EECS2200 Electric Circuits

Chapter 4

Techniques for Circuit Analysis

Objectives

- Know how to use node-voltage method to solve a circuit
- Know how to use mesh-current method to solve a circuit.
- Be able to choose the appropriate circuit analysis method to use for a particular circuit
- Know how to use source transformation to simplify a circuit.
- Be able to calculate the Thevenin and Norton equivalents for a circuit.
- Understand and be able to use the condition for maximum power transfer to a load.

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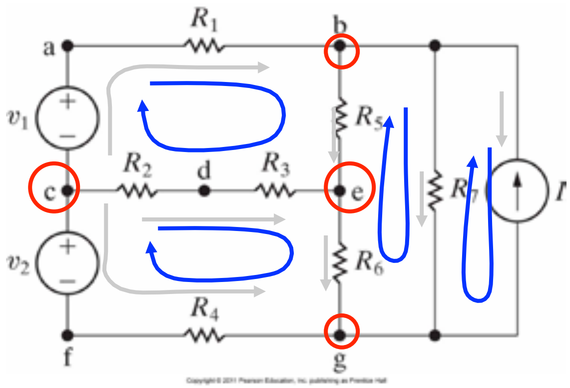
Terminology

Terminology

- **Node**: A point that connects two or more circuit elements are joined.
- **Essential node**: A node where 3 or more circuit elements are joined.
- **Path**: A trace of adjoining basic elements with no element included twice.
- **Branch**: A path that connects two nodes.
- **Essential path**: A path which connects two essential nodes without passing through an essential node.
- **Loop**: A path whose last node is the same as the first
- **Mesh**: A loop that does not contain any other loops
- **Planar circuit**: A circuit that could be drawn on a plane with no crossing branches.

Essential Nodes, Branches

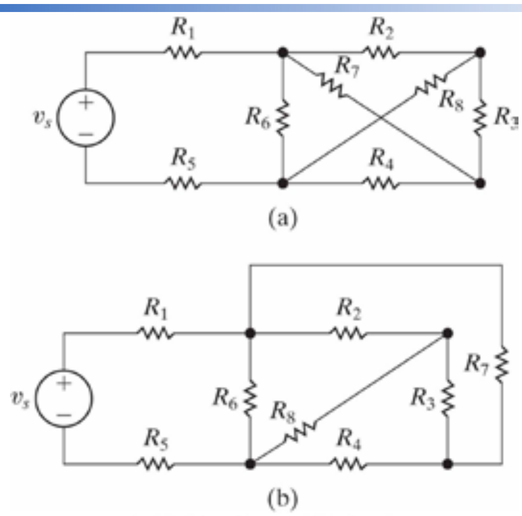


Essential nodes

Essential branches

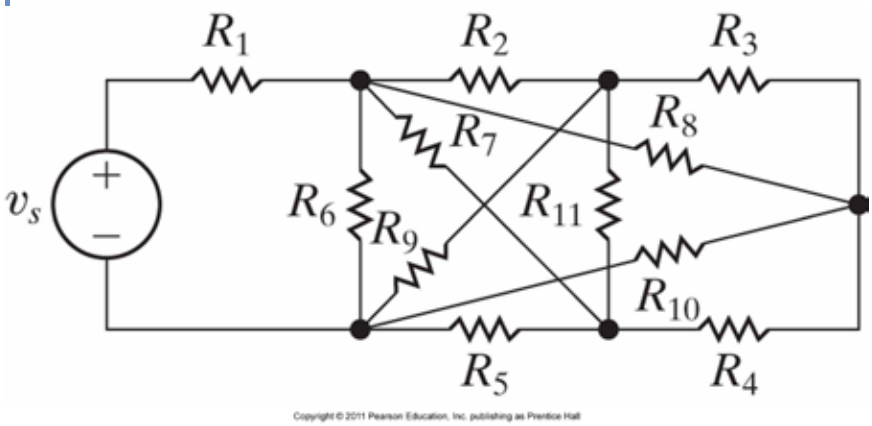
Meshes

Planar Circuit



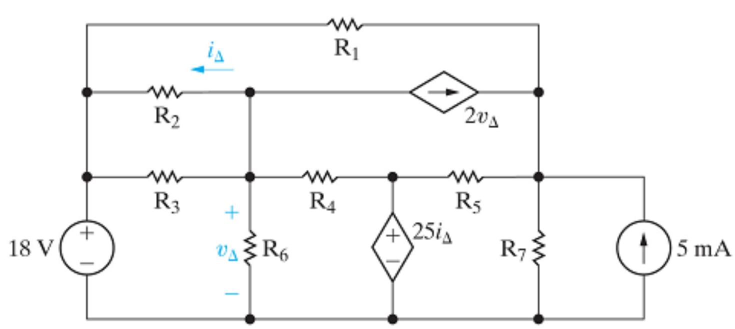
(a) and (b) are the same.

