EECS2200 Electric Circuits

Chapter 8 Part 2

Sinusoidal Steady State Power Calculation

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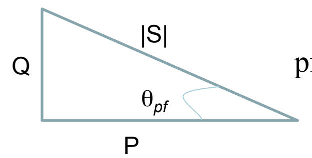
Complex Power

Complex Power

Complex power is the complex sum of average power and reactive power, or:

$$S = P + jQ = |S| \angle \theta_{pf}$$



$$pf = \cos\theta_{pf}$$

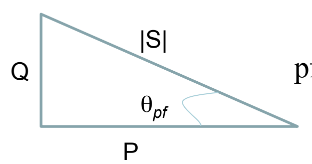
Notes:

- |S| is apparent power
- θ_{pf} is the power factor angle
- Units for both complex power and apparent power are volt-amperes [VA]

Complex Power

- Apparent power |S|

$$|S| = \sqrt{P^2 + Q^2}$$



$$pf = \cos\theta_{pf}$$

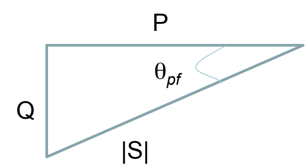
- Power factor angle

$$\frac{Q}{P} = \frac{(V_m I_m / 2)\sin(\theta_v - \theta_i)}{(V_m I_m / 2)\cos(\theta_v - \theta_i)} = \tan(\theta_v - \theta_i) = \tan\theta_{pf}$$

Activity 1

The power triangle that characterizes the impedance in a circuit is shown below. In its simplest form, the impedance is

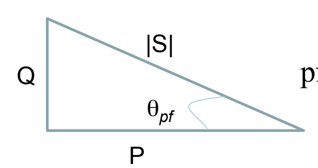
A. Purely resistive
B. RL
C. RC
D. Purely capacitive



Activity 2

The power triangle shown here represents the power for which type of load?

A. Purely resistive
B. RL
C. RC
D. Can't tell from the triangle



$\text{pf} = \cos\theta_{pf}$

Example

An electrical motor operates at 240 V rms. The average power is 8 kW at a lagging power factor of 0.8.

a. Calculate the complex power of the motor.

b. Calculate the impedance of the motor.

Solution

A. Calculate the complex power

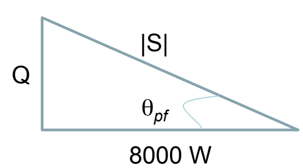
Given power factor is lagging, the load is inductive, i.e. Q>0.

$$\theta_{pf} = \cos^{-1} \text{pf} = \cos^{-1} 0.8 = 36.87°$$

$$P = 8000$$

$$Q/8000 = \tan 36.87°$$

$$\Rightarrow \quad Q = 6000 \text{ var}$$



$$|S| = \sqrt{P^2 + Q^2} = \sqrt{8000^2 + 6000^2}$$

$$= 10{,}000 \text{ VA}$$

$$S = 8000 + j6000 \text{ VA} = 10{,}000\angle 36.87° \text{ VA}$$

Solution

B. Calculate the load impedance

How to find the load impedance, let's start from its definition.

$$Z_L = \frac{\mathbf{V}_L}{\mathbf{I}_L} = \frac{|\mathbf{V}_L|}{|\mathbf{I}_L|}\angle(\theta_v - \theta_i) = \frac{|\mathbf{V}_L|}{|\mathbf{I}_L|}\angle\theta_{pf} = \frac{|\mathbf{V}_L|}{|\mathbf{I}_L|}\angle 36.87^\circ$$

$|\mathbf{V}_L| = 240\ \mathrm{V}_{rms}$ but what about $|\mathbf{I}_L| = I_{rms}$?

$$P = V_{rms} I_{rms} \mathrm{pf} \quad \Rightarrow \quad I_{rms} = \frac{P}{V_{rms}\mathrm{pf}} = \frac{8000}{(240)(0.8)} = 41.67\ \mathrm{A_{rms}}$$

$$\therefore |Z_L| = \frac{V_{rms}}{I_{rms}} = \frac{240}{41.67} = 5.76\Omega \quad \Rightarrow \quad Z_L = 5.76\angle 36.87^\circ \Omega = 4.61 + j3.46\Omega$$

