EECS2200 Electric Circuits

Chapter 8 Part 3

Sinusoidal Steady State Power Calculation

EECS2200 Electric Circuits

Transformer

Analysis of Linear Transformer Circuit

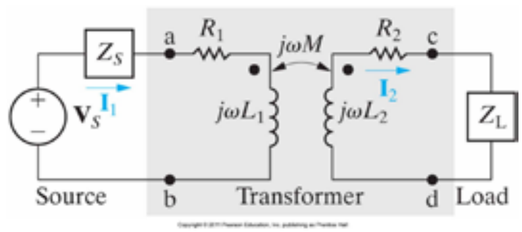
$$V_s = I_1(Z_s + R_1 + j\omega L_1) - I_2 j\omega M$$
$$0 = -j\omega M I_1 + I_2(R_2 + j\omega L_2 + Z_L)$$

$$j\omega M I_1 = I_2(R_2 + j\omega L_2 + Z_L)$$

$$I_2 = \frac{j\omega M I_1}{(R_2 + j\omega L_2 + Z_L)}$$

$$Z_{11} = Z_s + R_1 + j\omega L_1$$

$$Z_{22} = Z_L + R_2 + j\omega L_2$$



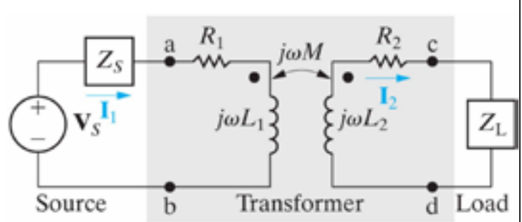
Analysis of Linear Transformer Circuit

$$V_s = I_1(Z_s + R_1 + j\omega L_1) - I_2 j\omega M$$
$$0 = -j\omega M I_1 + I_2(R_2 + j\omega L_2 + Z_L)$$

$$I_1 = \frac{Z_{22}}{Z_{11}Z_{22} + \omega^2 M^2} V_s$$

$$I_2 = \frac{j\omega M}{Z_{22}} I_1 = \frac{j\omega M}{Z_{11}Z_{22} + \omega^2 M^2} V_s$$

$$\frac{V_s}{I_1} = Z_{\text{int}} = Z_{11} + \frac{\omega^2 M^2}{Z_{22}}$$



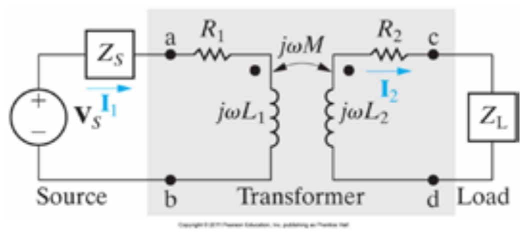
Analysis of Linear Transformer Circuit

$$Z_{ab} = Z_{\text{int}} - Z_s = Z_{11} + \frac{\omega^2 M^2}{Z_{22}} - Z_s$$

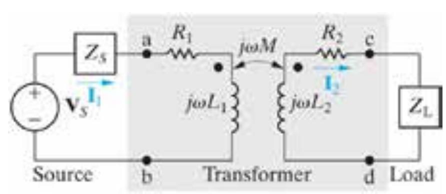
$$= Z_s + R_1 + j\omega L_1 + \frac{\omega^2 M^2}{Z_{22}} - Z_s$$

$$= R_1 + j\omega L_1 + \frac{\omega^2 M^2}{Z_{22}}$$

Reflected Impedance



Reflected Impedance



$$Z_{reflected} = \frac{\omega^2 M^2}{Z_{22}}$$

$$Z_{reflected} = \frac{\omega^2 M^2}{R_2 + j\omega L_2 + Z_L}$$

$$Z_{reflected} = \frac{\omega^2 M^2}{R_2 + j\omega L_2 + R_L + j\omega X_L}$$

$$Z_{reflected} = \frac{\omega^2 M^2}{(R_2 + R_L) + j\omega(L_2 + X_L)}$$

$$Z_{reflected} = \frac{\omega^2 M^2}{|Z_{22}|^2}\left[(R_2 + R_L) - j\omega(L_2 + X_L)\right]$$

The complex conjugate of the self impedance of the secondary circuit scaled by a factor

Example

The parameters of a linear transformer are R1=200Ω, R2=100Ω, L1=9H, L2=4H, k=0.5. The transformer couples an impedance consists of an 800Ω resister in series with a 1μF capacitor to a sinusoidal voltage source. The 300V(rms) source has an internal impedance of 500+j100Ω and a frequency of 400 rad/s.

