EECS2200 Electric Circuits

Chapter 6

RLC Circuit
Natural and Step Responses

Objectives

- Determine the response form of the circuit
- Natural response parallel RLC circuits
- Natural response series RLC circuits
- Step response of parallel and series RLC circuits

EECS2200 Electric Circuits

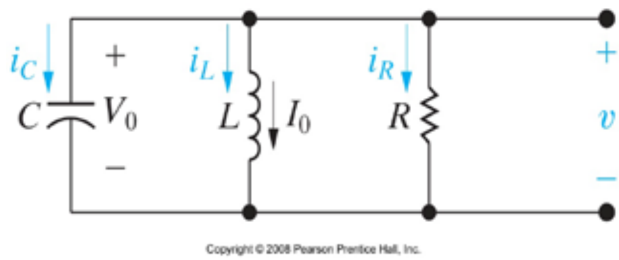
Natural Response of Parallel RLC Circuits

Steps in Solving RLC Circuits

- The first step is to write either KVL or KCL for the circuit.
- Take the derivative to remove any integration
- Solve the resulting differential equation

Natural Response of Parallel RLC Circuits

The problem – given initial energy stored in the inductor and/or capacitor, find *v(t)* for $t \geq 0$.



Activity 1

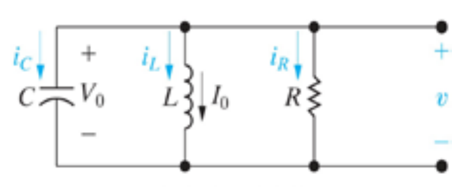
It is convenient to calculate *v(t)* for this circuit because:

A. The voltage must be continuous for all time.

B. The voltage is the same for all three components.

C. Once we have the voltage, it is pretty easy to calculate the branch current.

D. All of the above.



Natural Response of Parallel RLC Circuits

KCL: $$C\frac{dv(t)}{dt}+\frac{1}{L}\int_0^t v(x)\,dx+I_0+\frac{v(t)}{R}=0$$

Differentiate both sides to remove the integral:

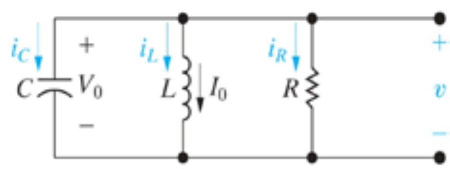
$$C\frac{d^2v(t)}{dt^2}+\frac{1}{L}v(t)+\frac{1}{R}\frac{dv(t)}{dt}=0$$

Divide both sides by C to place in standard form:

$$\frac{d^2v(t)}{dt^2}+\frac{1}{LC}v(t)+\frac{1}{RC}\frac{dv(t)}{dt}=0$$

Natural Response of Parallel RLC Circuits

The problem – given initial energy stored in the inductor and/or capacitor, find *v(t)* for *t* ≥ *0*.



Describing equation: $$\frac{d^2v(t)}{dt^2}+\frac{1}{LC}v(t)+\frac{1}{RC}\frac{dv(t)}{dt}=0$$

This equation is

- ✓Second order
- ✓Homogeneous
- ✓Ordinary differential equation
- ✓With constant coefficients

Natural Response of Parallel RLC Circuits

Describing equation: $$\frac{d^2v(t)}{dt^2}+\frac{1}{LC}v(t)+\frac{1}{RC}\frac{dv(t)}{dt}=0$$

The circuit has two initial conditions that must be satisfied, so the solution for v(t) must have two constants. Use

$v(t)=A_1e^{s_1t}+A_2e^{s_2t}$ V; Substitute:

$$(s_1^2A_1e^{s_1t}+s_2^2A_2e^{s_2t})+\frac{1}{RC}(s_1A_1e^{s_1t}+s_2A_2e^{s_2t})+\frac{1}{LC}(A_1e^{s_1t}+A_2e^{s_2t})=0$$

$$\Rightarrow[s_1^2+(1/RC)s_1+(1/LC)]A_1e^{s_1t}+[s_2^2+(1/RC)s_2+(1/LC)]A_2e^{s_2t}=0$$

For solution, either A_1=0 and A_2=0 or the quadratic part is 0. The quadratic part is called Characteristic equation:

$$s^2+(1/RC)s+(1/LC)=0$$

Natural Response of Parallel RLC Circuits

The two solutions to the characteristic equation s_1 and s_2 can be calculated using the quadratic formula:

$$s^2+\left(\frac{1}{RS}\right)s+\left(\frac{1}{LC}\right)=0;$$

$$s_{1,2}=-\left(\frac{1}{2RC}\right)\pm\sqrt{\left(\frac{1}{2RC}\right)^2-\left(\frac{1}{LC}\right)}=-\alpha\pm\sqrt{\alpha^2-\omega_0^2}$$

where $\alpha=\frac{1}{2RC}$ (the neper frequency in rad/s)

and $\omega_0=\sqrt{\frac{1}{LC}}$ (the resonant radian frequency in rad/s)

Parallel RLC Circuits

$$s_{1,2} = -\frac{1}{2RC} \pm \sqrt{\left(\frac{1}{2RC}\right)^2 - \frac{1}{LC}}$$

$$s_{1,2} = -\alpha \pm \sqrt{\alpha^2 - \omega_0^2}$$

$$\alpha = \frac{1}{2RC} \text{ rad/s}, \quad \omega_0 = \frac{1}{\sqrt{LC}} \text{ rad/s}$$

$$s_{1,2} = -\frac{1}{2RC} \pm \sqrt{\left(\frac{1}{2RC}\right)^2 - \frac{1}{LC}}$$

$$s_{1,2} = -\xi\omega_0 \pm \omega_0\sqrt{\xi^2 - 1}$$

$$\omega_0 = \frac{1}{\sqrt{LC}} \quad , \quad \xi = \frac{1}{2R}\sqrt{\frac{L}{C}}$$

Neper frequency
radian frequency

Resonant

Damping ratio

Solution to Parallel RLC Circuits

$$v(t) = A_1 e^{s_1 t} + A_2 e^{s_2 t}$$

- The solution of the differential equation depends on the values of s_1 and s_2, A_1 and A_2.
- s_1 and s_2 can be found from the characteristic equation, consider three cases