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Power Calculation

Power calculation

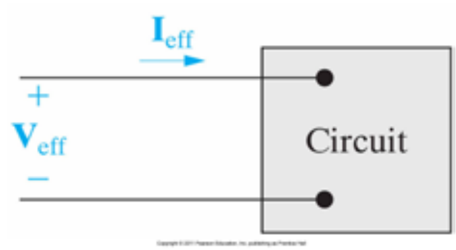
$$S = (V_m I_m / 2)\cos(\theta_v - \theta_i) + j(V_m I_m / 2)\sin(\theta_v - \theta_i)$$

$$S = \frac{V_m I_m}{2}\left[\cos(\theta_v - \theta_i) + j\sin(\theta_v - \theta_i)\right]$$

$$S = \frac{V_m I_m}{2} e^{j(\theta_v - \theta_i)} = \frac{V_m I_m}{2} \angle(\theta_v - \theta_i)$$

$$S = V_{rms}\angle\theta_v \times I_{rme}\angle - \theta_i$$

$$S = \mathbf{V}_{rms}\mathbf{I}^*_{rms} = \frac{1}{2}\mathbf{VI}^*$$



Alternative Forms of Complex Power

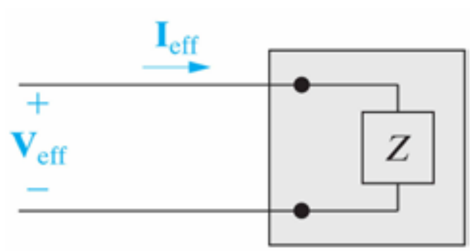
$$S = \mathbf{V}_{rms}\mathbf{I}^*_{rms}, V_{rms} = \mathbf{I}_{rms}Z$$

$$\rightarrow S = \mathbf{I}^*_{rms}\mathbf{I}_{rms}Z = |\mathbf{I}_{rms}|^2 Z$$

$$S = |\mathbf{I}_{rms}|^2 (R + jX) = |\mathbf{I}_{rms}|^2 R + j|\mathbf{I}_{rms}|^2 X = P + jQ$$

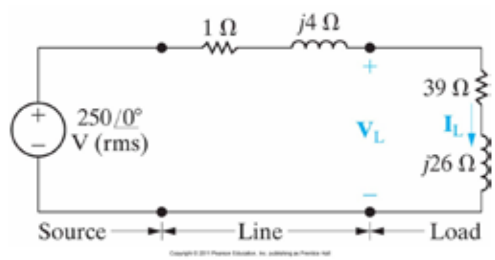
$$\rightarrow P = |\mathbf{I}_{rms}|^2 R = \frac{1}{2} I_m^2 R$$

$$\rightarrow Q = |\mathbf{I}_{rms}|^2 X = \frac{1}{2} I_m^2 X$$



Example

- Find $\mathbf{I}_L$ and $\mathbf{V}_L$
- Calculate S supplied by the source
- Calculate S delivered to the load
- Calculate S delivered to the line



Solution

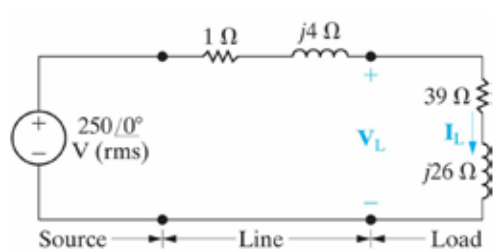
Find $\mathbf{I}_L$ and $\mathbf{V}_L$

$$\mathbf{I}_L = \frac{250\angle 0^\circ}{(1+j4)+(39+j26)} = \frac{250\angle 0^\circ}{40+j30}$$

$$= (4-j3)\text{ A} = 5\angle\text{-}36.87^\circ\text{ A(rms)}$$

$$\mathbf{V}_L = (39+j26)\mathbf{I}_L$$

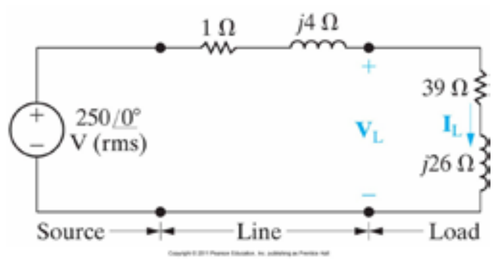
$$= (234-j13) = 234.36\angle\text{-}3.18^\circ\text{ V(rms)}$$



Solution

Calculate S delivered to the load

$$S = \mathbf{V}_L \mathbf{I}_L^* = (234 - j13)(4 + j3) = 975 + j650\,\text{VA}$$