Note: $M = k\sqrt{L_1 L_2}$

A. Construct a frequency-domain equivalent circuit.

B. Find the impedance seen looking into the primary terminals of the transformer.

C. Find the Thevenin equivalent with respect to the terminals of load.

Solution

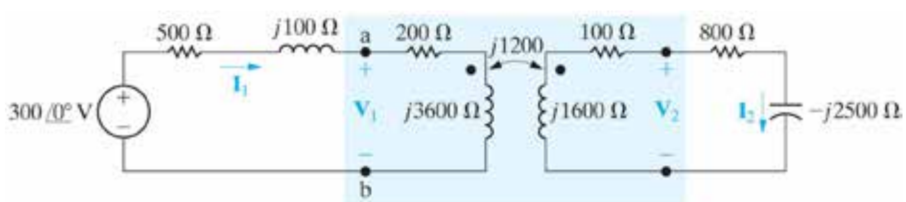
A. Frequency domain equivalent circuit.

Given: R1=200Ω, R2=100Ω, L1=9H, L2=4H, k=0.5, C=1μF, ω=400rad/s

$$j\omega L_1 = j400 \times 9 = j3600\Omega, j\omega L_2 = j400 \times 4 = j1600\Omega$$

$$M = k\sqrt{L_1 L_2} = 0.5\sqrt{9 \times 4} = 3H, j\omega M = j400 \times 3 = j1200\Omega$$

$$\frac{1}{j\omega C} = -j\frac{10^6}{400 \times 1} = -j2500\Omega$$



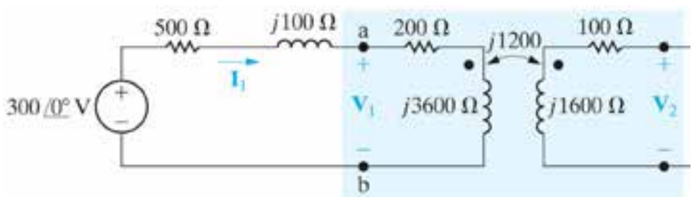
Solution

B. Find the Thevenin equivalent.

$$V_{Th} = V_2 = I_2 Z_{22} = \frac{j\omega M}{Z_{22}} I_1 Z_{22} = j\omega M I_1 = j1200 I_1$$

$$I_1 = \frac{300\angle 0^\circ}{700 + j3700} = 79.67\angle -79.29^\circ \, mA$$

$$\therefore V_{Th} = j1200 \times 79.67\angle -79.29^\circ \times 10^{-3} = 95.60\angle 10.71^\circ V$$



Solution

The Thevenin impedance will be equal to the impedance of secondary winding plus the impedance reflected from the primary when the voltage source is replaced by a short-circuit.

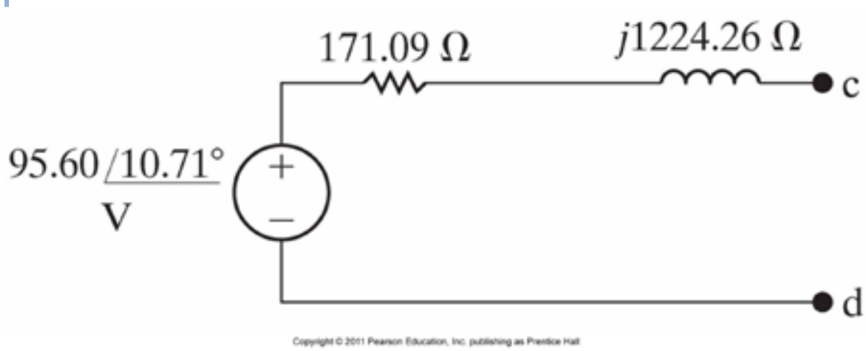
$$R_{Th} = R_2 + j\omega L_2 + \frac{\omega^2 M^2}{|Z_{11}|^2}$$