Non Planar Circuit



Activity 1

How many essential nodes does this circuit have?

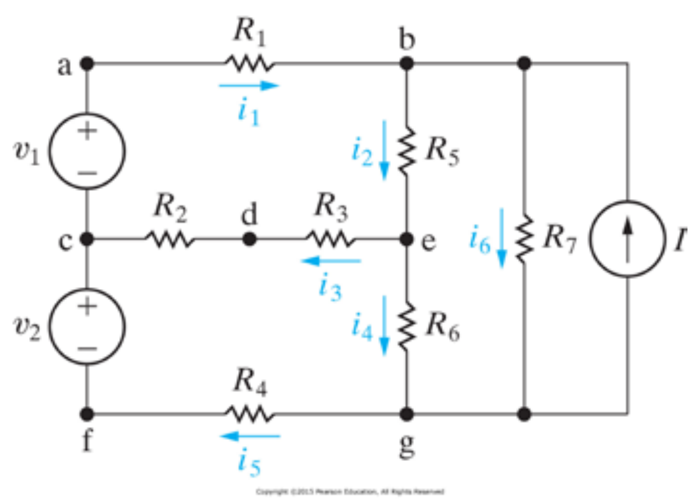


Simultaneous Equations

- In a circuit with *b* essential branches, and *n* essential nodes.
- We can get:
  - *n*-1 equations by applying KCL at *n*-1 nodes.
  - *b*-*n*+1 equations by applying KVL on loops or meshes.

Activity 2

Write simultaneous equations for below circuit.



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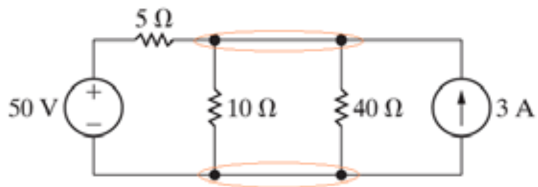
Node Voltage Method

Node-Voltage Method

- Based on writing KCL equations at essential nodes
- Solves for node voltages
- The "recipe":
  1. Identify the essential nodes
  2. Pick a reference node
  3. Label remaining essential nodes with voltage values
  4. Write a KCL equation at each non-reference essential node
  5. Put equations in standard form and solve
  6. Check your solutions by balancing power
  7. Calculate quantities of interest