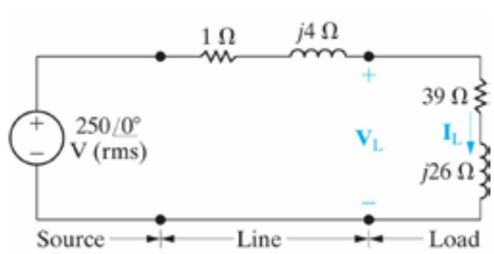
Solution

Calculate S delivered to the line

$$P = |\mathbf{I}_{rms}|^2 R = 5^2 \times 1 = 25W$$

$$Q = |\mathbf{I}_{rms}|^2 X = 5^2 \times 4 = 100VAR$$

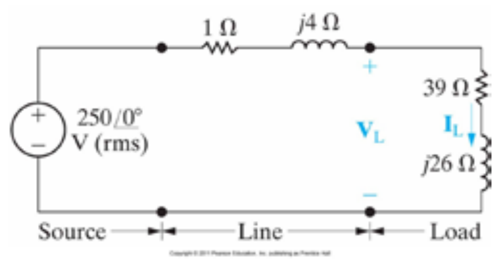
$$S = 25 + j100VA$$



Solution

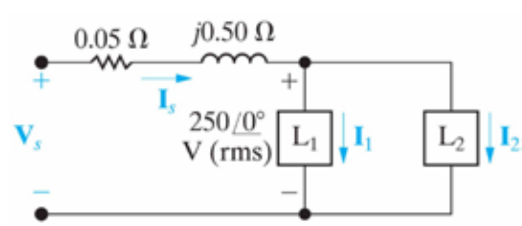
Calculate S supplied by the source

$$S = S_{load} + S_{line} = 975 + j650 + 25 + j100$$
$$= 1000 + j750\,\text{VA}$$



Activity 3

Load 1 absorbs an average power of 8 KW and leading pf 0.8. Load 2 absorbs 20 kVA at lagging pf 0.6. Assume effective values for all.



- Find the pf of the 2 loads in parallel.
- Find magnitude of I_s and the apparent power to supply the load.
- Assuming 60 Hz, what is the capacitor required to correct the power factor

Solution

1. Find pf of 2 loads in parallel

$$\mathbf{I}_s = \mathbf{I}_1 + \mathbf{I}_2$$

$$S = 250\mathbf{I}_s^* = 250(\mathbf{I}_1 + \mathbf{I}_2)^* = 250\mathbf{I}_1^* + 250\mathbf{I}_2^* = S_1 + S_2$$

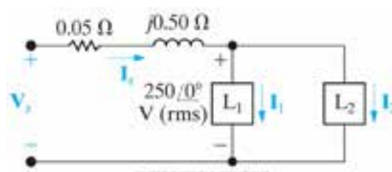
$$pf_1 = \cos\theta_1 = 0.8 \rightarrow \theta_1 = -36.87^\circ, \sin\theta_1 = -0.6$$

$$pf_2 = \cos\theta_2 = 0.6 \rightarrow \theta_2 = 53.13^\circ, \sin\theta_2 = 0.8$$

$$S_1 = P_1 + jQ_1 = 8000 + j\frac{8000 \times (-0.6)}{0.8} = 8000 - j6000VA$$

$$S_2 = P_2 + jQ_2 = 20000 \times 0.6 + j20000 \times 0.8$$

$$= 12000 + j16000VA$$



Solution

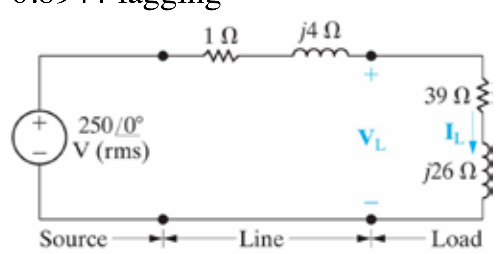
$$\because S_1 = 8000 - j6000VA, S_2 = 12000 + j16000VA$$

$$\therefore S = 20000 + j10000VA$$

$$\mathbf{I}_s^* = \frac{S}{250} = \frac{20000 + j10000}{250} = 80 + j40A$$

$$\mathbf{I}_s = 80 - j40A = 89.44\angle -26.57^\circ A$$

$$pf = \cos(0 - (-26.57^\circ)) = 0.8944 \text{ lagging}$$



Solution

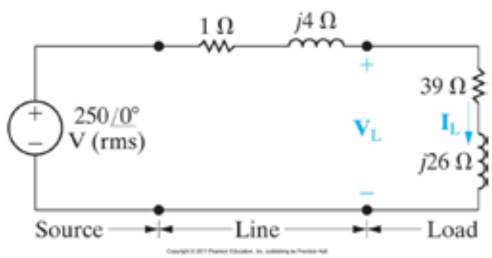
2. Find magnitude of I_s and apparent power,

$$\because \mathbf{I}_s = 80 - j40A$$

$$\therefore \left|\mathbf{I}_s\right| = |80 - j40| = \sqrt{80^2 + 40^2} = 89.44A$$

