

# EECS2200 Electric Circuits

## Chapter 5

### Capacitance, Inductance, and RC/RL circuits

#### Objectives (1)

- Understand how an inductor or a capacitor behaves in the presence of a constant current or constant voltage
- Know and being able to use the equations of current, voltage power and energy in an inductor and capacitor
- Be able to combine inductors (capacitors) with an initial condition in series and parallel to form a single equivalent conductor (capacitor).
- Understand the basic concept of mutual inductance and being able to write mesh-current equations for a circuit containing magnetically coupled coils using the dot convention correctly.

## Objective (2)

- Be able to determine the natural response of RL and RC circuits.
- Be able to determine the step response of RL and RC circuits.

## EECS2200 Electric Circuits

### Chapter 5 Part 1

#### Capacitance and RC circuits

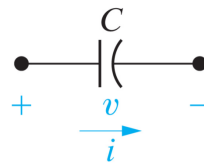
## EECS2200 Electric Circuits

### Capacitor

### Capacitor

- Parallel plates, separated by an insulator, so no charge flows between the plates. Impose a time-varying voltage drop.
- Capacitor equation:

$$i(t) = C \frac{dv(t)}{dt}$$



- Units:  $v(t)$  is volts,  $i(t)$  is amps, and  $C$  is farads [F]

## Activity 1

Look at the capacitor equation:

$$i(t) = C \frac{dv(t)}{dt}$$

Suppose  $v(t)$  is constant, what is the value of  $i(t)$ ?

- A. 0
- B.  $\infty$
- C. a constant

## Activity 2

if the voltage drop across the capacitor is constant, its current is 0, so the capacitor can be replaced by:

- A. A short circuit
- B. An open circuit
- C. a constant

## Capacitor

- If the voltage drop across a capacitor is constant, the current is 0, so the capacitor can be replaced by an OPEN CIRCUIT.
- Look at the capacitor equation again:  $i(t) = C \frac{dv(t)}{dt}$
- Suppose there is a discontinuity in  $v(t)$  – that is, at some value of  $t$ , the voltage jumps instantaneously. At this value of  $t$ , the derivative of the voltage is infinite. Therefore the current is infinite! NOT POSSIBLE.
- **Thus, the voltage drop across a capacitor is continuous for all time.**

## Voltage

- The equation for voltage in terms of current:

$$i(t) = C \frac{dv(t)}{dt} \quad \Rightarrow \quad i(t)dt = Cdv(t)$$

$$\Rightarrow \int_{t_o}^t i(\tau)d\tau = C \int_{v(t_o)}^{v(t)} dx$$

$$\Rightarrow v(t) = \frac{1}{C} \int_{t_o}^t i(\tau)d\tau + v(t_o)$$

## Power and Energy

### ■ Power and energy

$$p(t) = v(t)i(t) = Cv(t)\frac{dv(t)}{dt}$$

$$p(t) = \frac{dw(t)}{dt} = Cv(t)\frac{dv(t)}{dt}$$

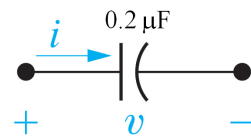
$$\Rightarrow dw(\tau) = Cv(\tau)dv(\tau)$$

