EECS2200 Electric Circuits

Chapter 8 Part 1

Sinusoidal Steady State Power Calculation

Objectives

- Understanding the difference between instantaneous power, average power reactive power, complex power and how to calculate them.
- Understanding power factor and how to calculate it.
- Understand the condition for a maximum real power delivered to the load.

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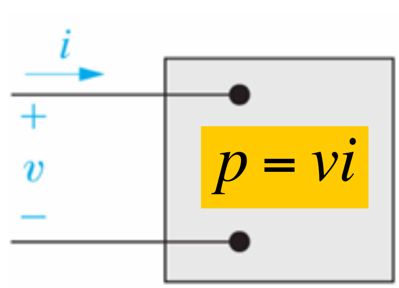
Instantaneous Power

Instantaneous Power

- The black box representation of a circuit used for calculating power.

$$v = V_m \cos(\omega t + \theta_v)$$
$$i = I_m \cos(\omega t + \theta_i)$$



$$v = V_m \cos(\omega t + \theta_v - \theta_i)$$
$$i = I_m \cos(\omega t)$$

Instantaneous Power

$v = V_m \cos(\omega t + \theta_v - \theta_i)$

$i = I_m \cos(\omega t)$

$p = I_m V_m \cos(\omega t + \theta_v - \theta_i)\cos(\omega t)$

$\because \cos(\alpha)\cos(\beta) = \frac{1}{2}\cos(\alpha - \beta) + \frac{1}{2}\cos(\alpha + \beta)$

Let $\alpha = \omega t + \theta_v - \theta_i, \beta = \omega t$

$\therefore p = \frac{I_m V_m}{2}\cos(\theta_v - \theta_i) + \frac{I_m V_m}{2}\cos(2\omega t + \theta_v - \theta_i)$

Instantaneous Power

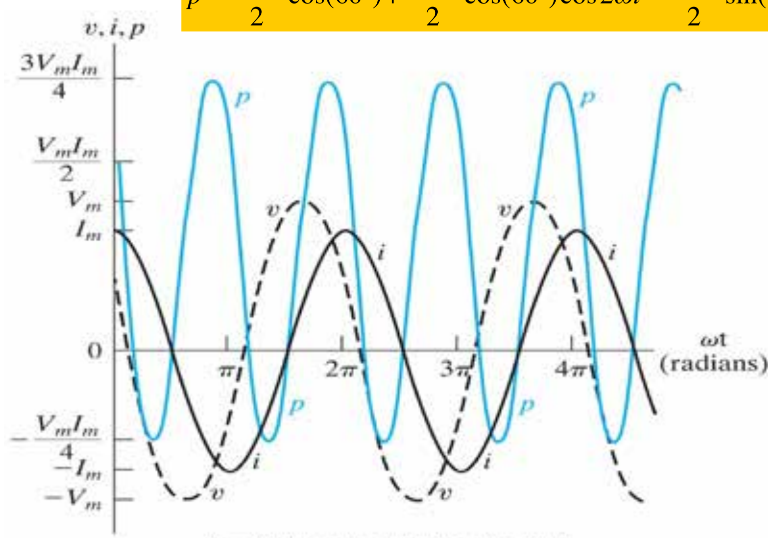
$p = \frac{I_m V_m}{2}\cos(\theta_v - \theta_i) + \frac{I_m V_m}{2}\cos(2\omega t + \theta_v - \theta_i)$

Given $\cos(\alpha + \beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$

$$p = \frac{I_m V_m}{2}\cos(\theta_v - \theta_i) + \frac{I_m V_m}{2}\cos(\theta_v - \theta_i)\cos 2\omega t - \frac{I_m V_m}{2}\sin(\theta_v - \theta_i)\sin 2\omega t$$

Assume: $\theta_v = 60^\circ, \theta_i = 0^\circ$

$$p = \frac{I_m V_m}{2}\cos(60^\circ) + \frac{I_m V_m}{2}\cos(60^\circ)\cos 2\omega t - \frac{I_m V_m}{2}\sin(60^\circ)\sin 2\omega t$$



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Average and Reactive Power

Average and Reactive Power

$$p = \frac{I_m V_m}{2}\cos(\theta_v - \theta_i) + \frac{I_m V_m}{2}\cos(\theta_v - \theta_i)\cos 2\omega t - \frac{I_m V_m}{2}\sin(\theta_v - \theta_i)\sin 2\omega t$$

$$p = P + P\cos 2\omega t - Q\sin 2\omega t$$

$$P = \frac{I_m V_m}{2}\cos(\theta_v - \theta_i), Q = \frac{I_m V_m}{2}\sin(\theta_v - \theta_i)$$

- P is the average power(or real power) with unit Watt. P is the power transformed from electric to nonelectric energy.
- Q is the reactive power with unit VAR (volt-amp reactive).