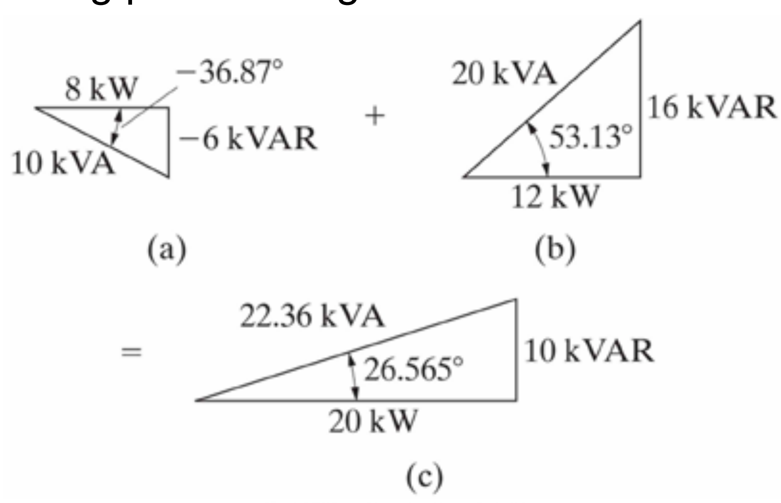
$$\because S = 20000 + j10000$$

$$\therefore |S| = |20000 + j10000| = \sqrt{20000^2 + 10000^2} = 22.36KVA$$



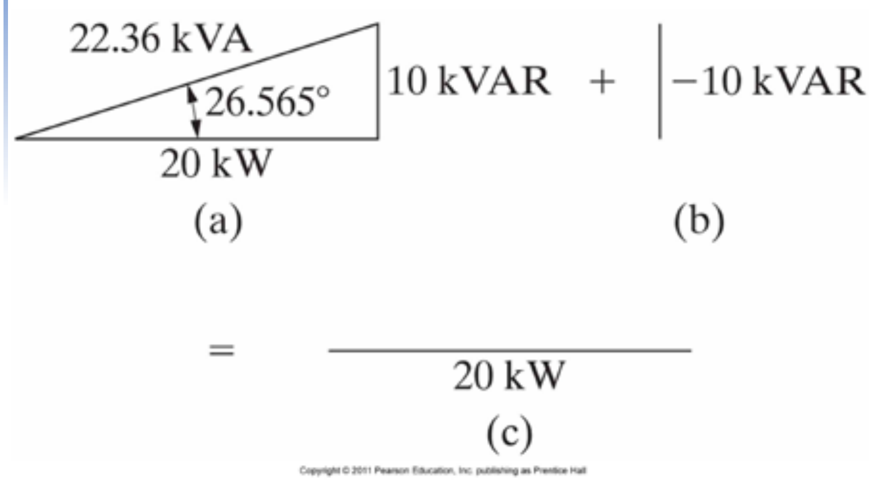
Solution

Using power triangle



Solution

3. Assuming 60 Hz, what is the capacitor required to correct the power factor



Solution

3. Assuming 60 Hz, what is the capacitor required to correct the power factor

$$Q = \frac{|V_{eff}|^2}{X} = -10000$$

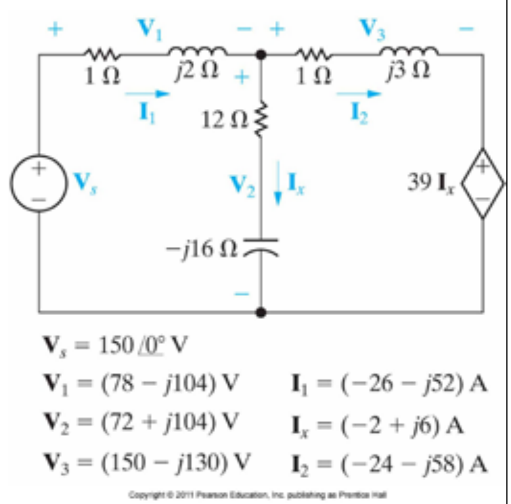
$$X = \frac{250^2}{-10000} = -6.25\Omega, X = \frac{-1}{\omega C}$$

$$C = \frac{-1}{\omega X} = \frac{-1}{2\pi f X} = \frac{1}{2 \times 3.1416 \times 60 \times 6.25} = 424.4\mu F$$