Case 1 - Overdamping

$\alpha > \omega_o$, $\zeta > 1$ → 2 real roots

$$s_{1,2} = -\frac{1}{2RC} \pm \sqrt{\left(\frac{1}{2RC}\right)^2 - \frac{1}{LC}}$$
$$s_{1,2} = -\alpha \pm \sqrt{\alpha^2 - \omega_0^2}$$
$$\alpha = \frac{1}{2RC} \text{ rad/s}, \quad \omega_0 = \frac{1}{\sqrt{LC}} \text{ rad/s}$$

$$s_{1,2} = -\frac{1}{2RC} \pm \sqrt{\left(\frac{1}{2RC}\right)^2 - \frac{1}{LC}}$$
$$s_{1,2} = -\xi\omega_0 \pm \omega_0\sqrt{\xi^2 - 1}$$
$$\omega_0 = \frac{1}{\sqrt{LC}} \quad , \quad \xi = \frac{1}{2R}\sqrt{\frac{L}{C}}$$

$$v(t) = A_1 e^{s_1 t} + A_2 e^{s_2 t}$$

Case 2: Underdamping

$\alpha < \omega_o$, $\zeta < 1$ → a pair of complex roots

$$s_{1,2} = -\frac{1}{2RC} \pm \sqrt{\left(\frac{1}{2RC}\right)^2 - \frac{1}{LC}}$$
$$s_{1,2} = -\alpha \pm \sqrt{\alpha^2 - \omega_0^2}$$
$$\alpha = \frac{1}{2RC} \text{ rad/s}, \quad \omega_0 = \frac{1}{\sqrt{LC}} \text{ rad/s}$$

$$s_{1,2} = -\frac{1}{2RC} \pm \sqrt{\left(\frac{1}{2RC}\right)^2 - \frac{1}{LC}}$$
$$s_{1,2} = -\xi\omega_0 \pm \omega_0\sqrt{\xi^2 - 1}$$
$$\omega_0 = \frac{1}{\sqrt{LC}} \quad , \quad \xi = \frac{1}{2R}\sqrt{\frac{L}{C}}$$

$$s = -\alpha \pm j\omega_d \qquad \omega_d = \sqrt{\omega_0^2 - \alpha}$$
$$v(t) = B_1 e^{-\alpha t}\cos(\omega_d t) + B_2 e^{-\alpha t}\sin(\omega_d t)$$

Case 3: Critical Damping

$\alpha = \omega_o$, $\zeta = 1$ → 2 equal roots

$$s_{1,2} = -\frac{1}{2RC} \pm \sqrt{\left(\frac{1}{2RC}\right)^2 - \frac{1}{LC}}$$

$$s_{1,2} = -\alpha \pm \sqrt{\alpha^2 - \omega_0^2}$$

$$\alpha = \frac{1}{2RC} \text{ rad/s}, \quad \omega_0 = \frac{1}{\sqrt{LC}} \text{ rad/s}$$

$$s_{1,2} = -\frac{1}{2RC} \pm \sqrt{\left(\frac{1}{2RC}\right)^2 - \frac{1}{LC}}$$

$$s_{1,2} = -\xi\omega_0 \pm \omega_0\sqrt{\xi^2 - 1}$$

$$\omega_0 = \frac{1}{\sqrt{LC}} \quad , \quad \xi = \frac{1}{2R}\sqrt{\frac{L}{C}}$$

$$s_1 = s_1 = -\alpha$$

$$v(t) = D_1 t e^{-\alpha t} + D_2 e^{-\alpha t}$$

Solution to Parallel RLC Circuits

$v(t) = A_1 e^{s_1 t} + A_2 e^{s_2 t}$ Overdamped

$v(t) = B_1 e^{-\alpha t} \cos \omega_d t + B_2 e^{-\alpha t} \sin \omega_d t$ Underdamped

$v(t) = D_1 t e^{-\alpha t} + D_2 e^{-\alpha t}$ Critically damped

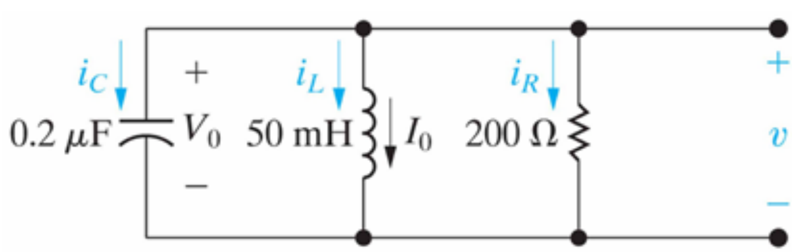
Determine the Coefficients

- We can calculate them from the *initial conditions*
- Keep in mind voltage across a capacitor and current in an inductor can not change instantaneously

$$v_C(0^-) = v_C(0) = v_C(0^+)$$
$$i_L(0^-) = i_L(0) = i_L(0^+)$$

Activity 2

Given $v(0^+)$=12V and $i_L(0^+)$=30mA,
Find v(t) for t>=0.



Solution

Step 1: determine the solution form.

Given R=200Ω,C=0.2μF, L=50mH

$$\alpha = \frac{1}{2RC} = \frac{1}{2\times 200\times 0.2\times 10^{-6}} = 0.0125\times 10^{6} rad/s$$

$$\omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{50\times 10^{-3}\times 0.2\times 10^{-6}}} = 10^{4} rad/s$$

$$\alpha > \omega_0 \qquad \text{overdamped}$$

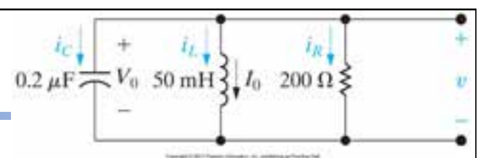
$$s_{1,2} = -\alpha \pm \sqrt{\alpha^2 - \omega_0^2} = -1.25\times 10^4 \pm \sqrt{1.5625-1}\times 10^4$$