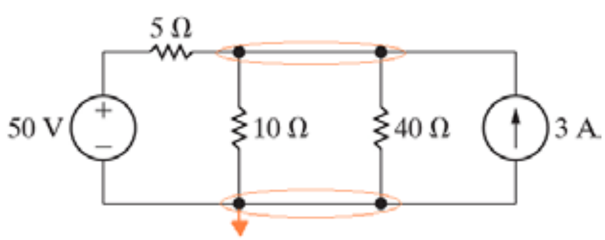
Node voltage method

Step 1 – identify the essential nodes



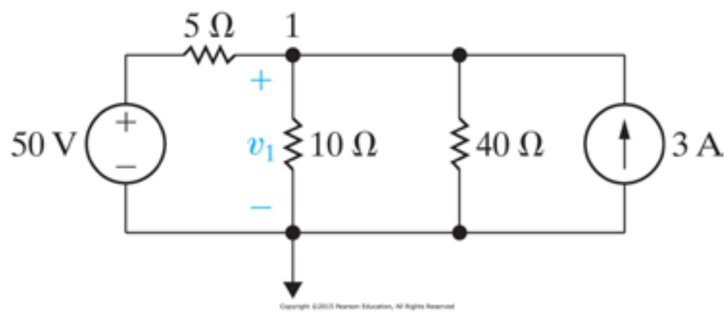
Node voltage method

Step 2 – pick a reference node



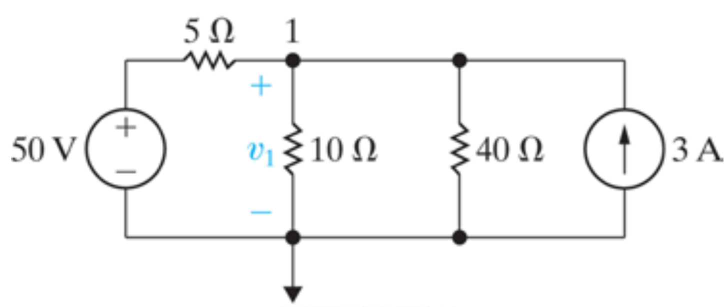
Node voltage method

Step 3 – label the remaining essential nodes with voltage values



Node voltage method

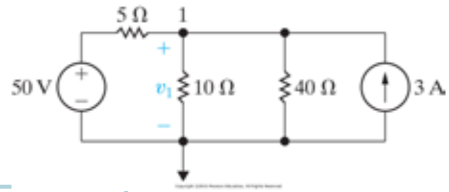
Step 4 – write a KCL equation at each non-reference essential node



At v: $\frac{v_1 - 50}{5} + \frac{v_1}{10} + \frac{v_1}{40} - 3 = 0$

Node voltage method

Step 5 – put the equations in standard form and solve



$$\frac{v_1-50}{5}+\frac{v_1}{10}+\frac{v_1}{40}-3=0$$

$$\Rightarrow \qquad v_1\left(\frac{1}{5}+\frac{1}{10}+\frac{1}{40}\right)=3+\frac{50}{5}$$

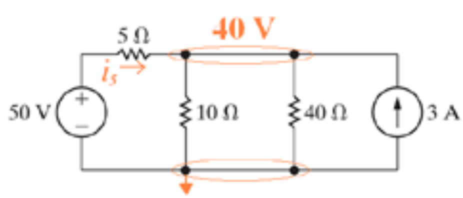
$$\Rightarrow \qquad (40)\left[v_1\left(\frac{1}{5}+\frac{1}{10}+\frac{1}{40}\right)\right]=(40)[3+10]$$

$$\Rightarrow \qquad v_1(8+4+1)=520$$

$$\Rightarrow \qquad v_1=520/13=40\text{ V}$$

Node voltage method

Step 6 – check your solutions



$$i_5=\frac{50-40}{5}=2\text{ A} \qquad p_{50}=-(50)(2)=-100\text{ W}$$

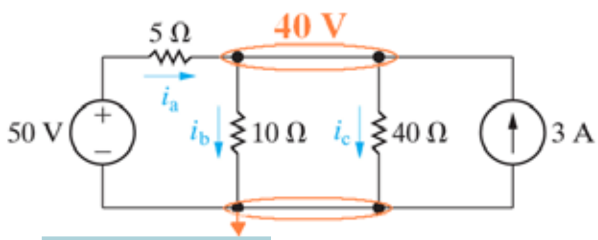
$$p_5=5(2)^2=20\text{ W} \qquad p_{10}=(40)^2/10=160\text{ W}$$

