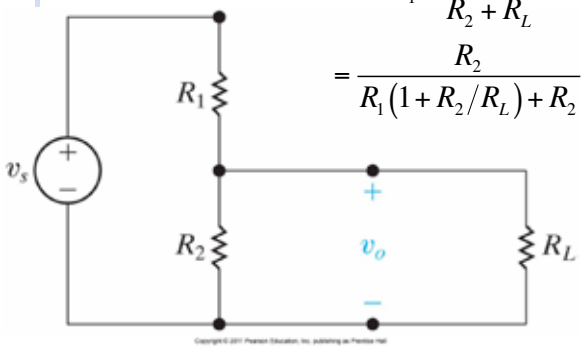


The effect of loading

$$v_o = \frac{R_{eq}}{R_1 + R_{eq}} v_s, R_{eq} = R_2 // R_L = \frac{R_2 R_L}{R_2 + R_L}$$

$$v_o = \frac{\frac{R_2 R_L}{R_2 + R_L}}{R_1 + \frac{R_2 R_L}{R_2 + R_L}} v_s = \frac{R_2 R_L}{R_1 (R_2 + R_L) + R_2 R_L} v_s$$

$$= \frac{R_2}{R_1 (1 + R_2 / R_L) + R_2} v_s$$

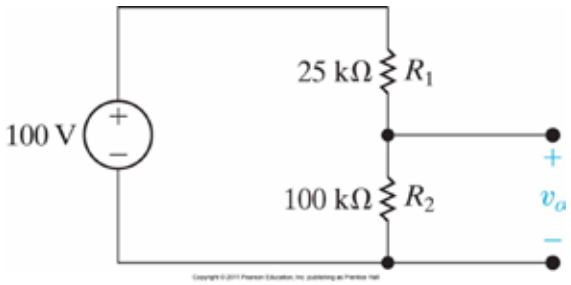


Activity 4

Find v_0

(1) if the load is 10Ω

(2) If the load is 10 M Ω

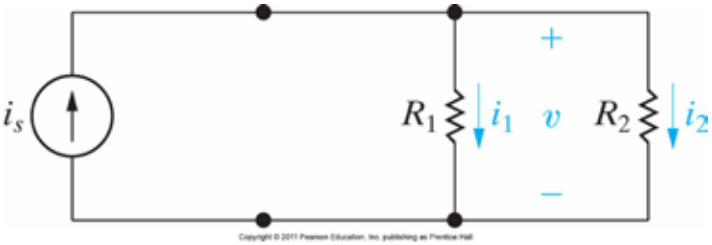


Current Divider

- Divide i_s between R_1 and R_2

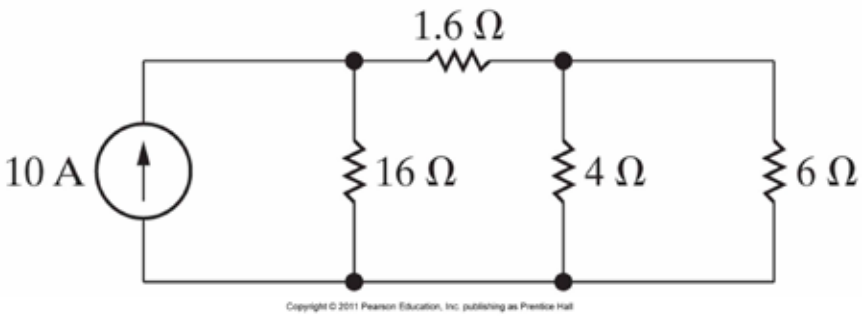
$$v = i_1 R_1 = i_2 R_2 = \frac{R_1 R_2}{R_1 + R_2} i_s$$

$$i_1 = \frac{R_2}{R_1 + R_2} i_s, i_2 = \frac{R_1}{R_1 + R_2} i_s$$



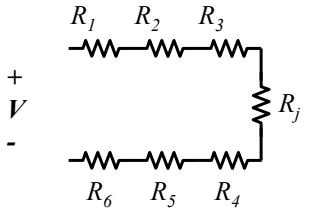
Activity 5

Find the power dissipated in the 6 Ω resistor.