Purely Resistive Circuits

- In purely resistive circuits, voltage and current are in phase, i.e. $\theta_v = \theta_i$

$$p = \frac{I_m V_m}{2}\cos(\theta_v - \theta_i) + \frac{I_m V_m}{2}\cos(\theta_v - \theta_i)\cos 2\omega t$$

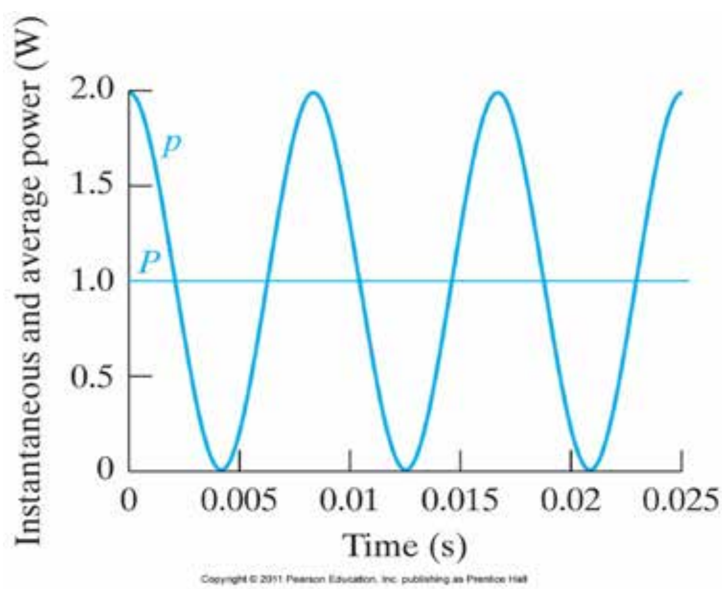
$$-\frac{I_m V_m}{2}\sin(\theta_v - \theta_i)\sin 2\omega t$$

$$\theta_v - \theta_i = 0$$

$$\rightarrow p = \frac{I_m V_m}{2} + \frac{I_m V_m}{2}\cos 2\omega t$$

Purely Resistive Circuits

$$p = \frac{I_m V_m}{2} + \frac{I_m V_m}{2}\cos 2\omega t$$



Purely Inductive Circuits

- Voltage and current are out of phase by 90 degree, i.e. voltage leads current by 90 degree, **θ_v-θ_i=90˚**

$$p = \frac{I_m V_m}{2}\cos(\theta_v - \theta_i) + \frac{I_m V_m}{2}\cos(\theta_v - \theta_i)\cos 2\omega t$$

$$-\frac{I_m V_m}{2}\sin(\theta_v - \theta_i)\sin 2\omega t$$

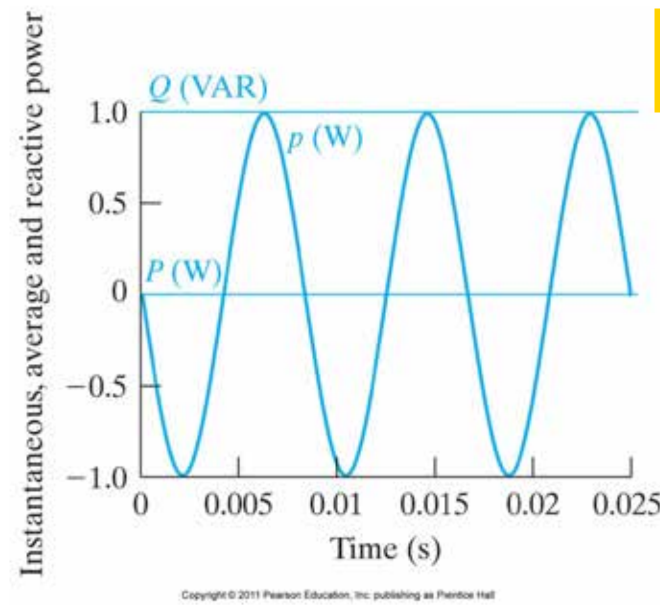
$$\theta_v - \theta_i = 90^\circ, Q = \frac{I_m V_m}{2}$$