$$p_{10}=(40)^2/40=40\text{ W} \qquad p_3=-(40)(3)=-120\text{ W}$$

$$\sum p=-100+20+160+40-120=0$$

Node voltage method

Step 7 – calculate any other quantities of interest



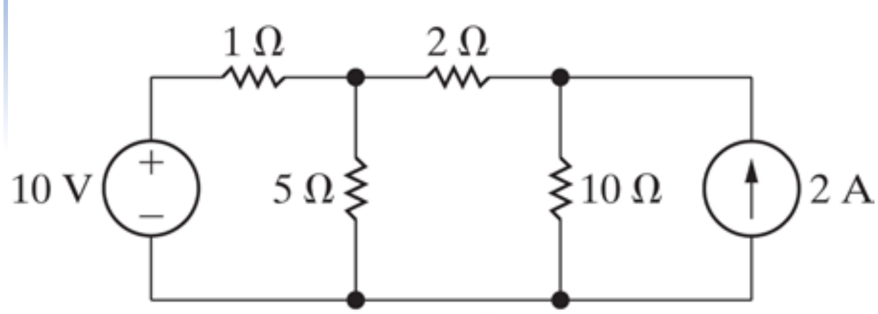
$$i_a = \frac{50-40}{5} = 2\ \text{A}$$

$$i_b = \frac{40}{10} = 4\ \text{A}$$

$$i_a = \frac{40}{40} = 1\ \text{A}$$

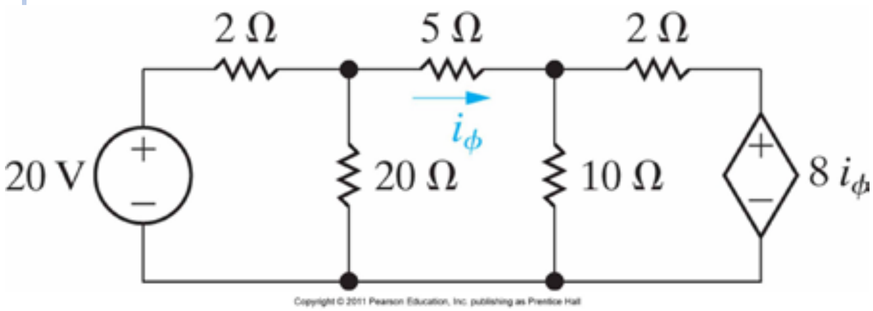
Activity 3

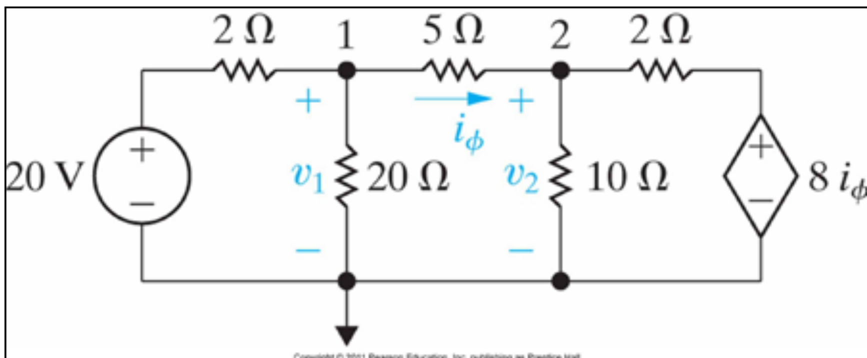
Find voltages and currents for each resistor.



Node-Voltage and Dependent Sources

- Another constraint is added by the dependent source





at Node 1: $\frac{v_1 - 20}{2} + \frac{v_1 - v_2}{5} + \frac{v_1}{20} = 0$

$i_\phi = \frac{v_1 - v_2}{5}$

at Node 2: $\frac{v_2 - 8i_\varphi}{2} - \frac{v_1 - v_2}{5} + \frac{v_2}{10} = 0$

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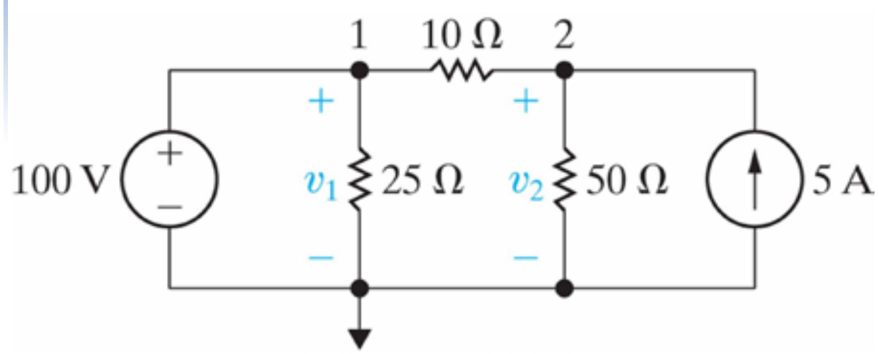
Node Voltage Method – Special Cases