$$s_1 = -12500 + 7500 = -5000 rad/s$$

$$s_2 = -12500 - 7500 = -20000 rad/s$$

$$\therefore v(t) = A_1 e^{-5000t} + A_2 e^{-20000t} V$$

Solution



Step 2, find A_1 and A_2

Given $v(0^+)$=12 V, $i_L(0^+)$=30 mA

and $v(t) = A_1 e^{-5,000t} + A_2 e^{-20,000t}$

$$v(0^+) = 12 = A_1 + A_2 \qquad (1)$$

$$\text{KCL: } i_C(0^+) = -i_R(0^+) - i_L(0^+), \text{ also } i_C(0^+) = C\frac{dv(0^+)}{dt}$$

$$C\frac{dv(0^+)}{dt} = -\frac{V_0}{R} - i_L(0^+)$$

$$\frac{dv(0^+)}{dt} = -\frac{V_0}{CR} - \frac{i_L(0^+)}{C} = -\frac{12}{0.2\times 10^{-6}\times 200} - \frac{30\times 10^{-3}}{0.2\times 10^{-6}}$$

$$= -0.3\times 10^6 - 0.15\times 10^6 = -0.45\times 10^6 V/s$$

$$\frac{dv(0^+)}{dt} = -450,000 = -5000A_1 - 20,000A_2 \qquad (2)$$

Solution

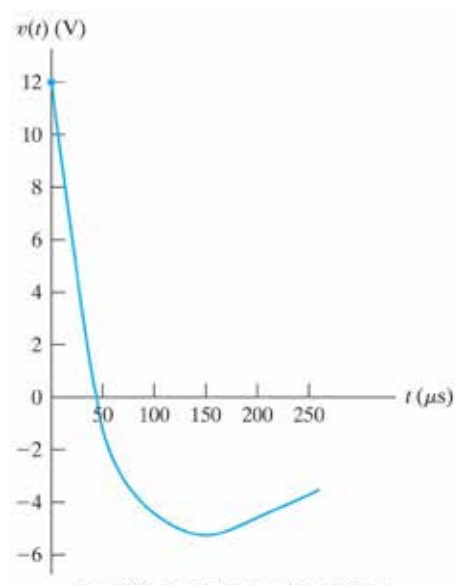
Solve for equations (1) and (2),

A_1=-14V and A_2=26V

$$\therefore v(t) = -14e^{-5,000t} + 26e^{-20,000t}V, \qquad t \geq 0$$

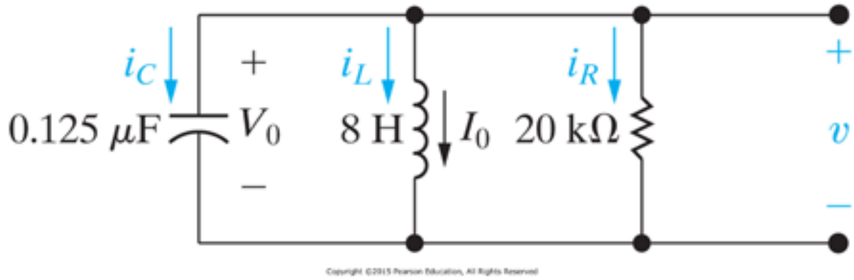
Solution

Plot v(t) from 0 to 250ms



Activity 3

Given V_0=0V, I_0=-12.25mA,
Find v(t) for t>=0.



Solution

Step 1: determine the solution form.
Given R=20kΩ,C=0.125μF, L=8H

$$\alpha = \frac{1}{2RC} = \frac{1}{2\times 20\times 10^3 \times 0.125\times 10^{-6}} = 200 rad/s$$

$$\omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{8\times 0.125\times 10^{-6}}} = 10^3 rad/s$$

$\alpha < \omega_0$ underdamped

$$\omega_d = \sqrt{\omega_0^2 - \alpha^2} = \sqrt{10^6 - 4\times 10^4} = 979.80 rad/s$$

$$s_1 = -\alpha + j\omega_d = -200 + j979.80 rad/s$$

$$s_2 = -\alpha - j\omega_d = -200 - j979.80 rad/s$$

$$v(t) = B_1 e^{-200t}\cos(979.80t) + B_2 e^{-200t}\sin(979.80t)$$

Solution

Step 2, find B_1 and B_2

Given $v_0=v(0^+)=0$ V, $i_R=0 \rightarrow i_C(0^+)=-I_0=12.25\text{mA}$

and $v(t) = B_1 e^{-200t}\cos(979.80t) + B_2 e^{-200t}\sin(979.80t)$

$v(0^+) = B_1 = 0 \qquad (1)$

KCL: $i_C(0^+) = -i_R(0^+) - i_L(0^+)$, also $i_C(0^+) = C\dfrac{dv(0^+)}{dt}$

$$\frac{dv(0^+)}{dt} = \frac{i_L(0^+)}{C} = \frac{12.25\times10^{-3}}{0.125\times10^{-6}} = 98000V/s$$

$$B_2 = \frac{dv(0^+)}{dt}\Big/\omega_d = \frac{98000}{979.80} \approx 100V \qquad (2)$$