$$\rightarrow p = -Q\sin 2\omega t$$

Purely Inductive Circuits

$p = -Q \sin 2\omega t$

$Q = \frac{I_m V_m}{2}$



Purely Capacitive Circuits

- Voltage and current are out of phase by 90 degree, i.e. voltage lags current by 90 degree, **$\theta_v - \theta_i = -90^\circ$** or **$\theta_i - \theta_v = 90^\circ$**

$$p = \frac{I_m V_m}{2}\cos(\theta_v - \theta_i) + \frac{I_m V_m}{2}\cos(\theta_v - \theta_i)\cos 2\omega t$$

$$-\frac{I_m V_m}{2}\sin(\theta_v - \theta_i)\sin 2\omega t$$

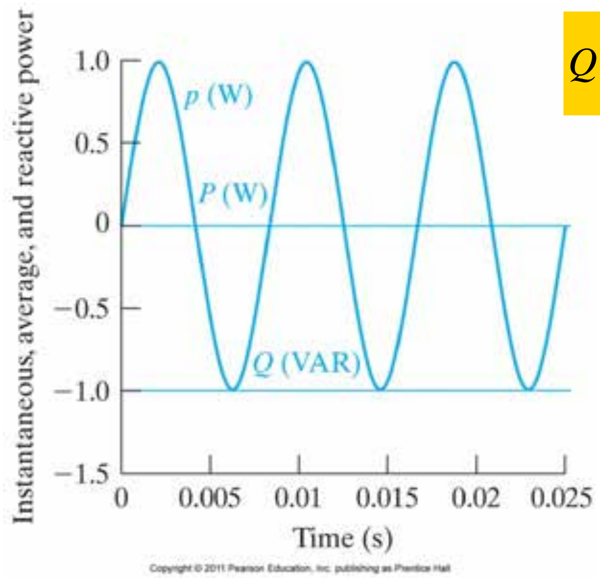
$$\theta_v - \theta_i = -90^\circ, Q = -\frac{I_m V_m}{2}$$

$$\rightarrow p = -Q \sin 2\omega t$$

Purely Capacitive Circuits

$$p = -Q \sin 2\omega t$$

$$Q = -\frac{I_m V_m}{2}$$



Summary

- Resistors P > 0, Q = 0
  - Resistors absorb real power and have no reactive power
- Inductors P = 0, Q > 0
  - Inductors absorb reactive power and have no real power
- Capacitors P = 0, Q < 0
  - Capacitors generate reactive power and have no real power

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Power Factor

Power Factor

- Power factor angle: $\theta_v - \theta_i$
- Power factor: $\text{pf} = \cos(\theta_v - \theta_i), \qquad 0 \le \text{pf} \le 1$
- This is a term that appears in the definition of average power, i.e.

$$P = \frac{I_m V_m}{2} \cos(\theta_v - \theta_i)$$

  - When pf = 1, the component is purely resistive.
  - When pf = 0, the component is purely reactive.

Power Factor

- Knowing the power factor does not tell you the value of the power factor angle,

$$\text{pf} = \cos(\theta_v - \theta_i) = \cos(\theta_i - \theta_v)$$

- To distinguish between inductive and capacitive reactance, we use the modifiers "leading" and "lagging":
  - When the pf is leading, the current leads the voltage – a capacitive load.
  - when pf is lagging, the current lags the voltage – an inductive load.

Activity 1

- Suppose the power factor of an impedance is 0.7 leading. This tells us that the simplest model of this impedance is comprised of

A. A capacitor

B. An inductor

C. A capacitor and a resistor

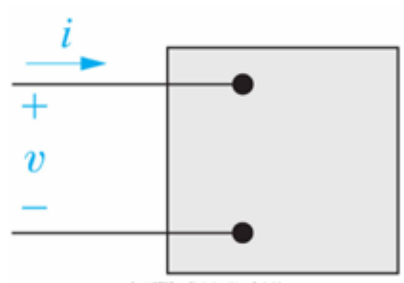
D. An inductor and a resistor

Example

Given:

$$v = 100\cos(\omega t + 15^\circ)\ \text{V}$$

$$i = 4\sin(\omega t - 15^\circ)\ \text{A}$$



Calculate the average power, reactive power.