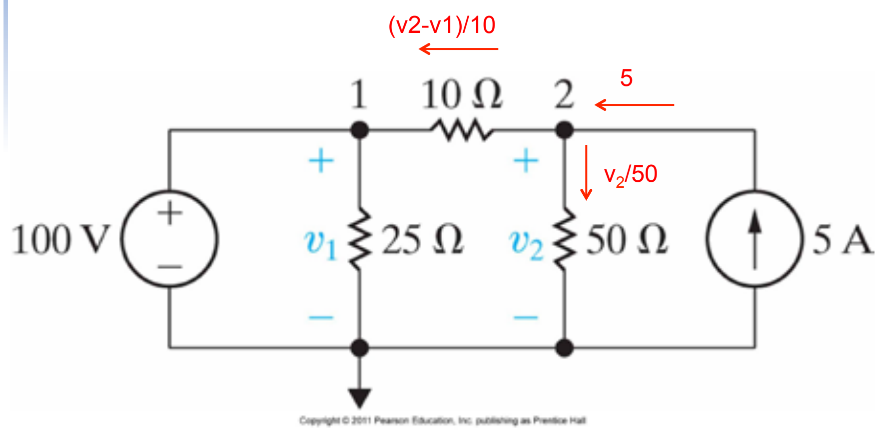
Special case 1

A voltage source is the only element between two essential nodes, node voltage method can be simplified.



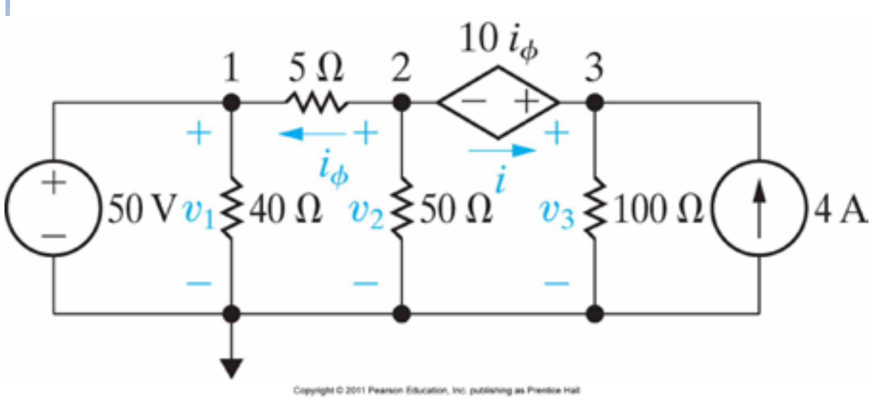
Special case 1

In the circuit below, v_1=100V, so we can write a KCL at Node 2 to find v_2.



Special case 2

A dependent voltage is between two non-reference essential nodes

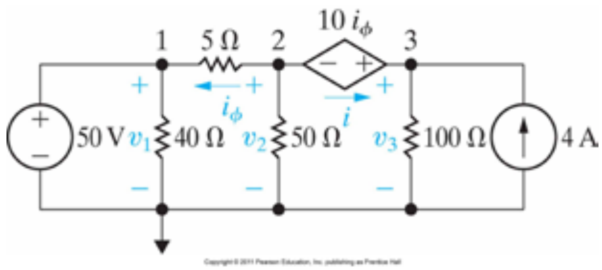


Special case 2

Applying node voltage method, we have two KCL equations at nodes 2 and 3.

at Node 2: $i + \frac{v_2 - 50}{5} + \frac{v_2}{50} = 0$, at Node 3: $\frac{v_3}{100} - 4 - i = 0$

adding two equations : $\frac{v_2 - 50}{5} + \frac{v_2}{50} + \frac{v_3}{100} - 4 = 0$



Special case 2

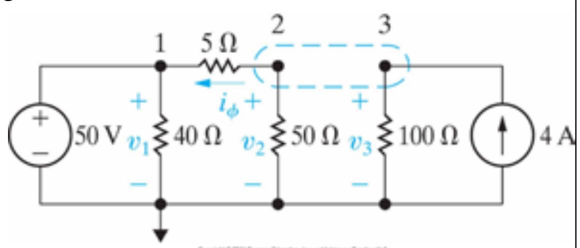
The concept of a Supernode:

- When a voltage source is between two essential nodes, we can combine those nodes to form a supernode → to write one KCL

$$\frac{v_2 - 50}{5} + \frac{v_2}{50} + \frac{v_3}{100} - 4 = 0$$

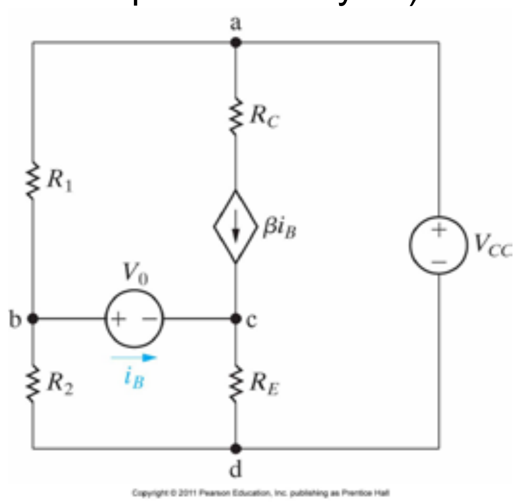
Need one more equation in v_2 and v_3 to be able to solve the circuit