$$= 100 + j1600 + \frac{(1200)^2}{|700 + j3700|^2}(700 - j$$

$$= 171.09 + j1224.26\Omega$$

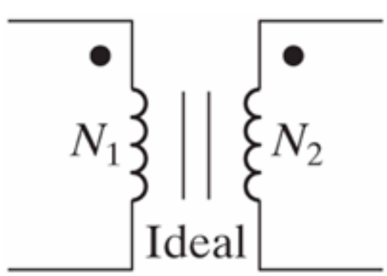
Solution

The Thevenin equivalent circuit



Ideal Transformer

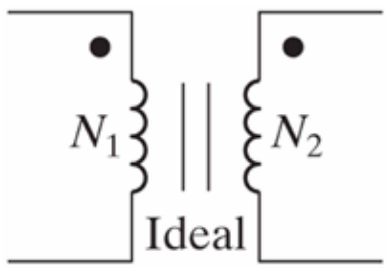
- Two magnetically coupled coils having N_1 and N_2 turns, respectively, and exhibiting three properties
  - The coefficient of coupling is unity (k=1)
  - The self-inductance of each coil is infinite ($L_1=L_2=\infty$)
  - The coil losses are negligible.



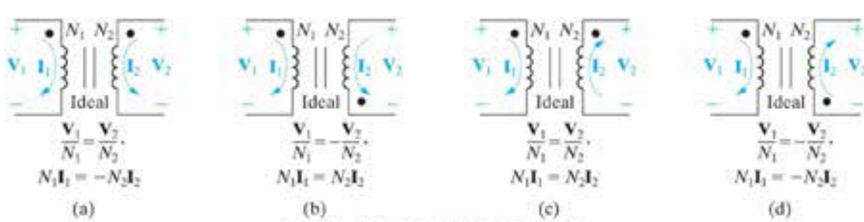
Voltage and Current Ratio

- Voltage ratio: $\dfrac{V_1}{N_1} = \dfrac{V_2}{N_2}$

- Current ratio: $I_1 N_1 = I_2 N_2$

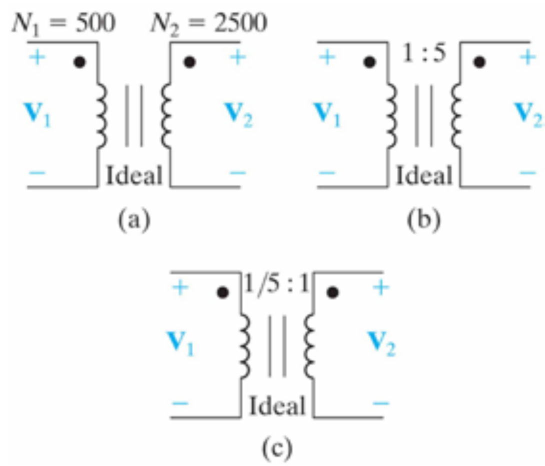


Polarity of the Voltage and Current



Example

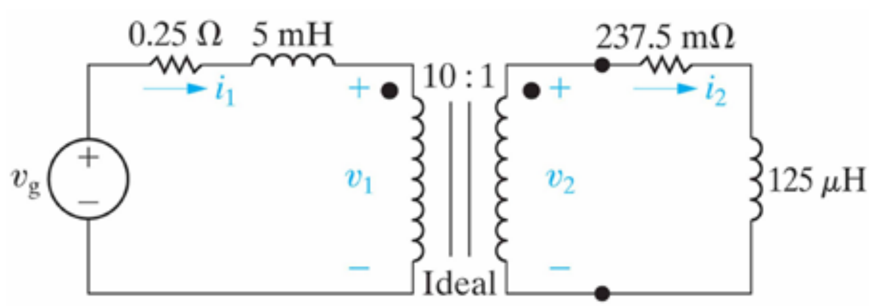
- Three ways to show that the turns ratio of an ideal transformer is 5.