Solution

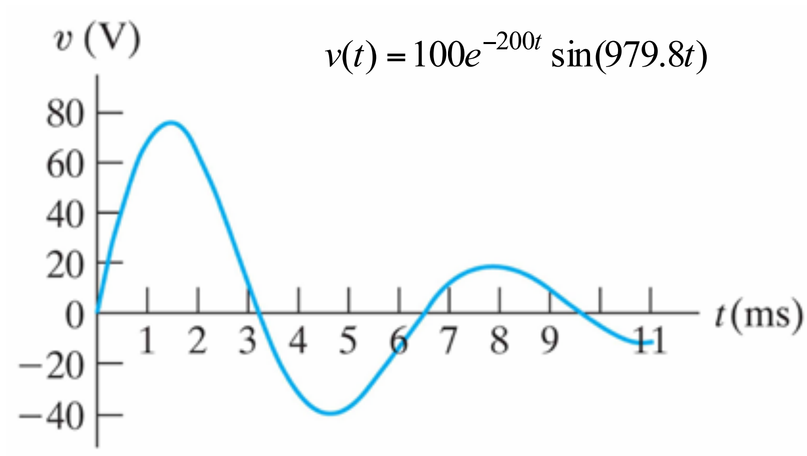
Solve for equations (1) and (2),

B_1=0V and B_2=100V

$$v(t) = 100e^{-200t}\sin(979.80t)V \qquad t \ge 0$$

Solution

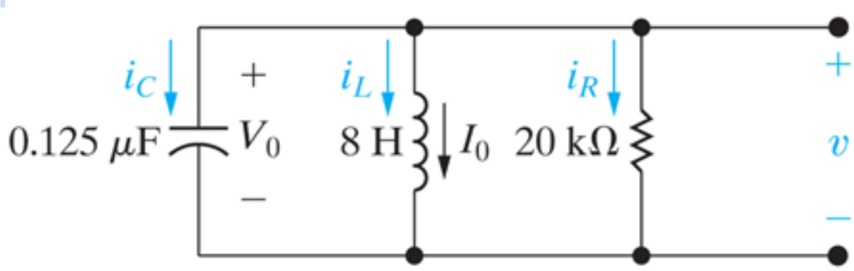
Plot v(t) from 0 to 11ms



Activity 4

In Activity 4, what is the value of R that results in a critically damped voltage response?

Find v(t) for t>=0.



Solution

Step 1: determine the solution form.

Given C=0.125μF, L=8H

$$\because \omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{8 \times 0.125 \times 10^{-6}}} = 10^3 rad/s$$

$$\therefore \alpha = \omega_0 = 10^3 = \frac{1}{2RC} \Rightarrow R = \frac{1}{2 \times 10^3 \times 0.125 \times 10^{-6}} = 4k\Omega$$

$$\therefore v(t) = D_1 t e^{-\alpha t} + D_2 e^{-\alpha t}$$

Solution

Step 2, find D_1 and D_2

Given $v_0 = v(0^+) = 0$ V, $dv(0^+)/dt = 98000$V/s

and $v(t) = D_1 t e^{-\alpha t} + D_2 e^{-\alpha t}$

$$v(0^+) = D_2 = 0 \qquad (1)$$

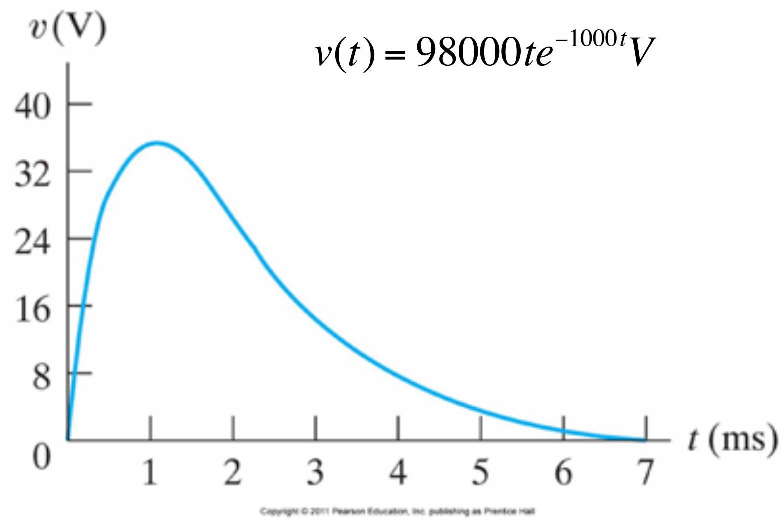
$$\frac{dv(t)}{dt} = D_1 e^{-\alpha t} + D_1 t \frac{e^{-\alpha t}}{-\alpha}$$

$$\frac{dv(0^+)}{dt} = D_1 = 98000V \qquad (2)$$

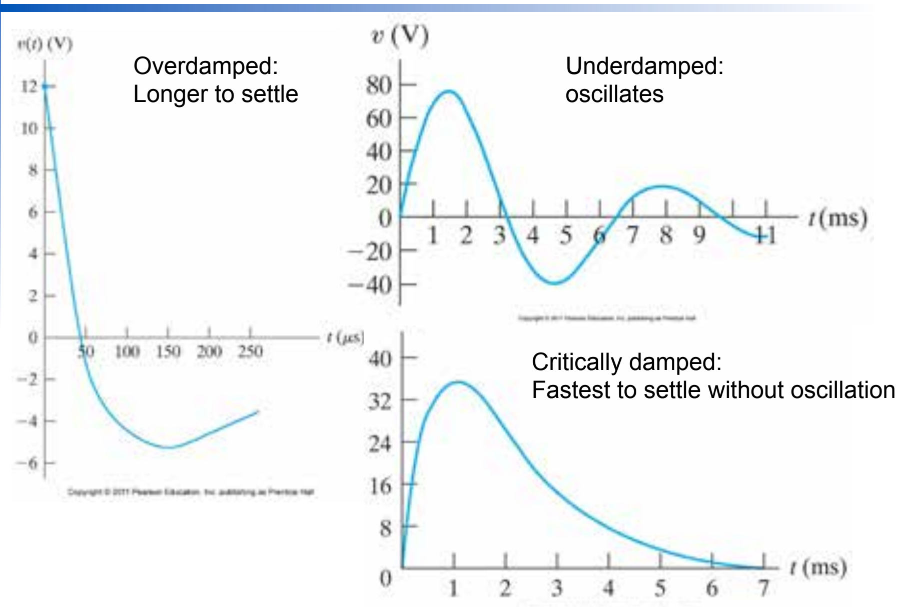
$$\therefore v(t) = 98000 t e^{-1000t} V \qquad t \geq 0$$

Solution

$$v(t) = 98000te^{-1000t}V$$



3 Cases



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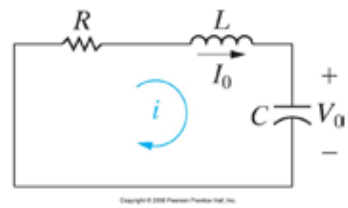
Natural Response of Series RLC Circuits

Series and Parallel RLC Circuits

- The difference(s) between the analysis of series RLC circuit and the parallel RLC circuit is/are:
  - A. The variable we calculate.
  - B. The describing differential equation.
  - C. The equations for satisfying the initial conditions

Natural Response of Series RLC Circuits

The problem – given initial energy stored in the inductor and/or capacitor, find *i*(*t*) for *t* ≥ *0*.



