EECS2200 Electric Circuits

Chapter 5

Capacitance, Inductance, and RC/RL circuits

Objectives (1)

- Understand how an inductor or a capacitor behaves in the presence of a constant current or constant voltage
- Know and being able to use the equations of current, voltage power and energy in an inductor and capacitor
- Be able to combine inductors (capacitors) with an initial condition in series and parallel to form a single equivalent conductor (capacitor).
- Understand the basic concept of mutual inductance and being able to write mesh-current equations for a circuit containing magnetically coupled coils using the dot convention correctly.

Objective (2)

- Be able to determine the natural response of RL and RC circuits.
- Be able to determine the step response of RL and RC circuits.
- Know how to analyze circuits with sequential switching.

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Chapter 5 Part 1

Capacitance and RC circuits

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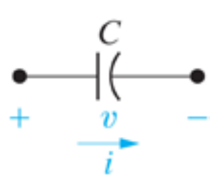
Capacitor

Capacitor

- Parallel plates, separated by an insulator, so no charge flows between the plates. Impose a time-varying voltage drop.
- Capacitor equation:

$$i(t) = C\frac{dv(t)}{dt}$$



- Units: v(t) is volts, i(t) is amps, and C is farads [F]

Activity 1

Look at the capacitor equation:

$$i(t) = C\frac{dv(t)}{dt}$$

Suppose *v(t)* is constant, what is the value of *i(t)*?

A. 0

B. ∞

C. a constant

Activity 2

if the voltage drop across the capacitor is constant, its current is 0, so the capacitor can be replaced by:

A. A short circuit

B. An open circuit

C. a constant

Capacitor

- If the voltage drop across a capacitor is constant, the current is 0, so the capacitor can be replaced by an OPEN CIRCUIT.
- Look at the capacitor equation again: $i(t) = C\dfrac{dv(t)}{dt}$
- Suppose there is a discontinuity in *v(t)* – that is, at some value of *t*, the voltage jumps instantaneously. At this value of *t*, the derivative of the voltage is infinite. Therefore the current is infinite! NOT POSSIBLE.
- **Thus, the voltage drop across a capacitor is continuous for all time.**

Voltage

- The equation for voltage in terms of current:

$$i(t) = C\frac{dv(t)}{dt} \qquad \Rightarrow \qquad i(t)dt = Cdv(t)$$

$$\Rightarrow \qquad \int_{t_o}^{t} i(\tau)d\tau = C\int_{v(t_o)}^{v(t)} dx$$

$$\Rightarrow \qquad v(t) = \frac{1}{C}\int_{t_o}^{t} i(\tau)d\tau + v(t_o)$$

Power and Energy

- Power and energy

$$p(t) = v(t)i(t) = Cv(t)\frac{dv(t)}{dt}$$

$$p(t) = \frac{dw(t)}{dt} = Cv(t)\frac{dv(t)}{dt}$$

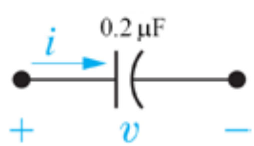
$$\Rightarrow \qquad dw(\tau) = Cv(\tau)dv(\tau)$$