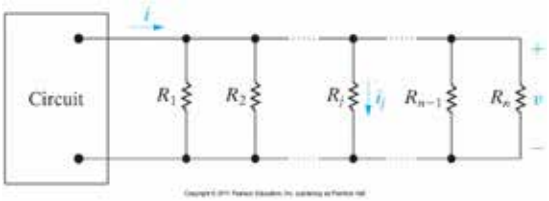
Voltage and Current Dividers

- Voltage divider



$$v_j = iR_j = \frac{R_j}{R_{eq}} v$$

- Current divider



$$i_j = \frac{v}{R_j} = \frac{R_{eq}}{Rj} i$$

EECS2200 Electric Circuits

Measuring Voltage and Current

Measuring Voltage and Current

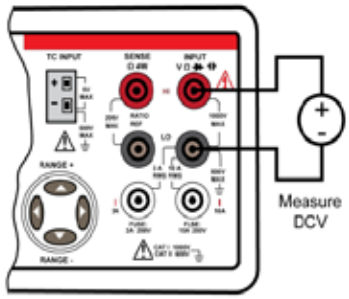
- Voltmeter is an instrument to measure voltage across a load
  - In ***parallel*** with the load
  - Ideal resistance of ∞
- Ammeter is an instrument to measure current in a load
  - In ***series*** with the load
  - Ideal resistance of 0

Digital Multi-meter

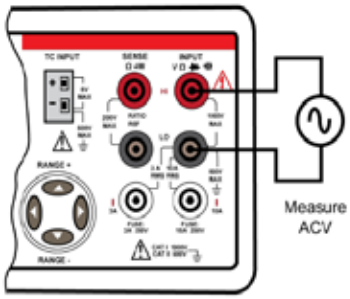
- Digital multi-meter is used to measure voltage, current, resistance, etc.

Measure Voltage

- Two terminals:
  - Red – positive
  - Black – negative



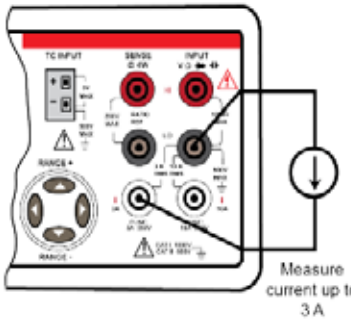
Input resistance:
10 MΩ on all ranges

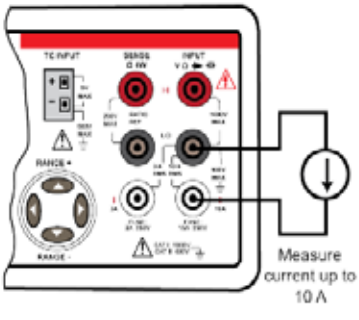


Input impedance:
1 MΩ in parallel with < 100 pF

Measure Current

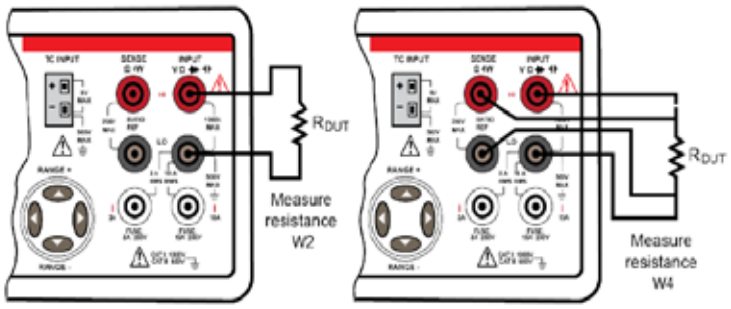
- Two sets of terminals:
  - One for current up to 3A
  - One for current up to 10A





Measure Resistance

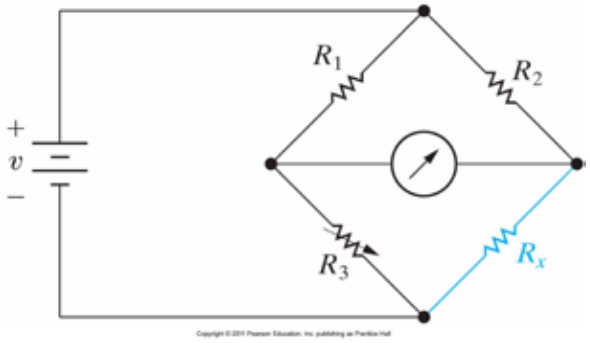
- Two methods
  - Two wires – for resistance > 100Ω
  - Four wires – for resistance < 100Ω
  - More info: http://www.keithley.com/data?asset=57571



Wheatstone Bridge Circuit.