KVL: $$L\frac{di(t)}{dt}+\frac{1}{C}\int_0^t i(x)\,dx+V_0+Ri(t)=0$$

Differentiate both sides to remove the integral:

$$L\frac{d^2i(t)}{dt^2}+\frac{1}{C}i(t)+R\frac{di(t)}{dt}=0$$

Divide both sides by L to place in standard form:

$$\frac{d^2i(t)}{dt^2}+\frac{R}{L}\frac{di(t)}{dt}+\frac{1}{LC}i(t)=0$$

Activity 5

The describing differential equation for the series RLC circuit is

$$\frac{d^2i(t)}{dt^2}+\frac{R}{L}\frac{di(t)}{dt}+\frac{1}{LC}i(t)=0$$

Therefore, the characteristic equation is

A. $s^2 + (1/RC)s + 1/LC = 0$

B. $s^2 + (R/L)s + 1/LC = 0$

C. $s^2 + (1/LC)s + 1/RC = 0$

Natural Response of Series RLC Circuits

The two solutions to the characteristic equation can be calculated using the quadratic formula:

$$s^2 + (R/L)s + (1/LC) = 0$$

$$s_{1,2} = -\alpha \pm \sqrt{\alpha^2 - \omega_0^2}$$

$$\alpha = \frac{R}{2L} \quad \text{(the neper frequency in rad/s)}$$

$$\omega_0 = \sqrt{\frac{1}{LC}} \quad \text{(the resonant radian frequency in rad/s)}$$

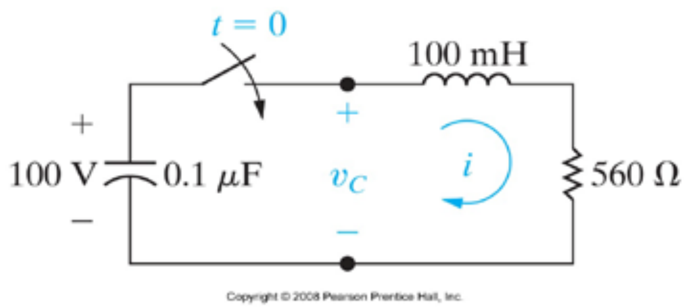
Natural Response of Series RLC Circuits

The solution are in the same form as in the parallel RLC circuits:

$i(t) = A_1 e^{s_1 t} + A_2 e^{s_2 t}$	Overdamped
$i(t) = B_1 e^{-\alpha t} \cos\omega_d t + B_2 e^{-\alpha t} \sin\omega_d t$	Underdamped
$i(t) = D_1 t e^{-\alpha t} + D_2 e^{-\alpha t}$	Critically damped

Activity 6

The capacitor is charged to 100 V and at t = 0, the switch closes. Find i(t) for t ≥ 0.



Solution

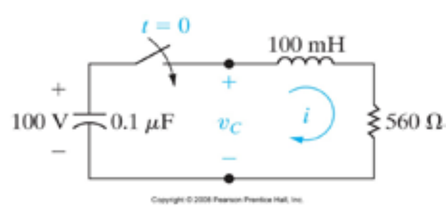
$$\alpha = \frac{R}{2L} = \frac{560}{2(0.1)} = 2800 \text{ rad/s}$$

$$\omega_0 = \sqrt{\frac{1}{LC}} = \sqrt{\frac{1}{(0.1)(0.1\mu)}} = 10{,}000 \text{ rad/s}$$

$\alpha^2 < \omega_o^2$ so this is the underdamped case!

$$\omega_d = \sqrt{\omega_0^2 - \alpha^2} = 9600 \text{ rad/s}$$

$$i(t) = B_1 e^{-2800t} \cos 9600t + B_2 e^{-2800t} \sin 9600t \text{ A}, t \geq 0$$



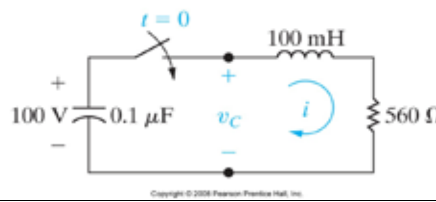
Solution

$i(t) = B_1 e^{-2800t}\cos 9600t + B_2 e^{-2800t}\sin 9600t$ A, $t \geq 0$

Now we must use the coefficients in the equation to satisfy the initial conditions in the circuit :

$$i(t)\Big|_{t=0} \text{ in the equation} = i(t)\Big|_{t=0} \text{ in the circuit}$$

$$\left.\frac{di(t)}{dt}\right|_{t=0} \text{ in the equation} = \left.\frac{di(t)}{dt}\right|_{t=0} \text{ in the circuit}$$



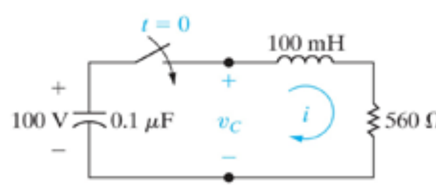
Solution

$i(t) = B_1 e^{-2800t}\cos 9600t + B_2 e^{-2800t}\sin 9600t$ A, $t \geq 0$

Equation: $i(0) = B_1$ (same as the parallel case!)

Circuit : $i(0) = I_0 = 0$

$\Rightarrow \quad B_1 = 0$



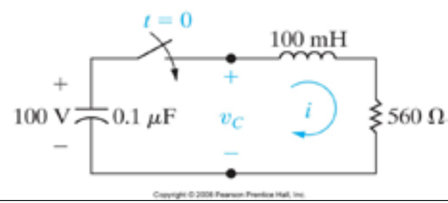
Solution

Equation: $$\frac{di(0^+)}{dt} = -\alpha B_1 + \omega_d B_2 \text{ (same as the parallel case!)}$$