$$\Rightarrow \qquad \int_0^{w(t)} dx = C\int_0^{v(t)} y(\tau)d\tau$$

$$\Rightarrow \qquad w(t) = \tfrac{1}{2}Cv(t)^2$$

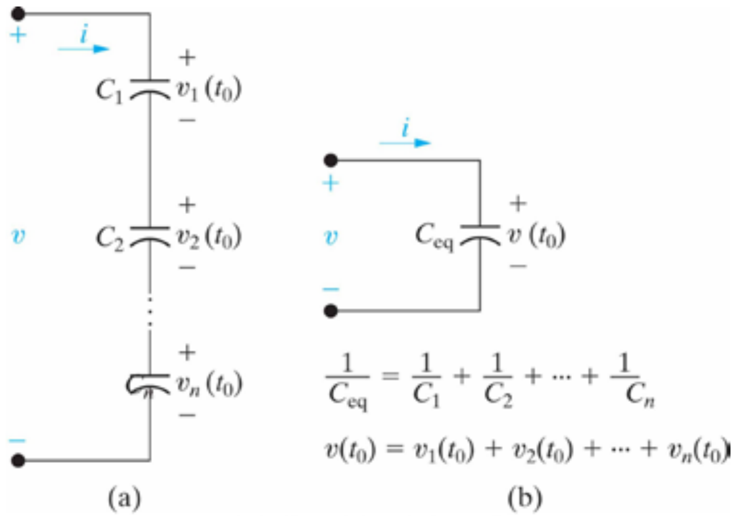
Activity 3

Find the voltage, power, and energy

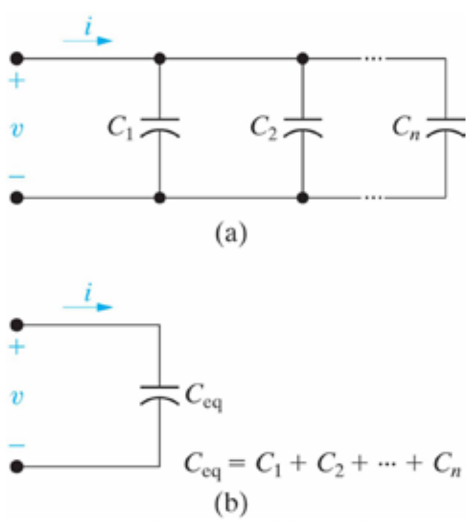


$$i(t) = \begin{cases} 0, & t \leq 0; \\ 5000t \text{ A}, & 0 \leq t \leq 20\ \mu\text{s}; \\ 0.2 - 5000t \text{ A}, & 20 \leq t \leq 40\ \mu\text{s}; \\ 0, & t \geq 40\ \mu\text{s}. \end{cases}$$

Capacitor in series



Capacitor in parallel



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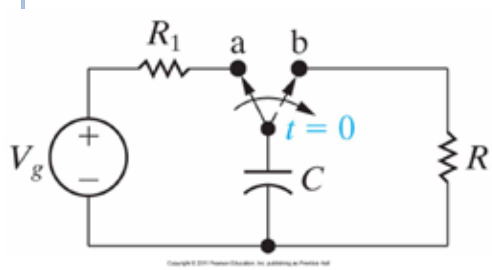
RC Natural Response

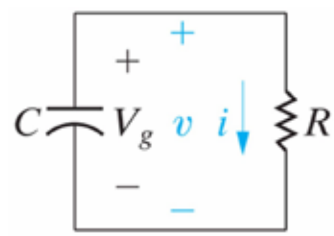
Introduction

- **Natural Response**: The current and voltages that arise when the stored energy in the L of C is released.
- **Step Response**: The current and voltage that arise when energy is being acquired by the L or C when a sudden application of voltage or current is applied to the circuit.

RC Natural Response

- Switch in position A for a long time
- The capacitor charged to V_g

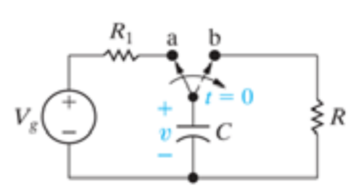




RC Natural Response

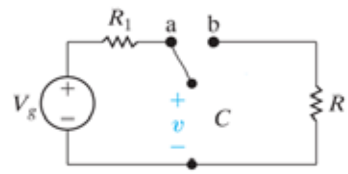
To evaluate the voltage drop across the capacitor for t < 0, put the switch in the "a" position and replace the capacitor with an open circuit. The voltage drop across the open circuit, positive at the top, is

A. V_g
B. $-V_g$
C. $RV_g / (R + R_1)$



RC Natural Response

$t < 0$:



There is an open circuit here, so the current in this circuit is 0, and the voltage drop across R_1 is 0. Thus,

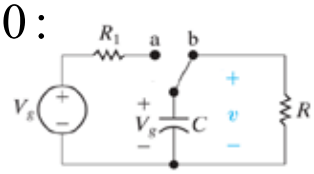
$$v = V_g, \qquad t < 0$$

Remember the continuity requirement for capacitors – the voltage drop across the capacitor must be continuous everywhere, so that the current through the capacitor remains finite. Thus,

$$v(0) = V_g$$

RC Natural Response

$t \geq 0$:



KCL at b: $C\dfrac{dv(t)}{dt} + \dfrac{v(t)}{R} = 0$

Let $\quad v(t) = e^{-at} \quad$ and $\quad \dfrac{dv(t)}{dt} = -ae^{-at}$

Substituting, $\quad C(-ae^{-at}) + e^{-at}/R = 0$

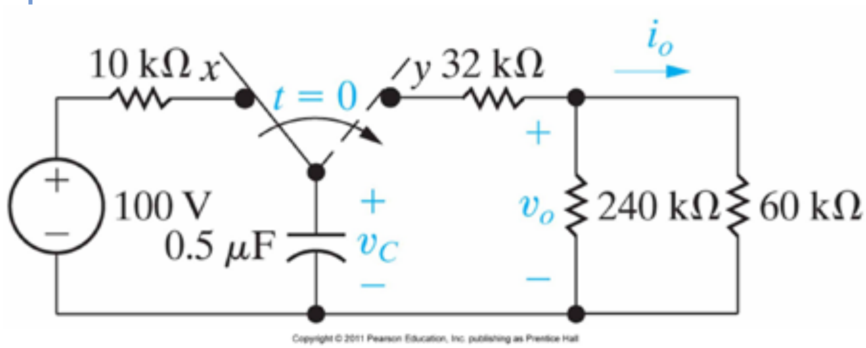
Rearranging, $\quad (1/R - aC)e^{-at} = 0 \quad$ so $\quad 1/R - aC = 0$