$v3=10i_\phi+v2$



Activity 4

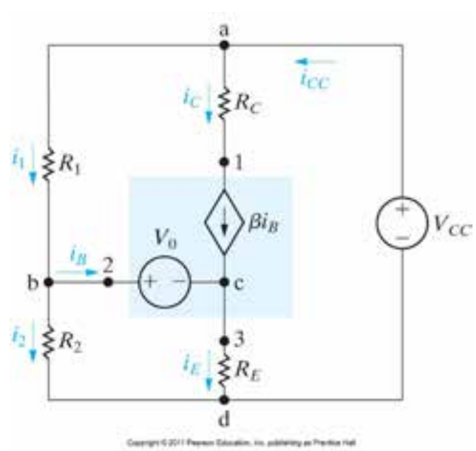
Use supernode concept to find i_B.
(Same circuit as in Chapter 2 Activity 10)



Chapter 2 Activity 10

The circuit represents a common configuration encountered in the analysis and design of transistor amplifiers. Assume that the values of R_1, R_2, R_C, R_E, V_{cc} and V_0 are known.

Find i_B in terms of the circuit element values.



Chapter 2 Solution

Apply KCL to nodes a, b, c, and 1, we have:

$$(1)\ i_1 + i_C - i_{CC} = 0$$
$$(2)\ i_B + i_2 - i_1 = 0$$
$$(3)\ i_E - i_B - i_C = 0$$
$$(4)\ i_C = \beta i_B$$

Apply KVL to 2 loops bcdb and badb, we have:

$$(5)\ V_0 + i_E R_E - i_2 R_2 = 0$$
$$(6)\ -i_1 R_1 + V_{CC} - i_2 R_2 = 0$$

Chapter 2 Solution

Solve Eq.(6) for i_1 and substitute i_1 into Eq. (2)

$$i_1 = \frac{V_{CC} - i_2 R_2}{R_1}$$

$$\frac{V_{CC} - i_2 R_2}{R_1} = i_B + i_2 \Rightarrow i_2 = \frac{V_{CC} - i_B R_1}{R_1 + R_2}$$

Substitute i_2 to Eq.(5), solve for i_E

$$\frac{V_0 + i_E R_E}{R_2} = \frac{V_{CC} - i_B R_1}{R_1 + R_2} \Rightarrow i_E = \left(\frac{(V_{CC} - i_B R_1) R_2}{(R_1 + R_2) R_E} - \frac{V_0}{R_E} \right)$$

Chapter 2 Solution

Substitute i_E into Eq. (3), and use Eq.(4) to eliminate i_c in Eq.(3), we have:

$$\frac{\left(V_{CC} - i_B R_1\right) R_2}{\left(R_1 + R_2\right) R_E} - \frac{V_0}{R_E} = i_B (1 + \beta)$$

$$\therefore i_B = \frac{V_{CC} R_2 / \left(R_1 + R_2\right) - V_0}{R_1 R_2 / \left(R_1 + R_2\right) + \left(1 + \beta\right) R_E}$$

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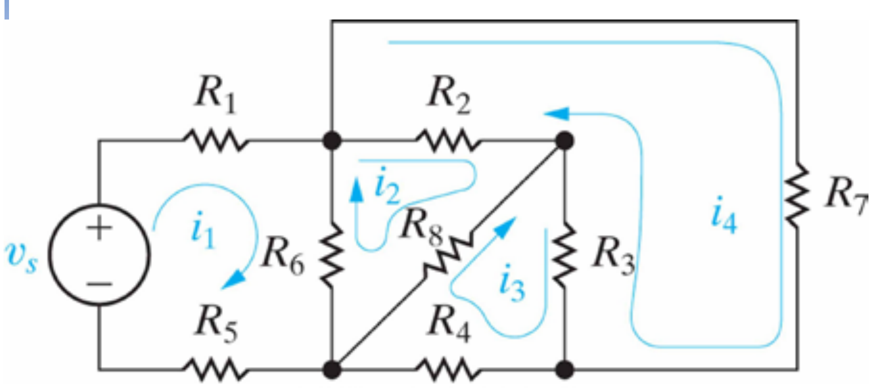
Mesh-Current Method