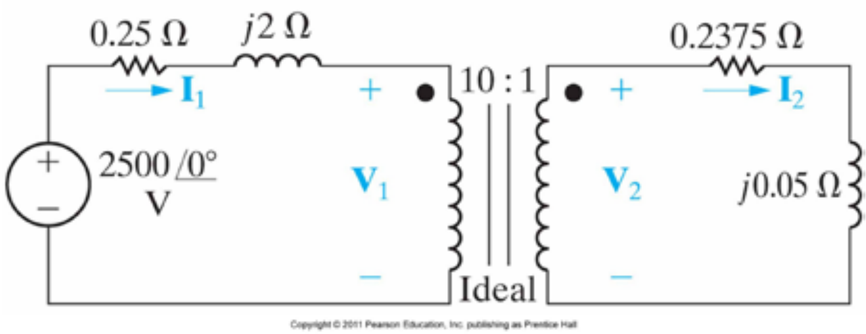
Example

Transfer the following circuit to phasor domain circuit

Given Vs=2500cos(400t) V

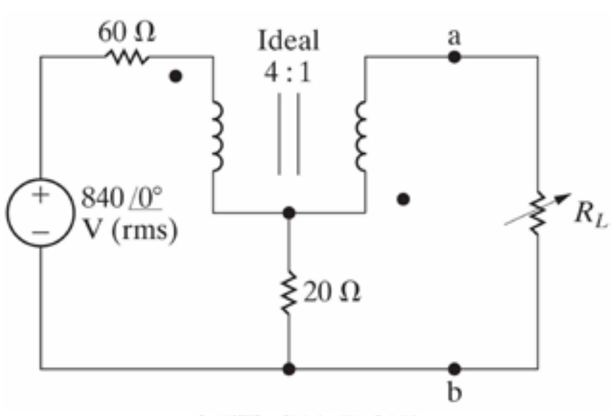


Example: Phasor Domain Circuit



Activity: Max Power Transfer

The variable resistor is adjustable until maximum average power is delivered to R_L. What is the value of R_L?



Solution

First find the Thévenin equivalent.
For ideal transformer, we have:

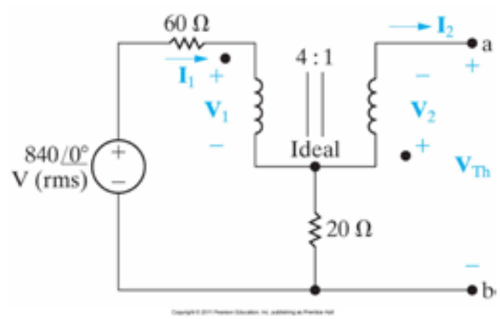
$$V_2 = \frac{1}{4}V_1, \qquad I_1 = -\frac{1}{4}I_2$$

Open circuit $I_2=0$, so $I_1=0$

$$\because V_1 = 840\angle 0^\circ V$$

$$\therefore V_2 = \frac{1}{4}V_1 = 210\angle 0^\circ V$$

$$V_{Th} = -V_2 = -210\angle 0^\circ V$$



Solution

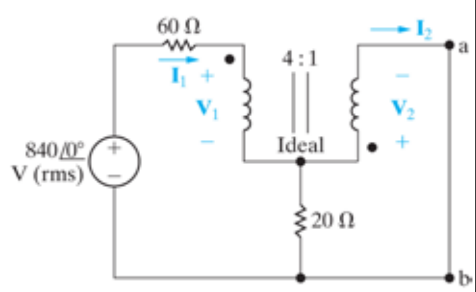
To find the Thévenin impedance, short-circuit “ab” to calculate I_2.

$$\begin{cases} 840\angle 0^\circ = 60I_1 + V_1 + 20(I_1 - I_2) \\ 0 = 20(I_2 - I_1) + V_2 \end{cases}$$

Note: $I_1 = -\frac{1}{4}I_2$

$$\begin{cases} 840\angle 0^\circ = -40I_2 + V_1 \\ 0 = 25I_2 + \dfrac{V_1}{4} \end{cases}$$

$$I_2 = -6A \rightarrow R_{Th} = \frac{V_{Th}}{I_2} = \frac{-210\angle 0^\circ}{-6} = 35\Omega$$



Solution

Max power will be delivered to R_L when R_L equals to 35Ω