Example: Power Balance

A. Find the total average and reactive power delivered to each impedance.

B. Find the average and reactive powers for each source.



Solution: Power Balance

A. Find the total average and reactive power delivered to each impedance.

$$S_1 = \frac{1}{2}\mathbf{V}_1\mathbf{I}_1^* = \frac{1}{2}(78 - j104)(-26 - j52)$$

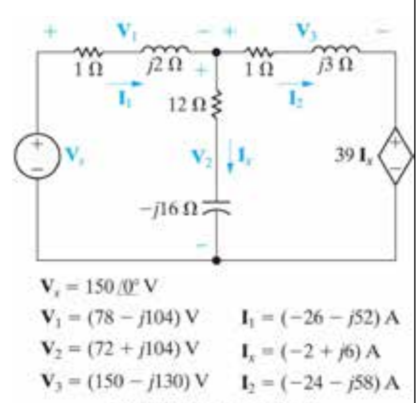
$$= 1690 + j3380VA$$

$$S_2 = \frac{1}{2}\mathbf{V}_2\mathbf{I}_x^* = \frac{1}{2}(72 + j104)(-2 + j6)$$

$$= 240 - j320VA$$

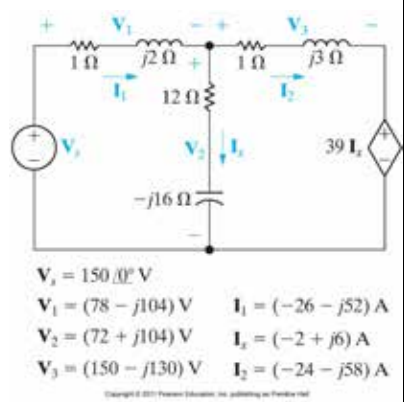
$$S_3 = \frac{1}{2}\mathbf{V}_3\mathbf{I}_2^* = \frac{1}{2}(150 - j130)(-24 + j58)$$

$$= 1970 + j5910VA$$



Solution: Power Balance

B. Find the average and reactive powers for each source.



$$S_s = -\frac{1}{2}\mathbf{V}_s\mathbf{I}_1^* = -\frac{1}{2}(150)(-26 - j52)$$

$$= 1950 - j3900VA$$

$$S_x = \frac{1}{2}(39\mathbf{I}_x)\mathbf{I}_2^*$$

$$= \frac{1}{2}(-78 + j234)(-24 + j58)$$

$$= -5850 - j5070VA$$

Solution: Power Balance

Verify power balance:

1. Total power absorbed by the passive impedances and the independent source.

$$P_{absorbed} = P_1 + P_2 + P_3 + P_s = 1690 + 240 + 1970 + 1950 = 5850W$$

2. Total power delivered (by dependent source)

$$P_{delivered} = P_x = -5850W$$

$$P_{absorbed} = P_{delivered}$$

3. Reactive power absorbed:

$$Q_{absorbed} = Q_1 + Q_3 = 3380 + 5910 = 9290VAR$$

4. Reactive power delivered:

$$Q_{delivered} = Q_2 + Q_s + Q_x = 320 + 3900 + 5070 = 9290VAR$$

$$Q_{absorbed} = Q_{delivered}$$

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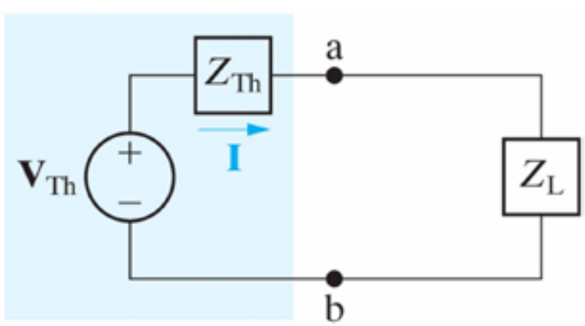
Maximum Power Transfer

Maximum Power Transfer

- Assume the source is replaced by its Thevenin equivalent circuits.
- V_{TH}, Z_{TH} and a load or Z_L is connected