Mesh-Current Method

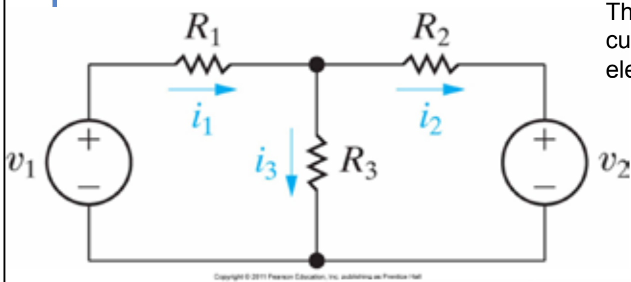
- Uses KVL equations around meshes
- Solves directly for currents
- Special cases for dependent sources and for current sources in a mesh
- The "recipe":
  1. Identify the meshes
  2. Label each with a mesh current
  3. Write a KVL equation around each mesh
  4. Put equations in standard form and solve
  5. Check your solutions by balancing power
  6. Calculate quantities of interest

Meshes in a circuit

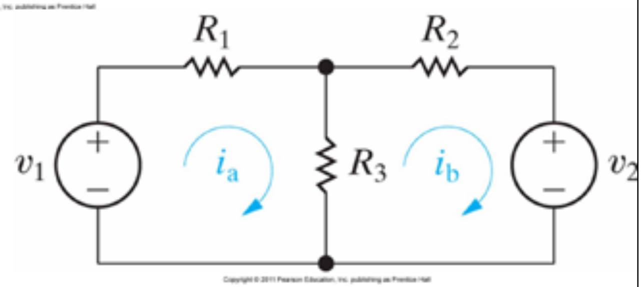
- 4 meshes versus 7 actual currents (for R_1~R_7)



Mesh currents & actual currents

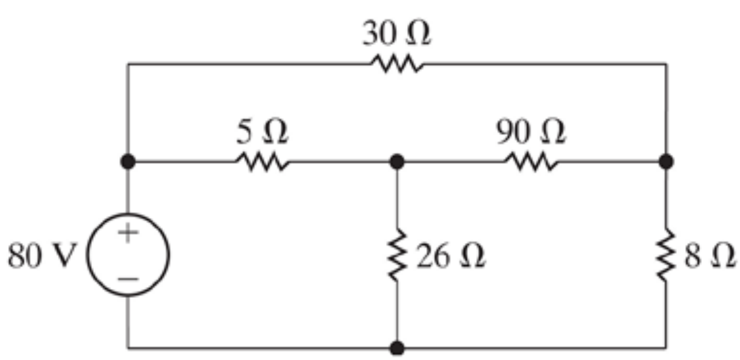


The relation between mesh currents and currents in a circuit element.



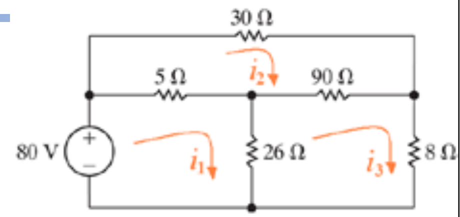
Mesh Example

- How many meshes does this circuit contain?



Mesh Example

- Find the power associated with the voltage source and the 8Ω resistor.



$$i_1 \text{ mesh}: -80 + 5(i_1 - i_2) + 26(i_1 - i_3) = 0$$

$$i_2 \text{ mesh}: 30(i_2) + 90(i_2 - i_3) + 5(i_2 - i_1) = 0$$

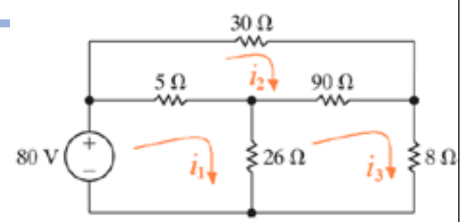
$$i_3 \text{ mesh}: 8(i_3) + 26(i_3 - i_1) + 90(i_3 - i_2) = 0$$

$$\text{Standard form}: \begin{aligned} i_1(5+26) + i_2(-5) + i_3(-26) &= 80 \\ i_1(-5) + i_2(30+90+5) + i_3(-90) &= 0 \\ i_1(-26) + i_2(-90) + i_3(8+26+90) &= 0 \end{aligned}$$

$$\text{Solution}: i_1 = 5\text{ A};\quad i_2 = 2\text{ A};\quad i_3 = 2.5\text{ A}$$