Solution

$$v = 100\cos(\omega t + 15^\circ)\ \text{V}$$

$$\because i = 4\sin(\omega t - 15^\circ)\ \text{A}$$

$$\therefore i = 4\cos(\omega t - 105^\circ)$$

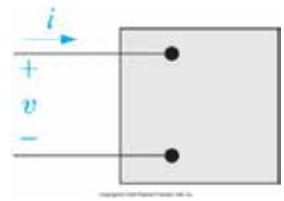
$$P = \frac{V_m I_m}{2}\cos(\theta_v - \theta_i) = \frac{100 \times 4}{2}\cos(15 - (-105)) = -100W$$

$$Q = \frac{V_m I_m}{2}\sin(\theta_v - \theta_i) = \frac{100 \times 4}{2}\sin(15 - (-105)) = 173.21VAR$$

Activity 2

Given

$$i(t) = 20\cos(\omega t + 15°)\ \text{A}$$
$$\text{v(t)} = 100\cos(\omega t - 45°)\ \text{V}$$

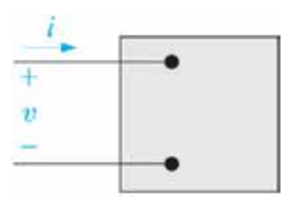


Find the average power, the reactive power, and the power factor.

Solution

Given

$$i(t) = 20\cos(\omega t + 15°)\ \text{A}$$
$$\text{v(t)} = 100\cos(\omega t - 45°)\ \text{V}$$



$$\text{P} = \frac{V_m I_m}{2}\cos(\theta_v - \theta_i) = \frac{(20)(100)}{2}\cos(-45° - 15°) = 500\ \text{W}$$

$$\text{Q} = \frac{V_m I_m}{2}\sin(\theta_v - \theta_i) = \frac{(20)(100)}{2}\sin(-45° - 15°) = -866\ \text{var}$$

$$\text{pf} = \cos(\theta_v - \theta_i) = \cos(-45° - 15°) = 0.5$$

Activity 3

For the previous example, we calculated a power factor of 0.5, but is it leading or lagging?

A. Leading
B. Lagging
C. Can't tell from the information

Activity 4

For the previous example, the average power for the circuit in the box is 500 W and the reactive power is -866 var. This means the circuit is

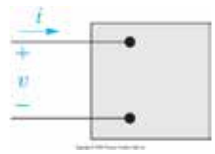
A. Generating P and generating Q
B. Generating P and absorbing Q
C. Absorbing P and generating Q
D. Absorbing P and absorbing Q

Activity 5

Given:

$i(t) = 20\cos(\omega t + 165°)\ \text{A}$

$\text{v(t)} = 100\cos(\omega t - 45°)\ \text{V}$



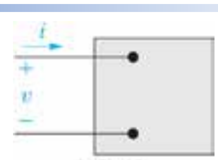
Find the average power, the reactive power, and the power factor.

Solution

Given:

$i(t) = 20\cos(\omega t + 165°)\ \text{A}$

$\text{v(t)} = 100\cos(\omega t - 45°)\ \text{V}$



$$\text{P} = \frac{V_m I_m}{2}\cos(\theta_v - \theta_i) = \frac{(20)(100)}{2}\cos(-45° - 165°) = -866\ \text{W}$$

$$\text{Q} = \frac{V_m I_m}{2}\sin(\theta_v - \theta_i) = \frac{(20)(100)}{2}\sin(-45° - 165°) = -500\ \text{var}$$

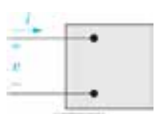
$$\text{pf} = \cos(\theta_v - \theta_i) = \cos(-45° - 165°) = 0.866\ \text{leading}$$

Activity 6

For the previous example, repeated below, P = -866 W and Q = -500 var. The simplest model of the circuit in the box is:

A. A resistor and a capacitor
B. A resistor and an inductor
C. None of the above

Activity 6 Solution



$$i(t) = 20\cos(\omega t + 165°)\text{ A}$$
$$v(t) = 100\cos(\omega t - 45°)\text{ V}$$

$$P = -866\text{ W},\qquad Q = -500\text{ var},\qquad pf = 0.866\text{ leading}$$

The circuit in the box is generating both average and reactive power.

- Capacitors generate reactive power,
- Only sources generate average power,
- The pf < 1, so there must also be a resistor.

Thus, the simplest circuit in the box has a source, a resistor, and a capacitor!