$$\Rightarrow \int_0^{w(t)} dx = C \int_0^{v(t)} y(\tau) d\tau$$

$$\Rightarrow w(t) = \frac{1}{2}Cv(t)^2$$

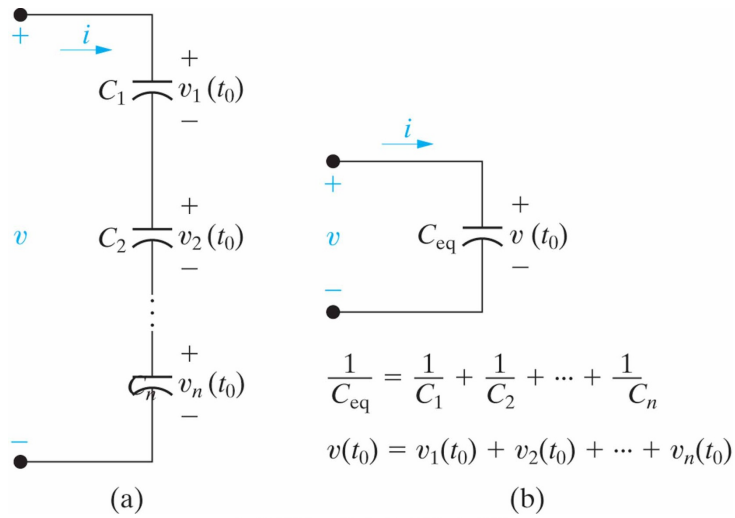
## Activity 3

Find the voltage, power, and energy



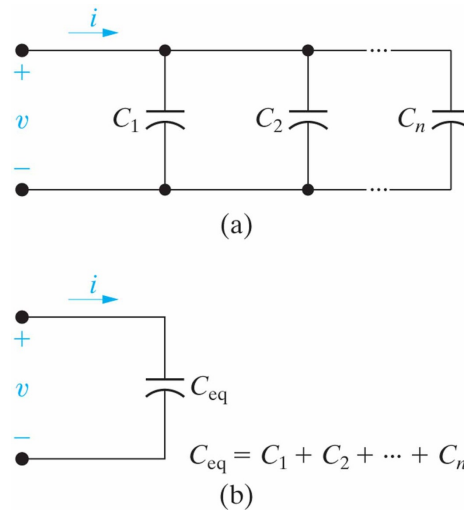
$$i(t) = \begin{cases} 0, & t \leq 0; \\ 5000t \text{ A}, & 0 \leq t \leq 20 \mu\text{s}; \\ 0.2 - 5000t \text{ A}, & 20 \leq t \leq 40 \mu\text{s}; \\ 0, & t \geq 40 \mu\text{s}. \end{cases}$$

## Capacitor in series



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## Capacitor in parallel



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### RC Natural Response

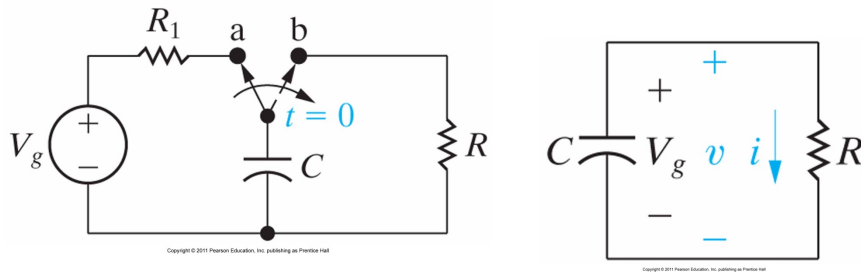
#### Introduction

- **Natural Response:** The current and voltages that arise when the stored energy in the L or C is released.
- **Step Response:** The current and voltage that arise when energy is being acquired by the L or C when a sudden application of voltage or current is applied to the circuit.



## RC Natural Response

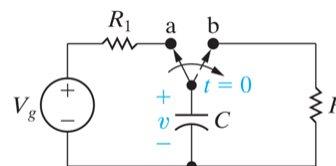
- Switch in position A for a long time
- The capacitor charged to  $V_g$



## RC Natural Response

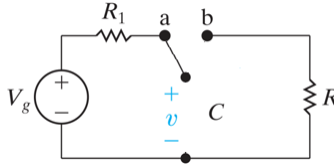
To evaluate the voltage drop across the capacitor for  $t < 0$ , put the switch in the “a” position and replace the capacitor with an open circuit. The voltage drop across the open circuit, positive at the top, is

- A.  $V_g$
- B.  $-V_g$
- C.  $RV_g / (R + R_1)$



## RC Natural Response

$t < 0$ :



There is an open circuit here, so the current in this circuit is 0, and the voltage drop across  $R_1$  is 0. Thus,

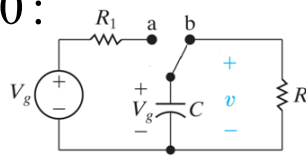
$$v = V_g, \quad t < 0$$

Remember the continuity requirement for capacitors – the voltage drop across the capacitor must be continuous everywhere, so that the current through the capacitor remains finite. Thus,

$$v(0) = V_g$$

## RC Natural Response

$t \geq 0$ :



$$\text{KCL at b: } C \frac{dv(t)}{dt} + \frac{v(t)}{R} = 0$$

$$\text{Let } v(t) = e^{-at} \quad \text{and} \quad \frac{dv(t)}{dt} = -ae^{-at}$$

$$\text{Substituting, } C(-ae^{-at}) + e^{-at}/R = 0$$

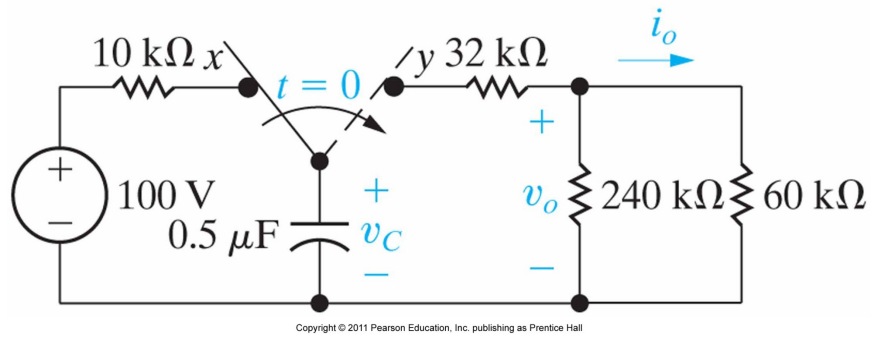
$$\text{Rearranging, } (1/R - aC)e^{-at} = 0 \quad \text{so} \quad 1/R - aC = 0$$

$$\text{Solving, } a = 1/RC$$

$$\text{But } v(0) = V_g \quad \text{so} \quad v(t) = V_g e^{-(1/RC)t}, \quad t \geq 0$$

