York University
Dept. of Electrical Engineering and Computer Science

A laboratory Manual for Electric Circuits Lab
EECS2200

Fall 2015-2016

ACKNOWLEDGEMENT

Prof Mokhtar Aboelaze developed this manual for EECS2200. Mr. Konstantin Bolshakov, who took this course in 2013, invested a lot of time and energy to improve this manual. Mr. Syed Islam updated this manual in Fall 2015.

PREFACE

This laboratory manual is intended for use in EECS2200 Electric Circuits. Every care was taken in preparing this manual, however no one is perfect. If you find any typos or errors in this manual, please contact the course director.

To the student:

The objective of this lab is to get you familiar with the instruments used in electric and electronic circuits measurements. It will introduce you to the concept of "lab book" and to how to design, implement and test simple electric circuits.
The lab will be done in groups of 2. Each lab consists of 2 parts. The prelab part will be done before you arrive to the lab. It will be submitted at the beginning of every lab. Then you have to do the experiment and record results. At the beginning of the next lab you should submit the lab report for the previous lab.
Each lab covers a specific topic in the course that will be clear from the lab title. It is your responsibility to read the theoretical part from the text book and the course notes before you go to the lab.
After you connect the circuit on the breadboard, check with the TA before connecting power. Please read the safety rules and troubleshooting hints before you start your first lab. Please be alert and use common sense during the experiment.
You have to maintain a laboratory book or journal, the TA must sign each page before you leave the lab. The journal will be checked once or twice during the term in order to be sure that you successfully did that part. Tips for maintaining a good journal are explained in this manual, please read "Appendix A: Laboratory Notebook" carefully.
The TA is there to help you, if you have any question ask the TA. A simple question might save you a lot of time and trouble later. Remember, you are dealing with expensive equipment.

To the TA:

Please read the experiment before you come to the lab. In the lab you have to approve the schematic diagram and the circuit connection before the students power up the experiment. Your job is to prevent any accidental mishaps that might injure students or destroy any equipment.

To the course Director:

The course director's responsibility is to be sure that the lab is properly equipped, the TA is qualified to run the lab, the marked reports are returned to the students in a timely manner, and supervising the TA.

Lab 3: Responses of RLC Circuits

OBJECTIVE:

- Understand the response of an RLC circuit.
- Understand the difference between over damped, under damped and critically damped circuit.
- Implement an RLC circuit and display the step response of the circuit on the scope

INTRODUCTION:

Consider the following Circuit

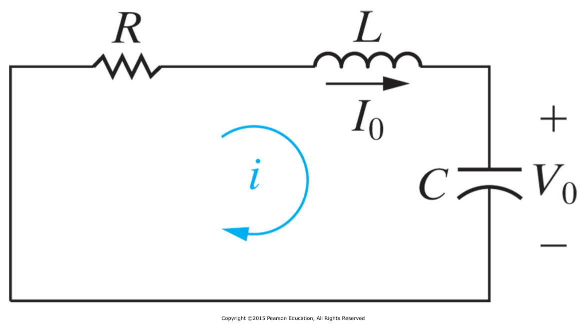

Figure L3.1 The natural response of an RLC circuit.

By applying KVL, we get

$$v_R + v_L + v_c = 0$$

$$Ri + L\frac{di}{dt} + \frac{1}{C}\int_0^t i\,d\tau + V_0 = 0$$

Differentiating with respect to t, we get

$$L\frac{d^2i}{dt^2} + R\frac{di}{dt} + \frac{1}{C}i = 0$$

$$\frac{d^2i}{dt^2} + \frac{R}{L}\frac{di}{dt} + \frac{1}{LC}i = 0$$

The characteristic equation for this differential equation is

$$s^2 + \frac{R}{L}s + \frac{1}{LC} = 0$$

$$s_{1,2} = -\frac{R}{2L} \pm \sqrt{\left(\frac{R}{2L}\right)^2 - \frac{1}{LC}}$$

$$s_{1,2} = -\alpha \pm \sqrt{\alpha^2 - \omega_0^2}$$

$$\alpha = \frac{R}{2L} \text{ rad/s}, \quad \omega_0 = \frac{1}{\sqrt{LC}} \text{ rad/s}$$

Another way to put this

$s_{1,2} = -\xi\omega_0 \pm \omega_0\sqrt{\xi^2 - 1}$ Where ω_0 is the resonant radian frequency, and ζ is the damping ratio, where

$$\omega_0 = \frac{1}{\sqrt{LC}} \quad , \quad \xi = \frac{R}{2}\sqrt{\frac{C}{L}}$$

The solution to the above equation depends on the value of ζ (relation of ω_0 to α)