Circuit: $$L\frac{di(0^+)}{dt} = V_0 \Rightarrow \frac{di(0^+)}{dt} = \frac{V_0}{L} = \frac{100}{100 \times 10^{-3}} = 1000 A/s$$

$$\because \quad B_1 = 0 \quad \Rightarrow \quad \frac{di(0^+)}{dt} = \omega_d B_2 = 9600 B_2 = 1000 \quad \Rightarrow \quad B_2 = 0.104$$

$$\therefore i(t) = 0.104 e^{-2800t} \sin 9600t \text{ A}, t \geq 0$$



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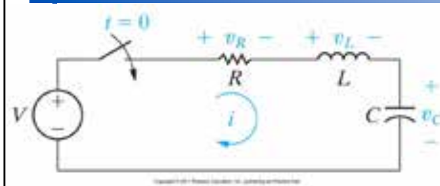
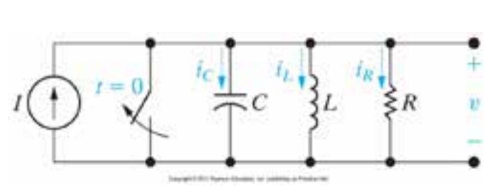
Step Response of RLC Circuits

Step Response of RLC Circuit





$$v_L + v_C + v_R = V$$

$$L\frac{di}{dt} + v_c + iR = V$$

$$i = C\frac{dv_C}{dt}$$

$$LC\frac{d^2v_C}{dt^2} + v_C + RC\frac{dv_C}{dt} = V$$

$$\frac{d^2v_C}{dt^2} + \frac{R}{L}\frac{dv_C}{dt} + \frac{v_C}{LC} = \frac{I}{LC}$$

$$i_L + i_C + i_R = I$$

$$i_L + C\frac{dv}{dt} + \frac{v}{R} = I$$

$$v = L\frac{di_L}{dt}$$

$$LC\frac{d^2i_L}{dt} + \frac{L}{R}\frac{di_L}{dt} + i_L = I$$

$$\frac{d^2i_L}{dt} + \frac{1}{RC}\frac{di_L}{dt} + \frac{i_L}{LC} = \frac{I}{LC}$$

Step Response of RLC Circuits

- A topic for a course in Math.
- Generally speaking, the solution of a second-order DE with a constant driving force equals the forced response plus the a response function identical to the natural response.

$$i = I_f + \begin{Bmatrix}\text{function of the same form} \\ \text{as natural response}\end{Bmatrix}$$

$$v = V_f + \begin{Bmatrix}\text{function of the same form} \\ \text{as natural response}\end{Bmatrix}$$

- I_f or V_f is the non-zero final value.

EECS2200 Electric Circuits

Step Response for Parallel RLC Circuits

Step Response of a Parallel *RLC Circuit*

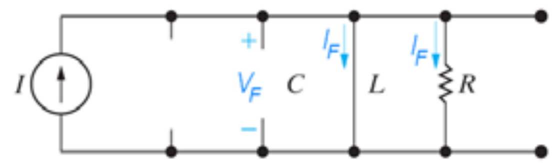
$i_L(t) = I_f + A_1 e^{s_1 t} + A_2 e^{s_2 t}$ Overdamped

$i_L(t) = I_f + B_1 e^{-\alpha t} \cos\omega_d t + B_2 e^{-\alpha t} \sin\omega_d t$ Underdamped

$i_L(t) = I_f + D_1 t e^{-\alpha t} + D_2 e^{-\alpha t}$ Critically damped

Step Response of RLC Circuit

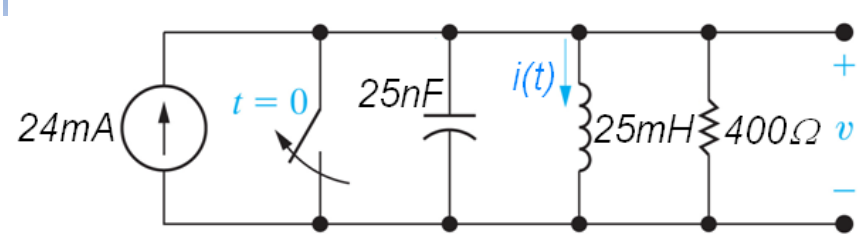
As $t \to \infty$:



The only component whose final value is NOT zero is the inductor, whose final current is the current supplied by the source.

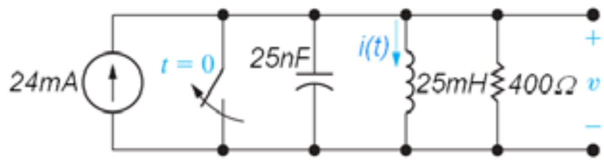
Activity 7

There is no initial energy stored in this circuit; find $i(t)$ for $t \geq 0$.



Solution

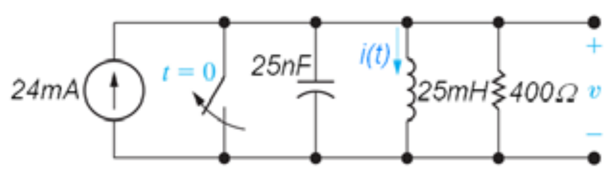
The problem – there is no initial energy stored in this circuit; find *i(t)* for $t \geq 0$.



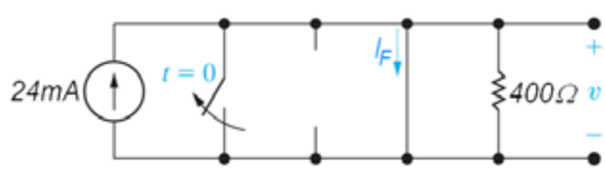
To begin, find the initial conditions and the final value. The initial conditions for this problem are both zero; the final value is found by analyzing the circuit as $t \rightarrow \infty$.

Solution

The problem – there is no initial energy stored in this circuit; find *i(t)* for $t \geq 0$.