Solving, $\quad a = 1/RC$

But $\quad v(0) = V_g \quad$ so $\quad v(t) = V_g e^{-(1/RC)t}, \qquad t \geq 0$

Activity 4

Find $V_c(t)$ for $t>=0$



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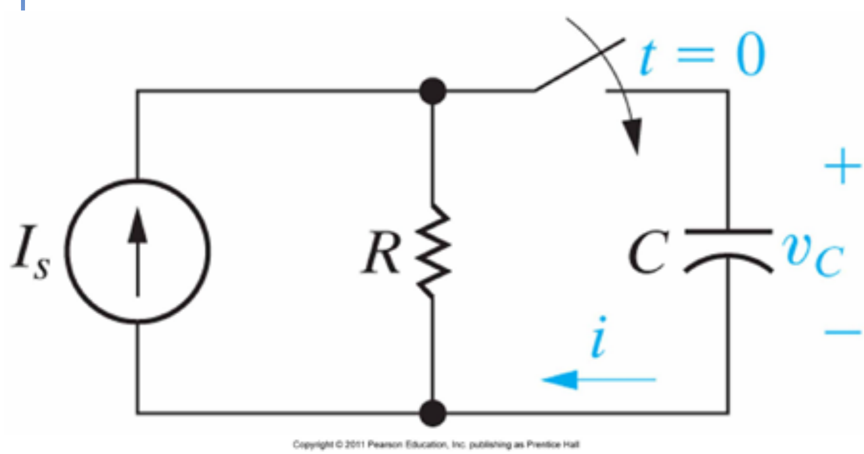
RC Step Response

Step Response of RC Circuits



Step Response of RC Circuit

$$C\frac{dv}{dt}+\frac{v}{R}=I_s$$

$$\frac{dv}{dt}=\frac{-v/R+I_s}{C}=\frac{-v+I_sR}{RC}$$

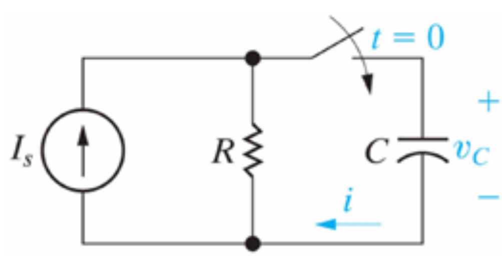
$$\frac{dv}{v-I_sR}=\frac{-1}{RC}dt$$

$$\int_{V_0}^{v(t)}\frac{dv}{v-I_sR}=\int_0^t\frac{-1}{RC}dt$$

$$\ln\frac{v(t)-(I_sR)}{V_0-(I_sR)}=\frac{-1}{RC}t$$

$$\frac{v(t)-(I_sR)}{V_0-(I_sR)}=e^{-t/RC}$$

$$v(t)=I_sR+(V_0-I_sR)e^{-t/RC}$$



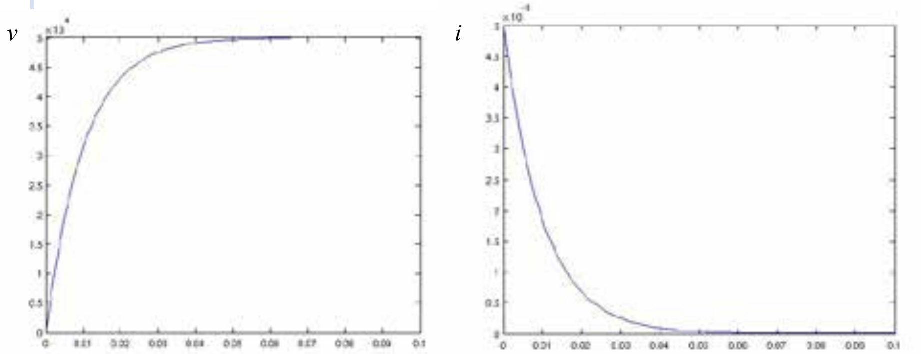
$$i=C\frac{dv}{dt}$$

$$i=C(V_0-I_sR)\left(\frac{-1}{RC}\right)e^{-t/RC}$$

$$i=\left(I_s-\frac{V_0}{R}\right)e^{-t/RC}$$

RC Step Response

- R=10 KΩ, C=1 μF
- I_s=5A, V_0=0



Activity 5

The switch has been in position 1 for a long time. At t=0, the switch moves to position 2. Find $v_0(t)$ for t>=0 and $i_o(t)$ for t>=0^+