Maximum Power Transfer

$$I = \frac{V_{TH}}{(R_{TH}+R_L)+j(X_{TH}+X_L)}$$

$$P = |I|^2 R_L$$

$$P = \frac{|V_{TH}|^2 R_L}{(R_{TH}+R_L)^2+(X_{TH}+X_L)^2}$$

$$\frac{\partial P}{\partial R_L} = 0, \frac{\partial P}{\partial X_L} = 0$$

$$X_L = X_{TH}, R_L = \sqrt{R_{TH}^2+(X_L+X_{TH})^2}$$

$$Z_L = Z_{TH}^*$$

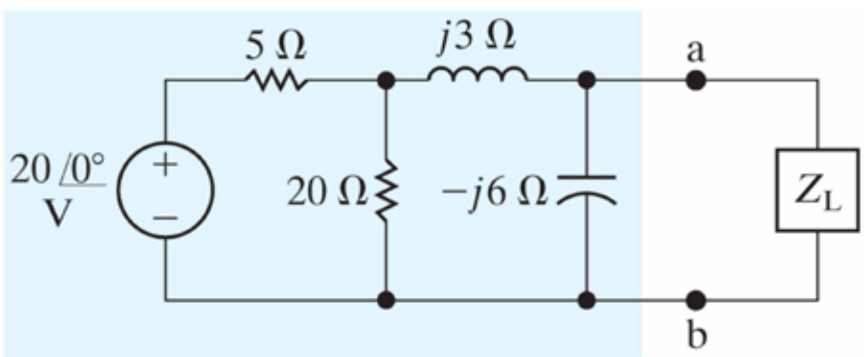
Restriction on Z_L

- R_L and X_L may restricted to a limited range of values.
  - Set X_L as near to $-X_{TH}$ as possible
  - Set R_L as close to $R_L = \sqrt{R_{TH}^2+(X_L+X_{TH})^2}$ as possible
- The magnitude of Z_L can be varied but its phase angle cannot.
  - Set $|Z_L|=|Z_{TH}|$

Example

Determine Z_L that results in maximum average power transferred to Z_L.

Find the maximum average power.



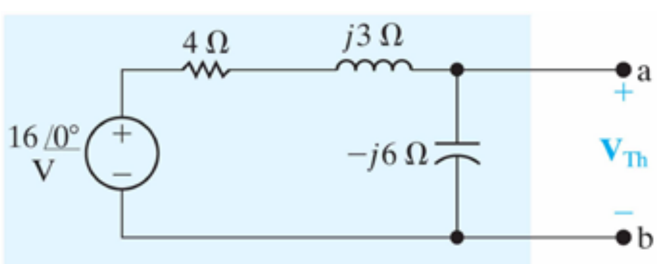
Solution

A. Determine the Z_L.

First, perform source transformation, then calculate V_{Th}

$$V_{Th} = \frac{-j6}{4 + j3 - j6} \times 16 = \frac{16(18 - j24)}{25} = \frac{96}{5}(0.6 - j0.8)$$

$$= 19.2\angle -53.13^{\circ} = 11.52 - j15.36V$$



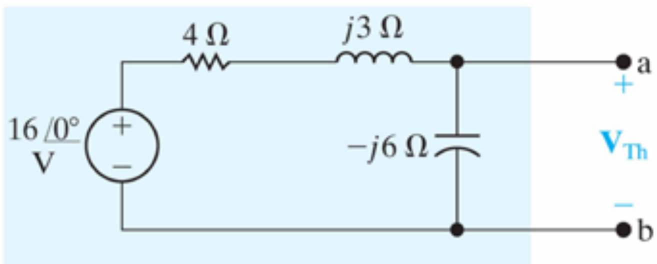
Solution

A. Determine the $Z_{L.}$

Calculate $Z_{Th,}$ deactivate voltage source

$$Z_{Th} = (4 + j3) // (-j6) = \frac{(4 + j3)(-j6)}{4 + j3 - j6} = \frac{6(24 - j7)}{25}$$