$t \rightarrow \infty$:



$$I_F = 24\text{mA}$$

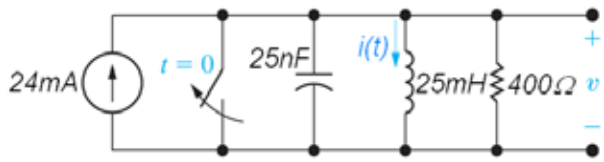
Solution

Next, calculate the values of α and ω_0 and determine the form of the response:

$$\alpha = \frac{1}{2RC} = \frac{1}{2(400)(25\text{n})} = 50{,}000 \text{ rad/s}$$

$$\omega_0 = \sqrt{1/LC} = \sqrt{1/(25\text{m})(25\text{n})} = 40{,}000 \text{ rad/s}$$

Overdamped



Solution

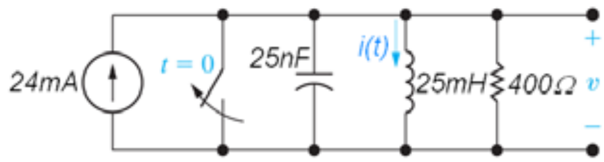
Since the response form is overdamped, calculate the values of s_1 and s_2:

$$s_{1,2} = -\alpha \pm \sqrt{\alpha^2 - \omega_0^2} = -50{,}000 \pm \sqrt{50{,}000^2 - 40{,}000^2}$$

$$= -50{,}000 \pm 30{,}000 \text{ rad/s}$$

$$\therefore \quad s_1 = -20{,}000 \text{ rad/s} \text{ and } s_2 = -80{,}000 \text{ rad/s}$$

$$\Rightarrow \quad i_L(t) = 0.024 + A_1 e^{-20{,}000t} + A_2 e^{-80{,}000t} \text{ A}, t \geq 0$$



Solution

$$i_L(t) = 0.024 + A_1 e^{-20,000t} + A_2 e^{-80,000t} \text{ A}, t \geq 0$$

Next, set the values of *i(0)* and *di(0)/dt* from the equation equal to the values of *i(0)* and di(0)/dt from the circuit.

From the equation: $i_L(0) = 0.024 + A_1 + A_2$

From the circuit: $i_L(0) = I_0 = 0$

From the equation: $\frac{di_L(0)}{dt} = -20,000A_1 - 80,000A_2$

From the circuit: $\frac{di_L(0)}{dt} = \frac{v_L(0)}{L} = \frac{V_0}{L} = 0$

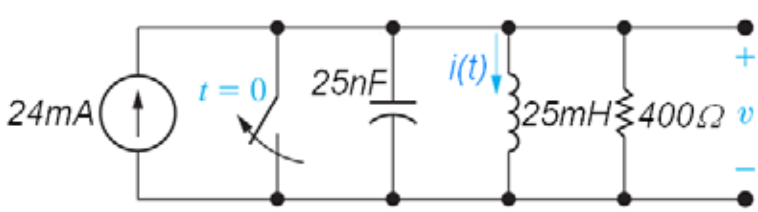
Solution

Solve: $0.024 + A_1 + A_2 = 0$

and $-20,000A_1 - 80,000A_2 = 0$

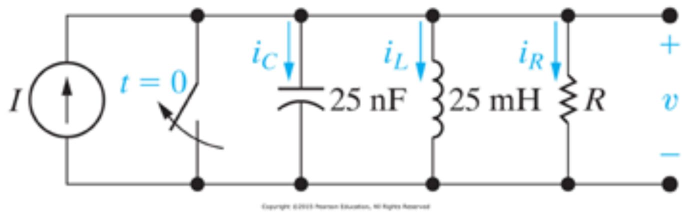
$\therefore$ $A_1 = -32$ mA; $A_2 = 8$ mA

$\Rightarrow$ $i_L(t) = 24 - 32e^{-20,000t} + 8e^{-80,000t}$ mA, $t \geq 0$



Activity 8

If the resistor value is changed to 625Ω, find *i(t)* for *t ≥ 0*.



Solution

$$\because R = 625\Omega, C = 25nF,$$

$$\alpha = \frac{1}{2RC} = \frac{1}{2(625)(25\text{n})} = 32{,}000 \text{ rad/s}$$

$$\omega_0 = \sqrt{1/LC} = \sqrt{1/(25\text{m})(25\text{n})} = 40{,}000 \text{ rad/s}$$

$$\alpha < \omega_0 \Rightarrow \text{Underdamped}$$

$$\omega_d = \sqrt{\omega_0^2 - \alpha^2} = \sqrt{(16 - 10.24)\times 10^8} = 24000\text{rad/s}$$

$$s_1 = -32000 + j24000 rad/s$$

$$s_2 = -32000 - j24000 rad/s$$

$$i_L(t) = I_f + B_1 e^{-32000t}\cos(24000t) + B_2 e^{-32000t}\sin(24000t)$$

Solution

Next, set the values of *i(0)* and *di(0)/dt* from the equation equal to the values of *i(0)* and di(0)/dt from the circuit.

$$i_L(0) = 0.024 + B_1 = 0$$

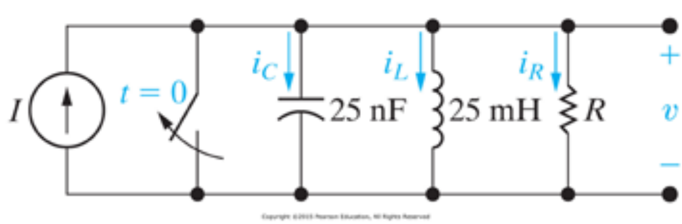
$$\frac{di_L(0)}{dt} = \omega_d B_2 - \alpha B_1 = 0$$

$$B_1 = -24mA, B_2 = 32mA$$

$$i_L(t) = 24 - 24e^{-32000t}\cos(24000t)$$
$$-32e^{-32000t}\sin(24000t)mA \qquad t \geq 0$$

Activity 9

If the resistor value in is changed to 500Ω, find *i(t)* for $t \geq 0$.