TABLE 10.1 Annual Energy Requirements of Electric Household Appliances

Appliance	Average Wattage	Est. kWh Consumed Annually[a]
Food preparation		
Coffeemaker	1,200	140
Dishwasher	1,201	165
Egg cooker	516	14
Frying pan	1,196	100
Mixer	127	2
Oven, microwave (only)	1,450	190
Range, with oven	12,200	596
Toaster	1,146	39
Laundry		
Clothes dryer	4,856	993
Washing machine, automatic	512	103
Water heater	2,475	4,219
Quick recovery type	4,474	4,811
Comfort conditioning		
Air conditioner (room)	860	860[b]
Dehumidifier	257	377
Fan (circulating)	88	43
Heater (portable)	1,322	176
Health and beauty		
Hair dryer	600	25
Shaver	15	0.5
Sunlamp	279	16
Home entertainment		
Radio	71	86
Television, color, tube type	240	528
Solid-state type	145	320
Housewares		
Clock	2	17
Vacuum cleaner	630	46

a) Based on normal usage. When using these figures for projections, such factors as the size of the specific appliance, the geographical area of use, and individual usage should be taken into consideration. Note that the wattages are not additive, since all units are normally not in operation at the same time.

b) Based on 1000 hours of operation per year. This figure will vary widely depending on the area and the specific size of the unit. See EEI-Pub #76-2, "Air Conditioning Usage Study," for an estimate for your location.

Source: Edison Electric Institute.

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The RMS Value and Power Calculation

RMS value (Chapter 7)

- RMS (root mean square) value of a periodic function is the square root of the mean value of the squared function

$$V_{rms} = \sqrt{\frac{1}{T}\int_{t_0}^{t_0+T} V_m^2 \cos^2(\omega t + \phi)\,dt}$$

$$V_{rms} = \frac{V_m}{\sqrt{2}}$$

RMS Value and Power Calculation

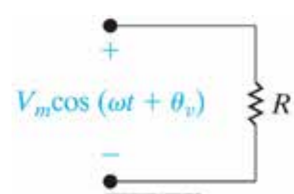
- A sinusoidal voltage applied to the terminals of a resistor.

$$P = \frac{1}{T}\int_{t_0}^{t_0+T} \frac{V_m^2 \cos^2(\omega t + \theta_v)}{R}\,dt$$

$$P = \frac{1}{R}\left[\frac{1}{T}\int_{t_0}^{t_0+T} V_m^2 \cos^2(\omega t + \theta_v)\,dt\right]$$

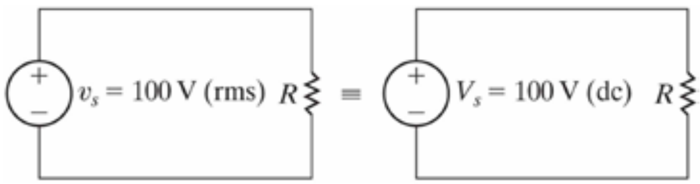
$$P = \frac{V_{RMS}^2}{R}$$

$$P = I_{RMS}^2 R$$



Effective Value

- The rms value is also referred to as the **effective value** of sinusoidal voltage (current).
- Given equivalent load and time, the rms value of a sinusoidal source delivers the same energy to R as does a DC source of the same value.



Effective Value

$$P = \frac{V_m I_m}{2}\cos(\theta_v - \theta_i) = \frac{V_m}{\sqrt{2}}\frac{I_m}{\sqrt{2}}\cos(\theta_v - \theta_i)$$

$$= V_{eff} I_{eff} \cos(\theta_v - \theta_i)$$

$$Q = \frac{V_m I_m}{2}\sin(\theta_v - \theta_i) = \frac{V_m}{\sqrt{2}}\frac{I_m}{\sqrt{2}}\sin(\theta_v - \theta_i)$$

$$= V_{eff} I_{eff} \sin(\theta_v - \theta_i)$$

Activity 7

- Given a 120V, 100W lamp, what are the resistance, effective current, and peak current?

Solution

$$V_{rms} = 120V, P = 100W$$

$$P = \frac{V_{rms}^2}{R} \rightarrow R = \frac{V_{rms}^2}{P} = \frac{120^2}{100} = 144\Omega$$

$$I_{rms} = \frac{V_{rms}}{R} = \frac{120}{144} = 0.833A$$

$$I_m = \sqrt{2} I_{rms} = 1.414 \times 0.833 = 1.18A$$