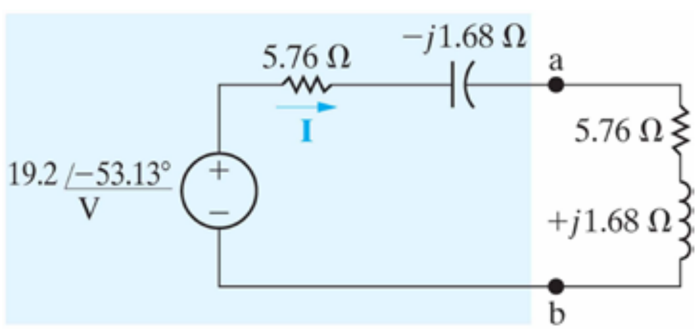
$$= 5.76 - j1.68\Omega$$



Solution

$$\because Z_{Th} = 5.76 - j1.68\Omega$$

$$\therefore Z_L = Z_{Th}^{*} = 5.76 + j1.68\Omega$$

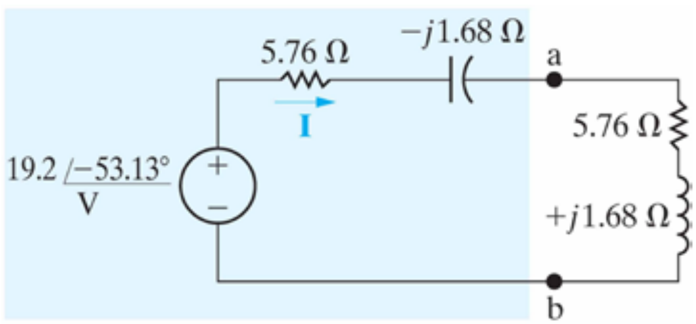


Solution

B. Find the maximum power

$$I_{eff} = \frac{19.2/\sqrt{2}}{5.76+5.76} A$$

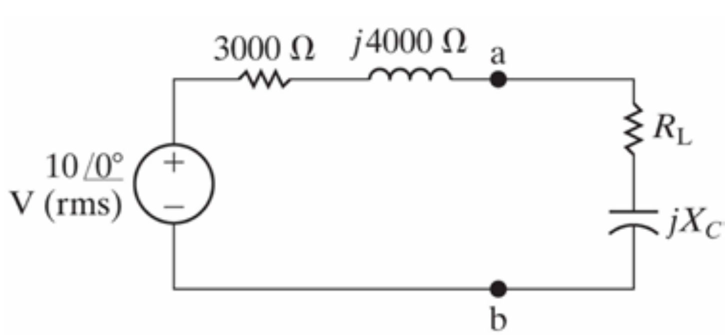
$$P = I_{eff}^2(5.76) = \frac{19.2^2}{2\times 4\times 5.76} = 8W$$



Example: Load with Restriction

A. Find Z_L for maximum average power.

B. If R_L can be varied from 0 to 4000Ω and X_C is restricted to 0~-2000Ω, what settings of R_L and X_C transfer the most average power?



Solution: Load with Restriction

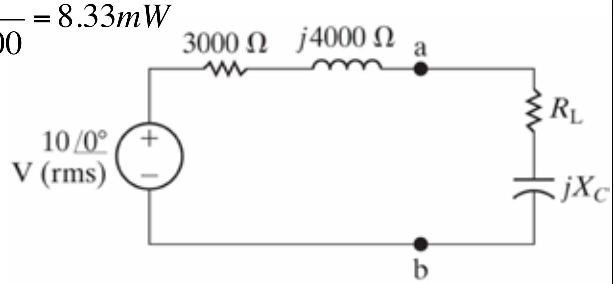
A. Find Z_L for maximum average power.

$$\because Z_{Th} = (3000 + j4000)\Omega$$

$$\therefore Z_L = Z_{Th}^* = 3000 - j4000\Omega$$

$$I_{eff} = \frac{10}{2 \times 3000} A$$

$$P_{\max} = \frac{I_{eff}^2}{3000} = \frac{10^2}{4 \times 3000} = 8.33mW$$



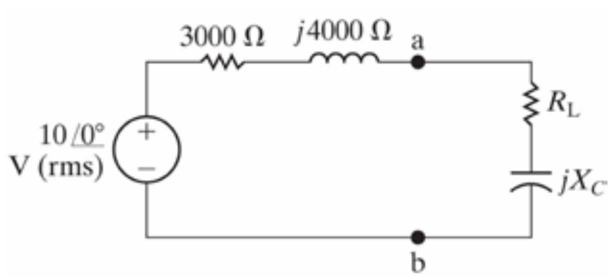
Solution: Load with Restriction

B. If R_L can be varied from 0 to 4000 ohm and X_C is restricted to 0~-2000ohm, what settings of R_L and X_C transfer the most average power?

First, set X_L as close to -4000Ω as possible, XL=-2000Ω

$$R_L = \sqrt{R_{Th}^2 + (X_L + X_{Th})^2} = \sqrt{3000^2 + (-2000 + 4000)^2} = 3605.55\Omega$$

$$Z_L = 3605.55 - j2000\Omega$$



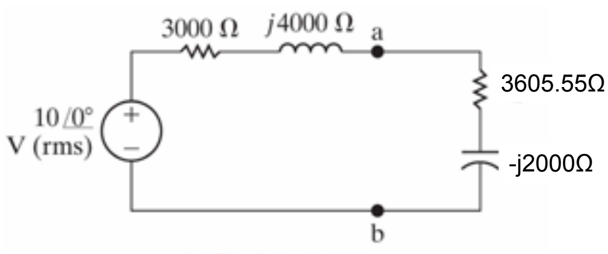
Solution: Load with Restriction

B. If R_L can be varied from 0 to 4000 ohm and X_C is restricted to 0~-2000ohm, what settings of R_L and X_C transfer the most average power?