Solution

$\because R = 500\Omega, C = 25nF,$

$$\alpha = \frac{1}{2RC} = \frac{1}{2(500)(25\text{n})} = 40{,}000 \text{ rad/s}$$

$$\omega_0 = \sqrt{1/LC} = \sqrt{1/(25\text{m})(25\text{n})} = 40{,}000 \text{ rad/s}$$

$$\alpha = \omega_0 \Rightarrow \text{Critically damped}$$

$$s_1 = s_2 = -\alpha = -40000 rad/s$$

$$i_L(t) = I_f + D_1 te^{-40000t} + D_2 e^{-40000t}$$

Solution

Next, set the values of *i(0)* and *di(0)/dt* from the equation equal to the values of *i(0)* and di(0)/dt from the circuit.

$$i_L(0) = 0.024 + D_2 = 0$$

$$\frac{di_L(0)}{dt} = D_1 - \alpha D_2 = 0$$

$$D_1 = -960000 mA/s, D_2 = -24 mA$$

$$i_L(t) = 24 - 960000te^{-40000t} - 24e^{-40000t} \quad mA \quad t \geq 0$$

Plot Responses of 3 Cases

The overdamped, underdamped, and critically damped responses of Activities 7-9 are given below:

Overdamped:

$$i_L(t) = 24 - 32e^{-20,000t} + 8e^{-80,000t} \text{ mA}, t \geq 0$$

Underdamped:

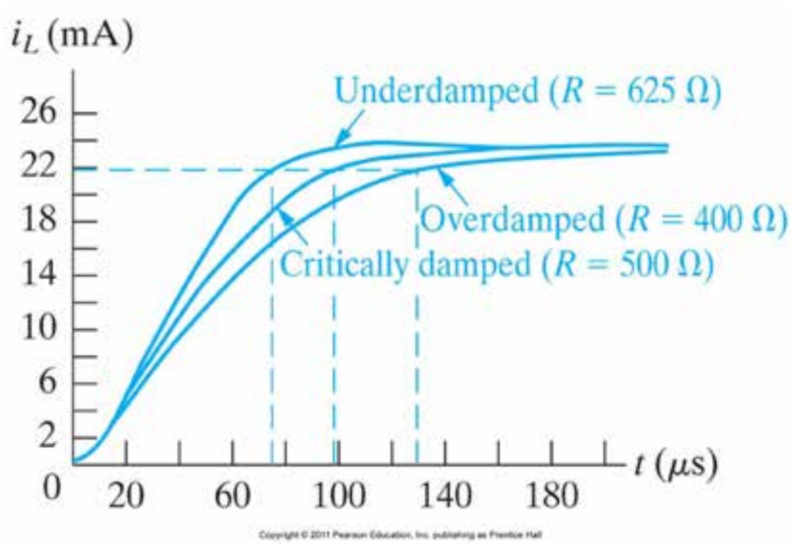
$$i_L(t) = 24 - 24e^{-32000t}\cos(24000t)$$

$$-32e^{-32000t}\sin(24000t) mA, \quad t \geq 0$$

Critically damped:

$$i_L(t) = 24 - 960000te^{-40000t} - 24e^{-40000t} \quad mA, \quad t \geq 0$$

The current plots



EECS2200 Electric Circuits

Step Response for Series RLC Circuits

Step Response of a Series *RLC Circuit*

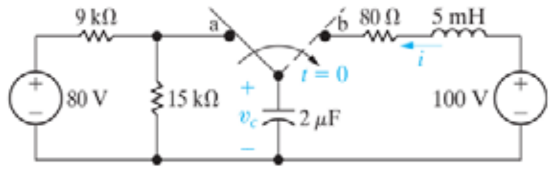
$v_c = V_f + A_1 e^{s_1 t} + A_2 e^{s_2 t}$ Overdamped

$v_c = V_f + B_1 e^{-\alpha t} \cos\omega_d t + B_2 e^{-\alpha t} \sin\omega_d t$ Underdamped

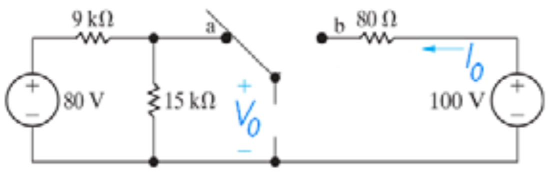
$v_c = V_f + D_1 t e^{-\alpha t} + D_2 e^{-\alpha t}$ Critically damped

Step Response for Series RLC Circuit

The problem – find $v_C(t)$ for $t \geq 0$.



Find the initial conditions by analyzing the circuit for $t < 0$:

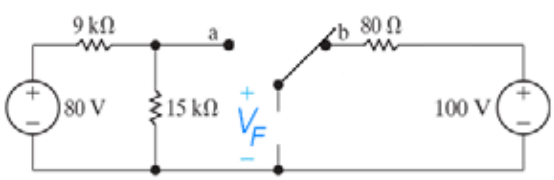


$$V_0 = \frac{15\,\text{k}}{9\,\text{k} + 15\,\text{k}}(80\text{ V}) = 50\text{ V}$$

$$I_0 = 0\text{ A}$$

Step Response for Series RLC Circuit

Find the final value of the capacitor voltage by analyzing the circuit as $t \to \infty$:



$$V_F = 100\text{V}$$

Step Response for Series RLC Circuit

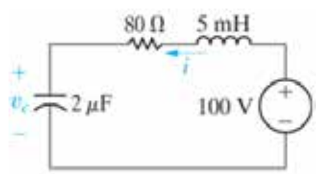
Use the circuit for $t \geq 0$ to find the values of α and ω_0:

$$\alpha = R/2L = 80/2(0.005) = 8000\,\text{rad/s}$$

$$\omega_0 = \sqrt{1/LC} = \sqrt{1/(0.005)(2\mu)}$$
$$= 10{,}000\,\text{rad/s}$$

$$\alpha^2 < \omega_0^2 \quad \Rightarrow \quad \text{underdamped}$$

$$\omega_d = \sqrt{\omega_0^2 - \alpha^2} = \sqrt{10{,}000^2 - 8000^2}$$
$$= 6000\,\text{rad/s}$$



Step Response for Series RLC Circuit

Write the equation for the response and solve for the unknown coefficients:

$$v_C(t) = 100 + B_1 e^{-8000t}\cos 6000t + B_2 e^{-8000t}\sin 6000t\,\text{V}, t \geq 0$$

$$v_C(0) = V_F + B_1 = V_0 \qquad \therefore \qquad 100 + B_1 = 50$$

$$\frac{dv_C(0)}{dt} = -\alpha B_1 + \omega_d B_2 = \frac{I_0}{C} \qquad \therefore \qquad -8000B_1 + 6000B_2 = 0$$

$$\Rightarrow \qquad B_1 = -50\,\text{V}, \qquad B_2 = 66.67\,\text{V}$$

$$v_C(t) = 100 - 50e^{-8000t}\cos 6000t + 66.67e^{-8000t}\sin 6000t\,\text{V}, t \geq 0$$