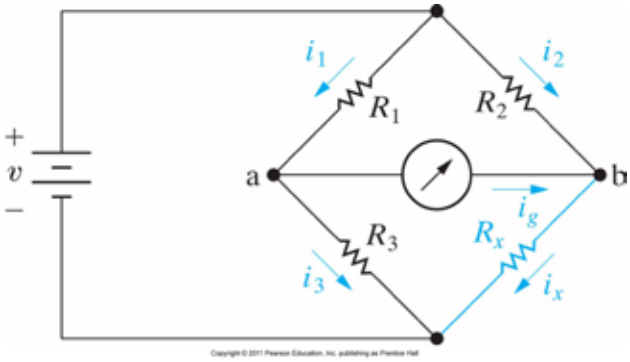
- For measuring resistance
- Adjust variable resistor R_3 until there is no current in the current meter, we have:

$$R_x = \frac{R_2}{R_1} R_3$$



Activity 6

Find Rx in a balanced Wheatstone bridge (i_g = 0). Assume R_1, R_2, and R_3 are known.

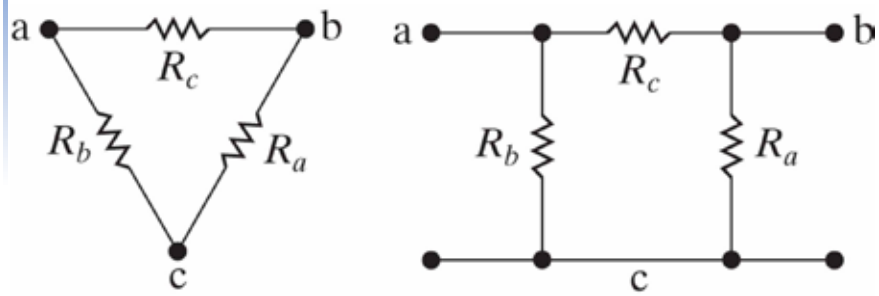


EECS2200 Electric Circuits

Δ to Y (π to T) Conversion

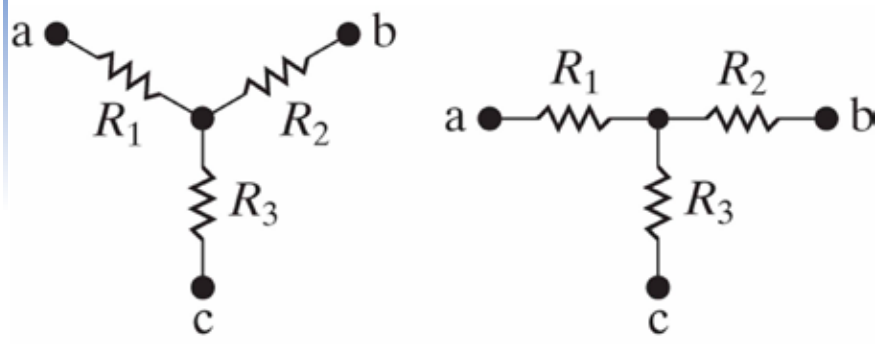
Δ and π Configurations

- A Δ configuration viewed as a π configuration



Y and T Configurations

- A Y structure viewed as a T structure.



Δ-to-Y (π-to-T) transformation

$$R_a = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_1},$$

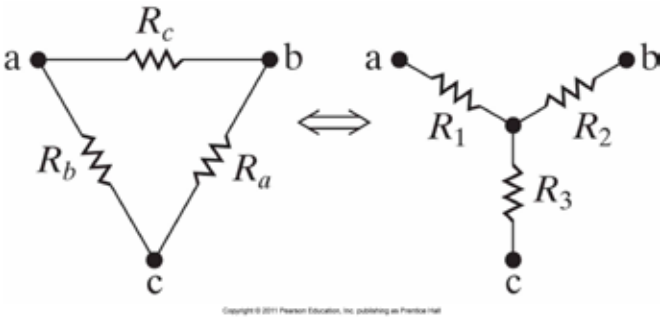
$$R_b = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_2},$$

$$R_c = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_3}$$

$$R_1 = \frac{R_bR_c}{R_a + R_b + R_c},$$

$$R_2 = \frac{R_cR_a}{R_a + R_b + R_c},$$

$$R_3 = \frac{R_aR_b}{R_a + R_b + R_c}$$



Activity 7

Find the current and power supplied by the 40V source .