## Activity 4

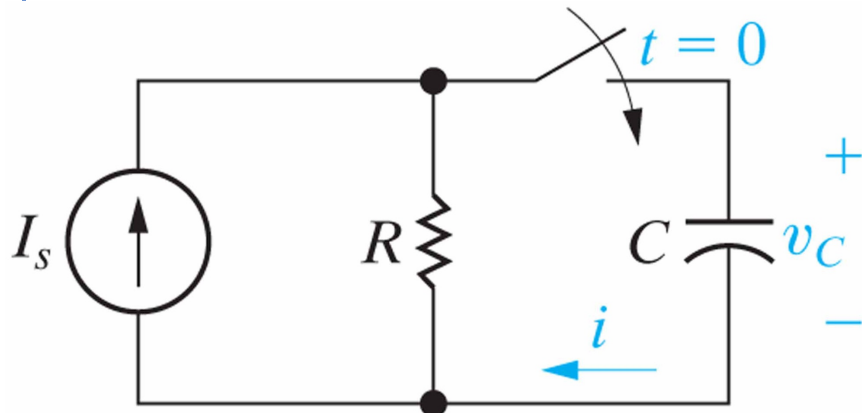
Find  $V_c(t)$  for  $t \geq 0$



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### RC Step Response

## Step Response of RC Circuits



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## Step Response of RC Circuit

$$C \frac{dv}{dt} + \frac{v}{R} = I_s$$

$$\frac{dv}{dt} = \frac{-v/R + I_s}{C} = \frac{-v + I_s R}{RC}$$

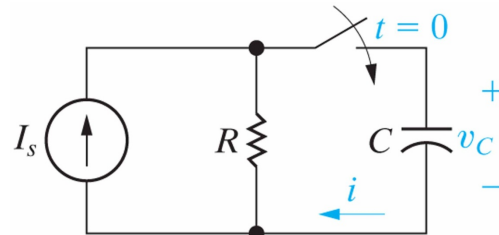
$$\frac{dv}{v - I_s R} = \frac{-1}{RC} dt$$

$$\int_{v_0}^{v(t)} \frac{dv}{v - I_s R} = \int_0^t \frac{-1}{RC} dt$$

$$\ln \frac{v(t) - (I_s R)}{V_0 - (I_s R)} = \frac{-1}{RC} t$$

$$\frac{v(t) - (I_s R)}{V_0 - (I_s R)} = e^{-t/RC}$$

$$v(t) = I_s R + (V_0 - I_s R) e^{-t/RC}$$



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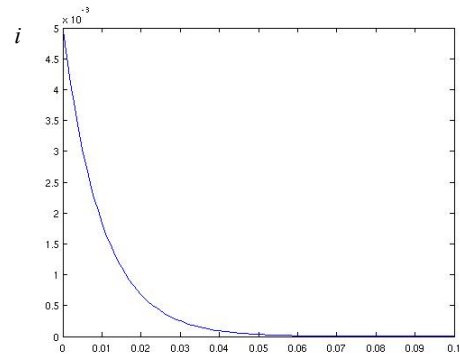
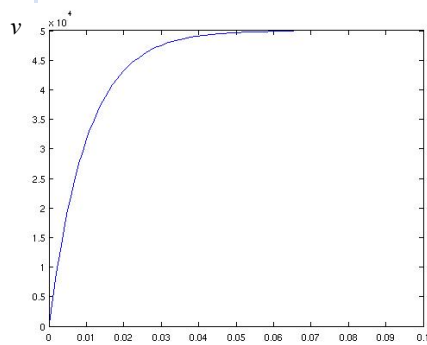
$$i = C \frac{dv}{dt}$$

$$i = C(V_0 - I_s R) \left( \frac{-1}{RC} \right) e^{-t/RC}$$

$$i = \left( I_s - \frac{V_0}{R} \right) e^{-t/RC}$$

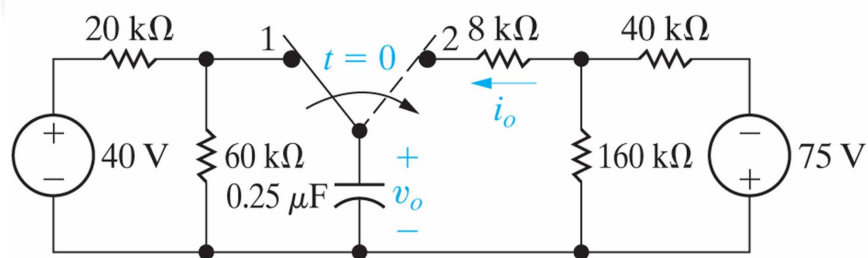
## RC Step Response

- $R=10\text{ K}\Omega$ ,  $C=1\text{ }\mu\text{F}$
- $I_s=5\text{ A}$ ,  $V_0=0$



## Activity 5

The switch has been in position 1 for a long time. At  $t=0$ , the switch moves to position 2. Find  $v_o(t)$  for  $t \geq 0$  and  $i_o(t)$  for  $t \geq 0^+$



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