CASE I (OVER DAMPED) ζ>1 or ($\alpha>\omega_0$)

The two roots are real numbers (s_1,s_2), the solution is

$i(t) = A_1e^{s_1t} + A_2e^{s_2t}$

CASE II (CRITICALLY DAMPED) ζ=1 or ($\alpha=\omega_0$)

Two identical real roots (σ)

$i(t) = D_1e^{\sigma t} + D_2te^{\sigma t}$

CASE III (Under DAMPED) ζ<1 or ($\alpha<\omega_0$)

Two complex conjugate roots, $s_{1,2} = -\xi\omega_0 \pm j\omega_0\sqrt{1-\xi^2} = -\sigma \pm j\omega_d$

$i(t) = e^{-\sigma t}\left(B_1\cos(\omega_d t) + B_2\sin(\omega_d t)\right)$

Once we know $i(t)$, the voltages across the different elements are easy to calculate, $v_R = i(t)R$, $v_L = L\dfrac{di}{dt}$, $v_C = -v_R - v_L$ or $v_C = -\dfrac{1}{C}\int_0^t i(\tau)d\tau + V_0$

We still need to determine the constants in the above equations (A1, A2, B1, B2, D1, D2). Determining the constants can be done from the initial conditions.

Two important rules, voltage across a capacitor and current in an inductor cannot change suddenly, that is to say

$v_C(0^-) = v_C(0^+)$, $i_L(0^-) = i_L(0^+)$

From these initial conditions we can solve to find the value of the 2 constants in the current equation.

For the natural response of an RLC circuit, the voltage across the capacitor can be found similarly.

$$v_c(t) = \begin{cases} A_1e^{s_1t} + A_2e^{s_2t} & \text{Overdamped} \\ e^{-\sigma t}\left(B_1\cos(\omega_d t) + B_1\sin(\omega_d t)\right) & \text{Underdamped} \\ D_1te^{-\alpha t} + D_2e^{-\alpha t} & \text{Critically damped} \end{cases}$$

For the step response, the solution is the same as the natural response with an added constant VF to reflect the forced response

$$v_c(t) = \begin{cases} V_f + A_1e^{s_1t} + A_2e^{s_2t} & \text{Overdamped} \\ V_f + e^{-\sigma t}\left(B_1\cos(\omega_d t) + B_1\sin(\omega_d t)\right) & \text{Underdamped} \\ V_f + D_1te^{-\alpha t} + D_2e^{-\alpha t} & \text{Critically damped} \end{cases}$$

PRELAB

Consider the circuit shown in Fig L3.2.

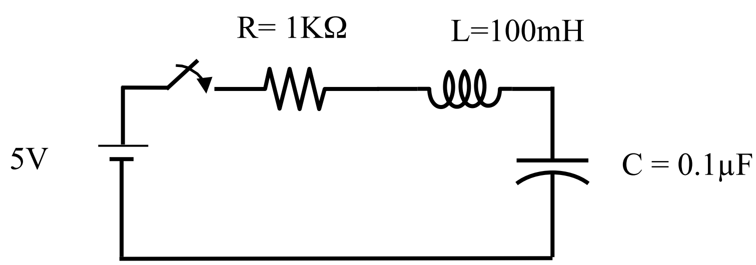

Figure L3.2 RLC Circuit

1. Solve the circuit to determine the voltage across the capacitor assuming no voltage or current at t=0. (Refer to Example 8.12 in textbook on page 288).
2. Using Matlab plot the voltage as a function of time.
3. What type of damping is that?
4. How long will it take until the voltage reaches: (1) 90% of the final value, and (2) 99% of the final value?
5. If you want to replace the voltage source and the switch by a square wave signal, what should be the frequency of the square wave?
6. What values for R will you suggest for the response to be: (1) overdamping (underdamping), (2) critically damped?

LAB:

In this part, you will construct an RLC circuit to understand the step response of an RLC circuit. Please follow the following steps.

1. Construct the circuit, as shown in Figure L3.2, on a breadboard. Replace the voltage source and switch with a square wave.
2. Based on your calculation in prelab, determine the frequency of the square wave.
3. Use the oscilloscope to observe the voltage across the capacitor.
4. What kind of damping is it?
5. Sketch the waveform in your notebook.
6. Change the resister value to make the circuit overdamped (or underdamped if the circuit you constructed in Step 1 is overdamped) based on your calculation in prelab. What is the value of the resister?
7. Observe the voltage across the capacitor using the oscilloscope. Is it overdamped (or underdamped)? Sketch the waveform in your notebook. (Note that you may need to adjust the frequency of square wave).