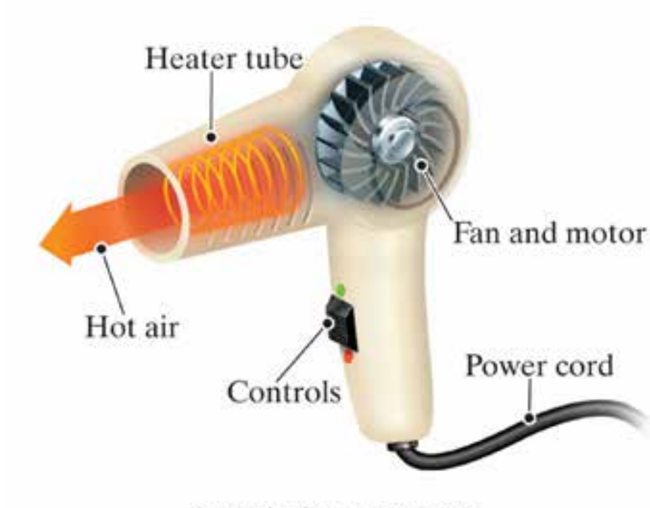
First, set X_L as close to -4000Ω as possible, XL=-2000Ω

$$I_{eff} = \frac{10\angle 0^\circ}{6605.55 + j2000} = 1.4489\angle -16.85^\circ mA$$

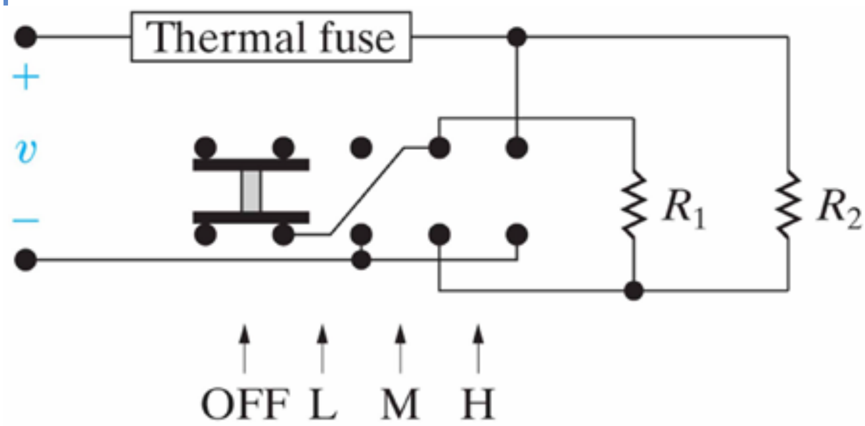
$$P = (1.4489\angle -16.85^\circ)^2 \times 3605.55 \approx 7.57mW$$



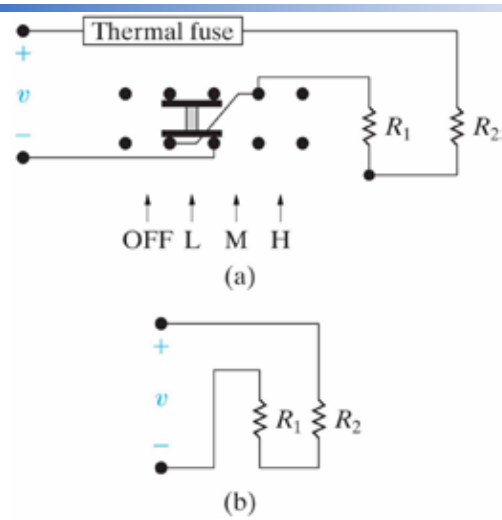
A Handheld Hair Dryer



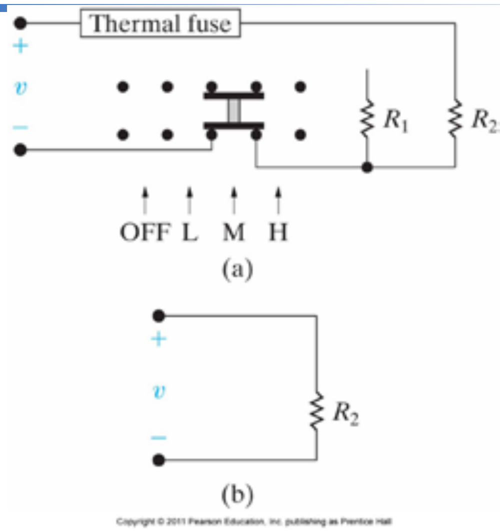
A circuit diagram for the hair dryer



Equivalent Circuit



Equivalent Circuit



Equivalent Circuit