Mesh Example

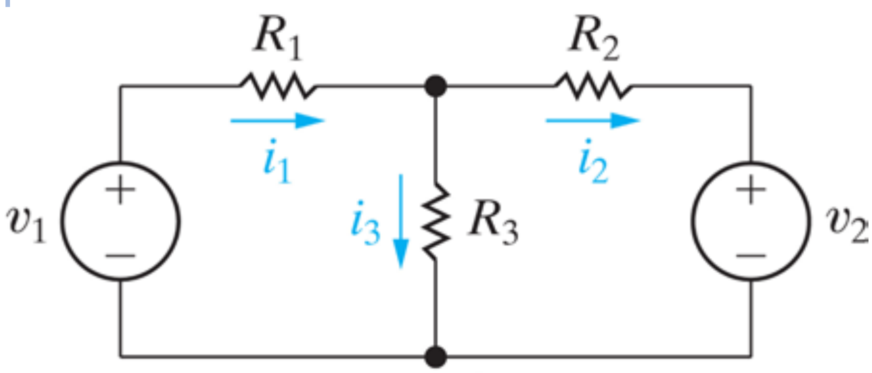
- Power balance



Component	Equation	p [W]
80 V	-(5)(80)	-400
5 Ω	$(5 - 2)^2(5)$	45
90 Ω	$(2.5 - 2)^2(90)$	22.5
30 Ω	$(2)^2(30)$	120
26 Ω	$(5 - 2.5)^2(26)$	162.5
8 Ω	$(2.5)^2(8)$	50

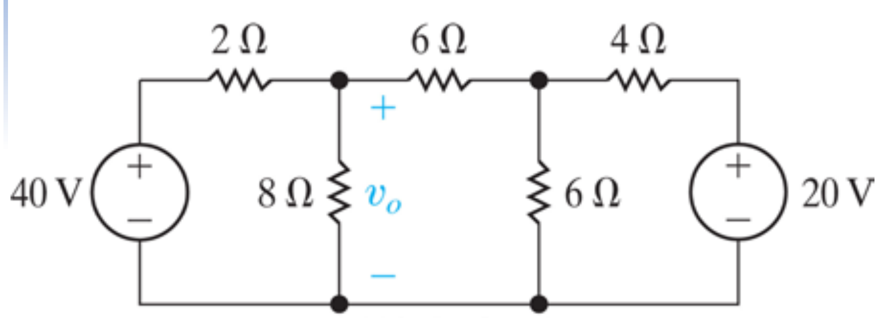
Activity 5

- Find currents in i_1, i_2, and i_3.



Activity 6

- Use the mesh-current method to determine the power associated with each voltage source in the circuit.



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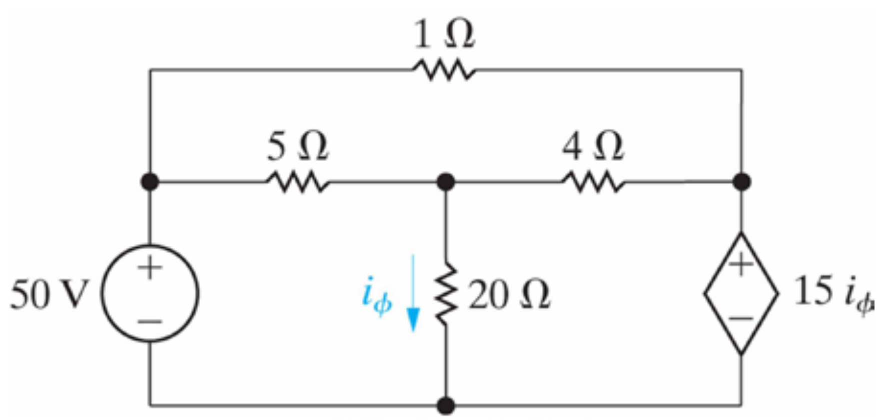
Mesh-Current Method: Special Cases

Special Case 1

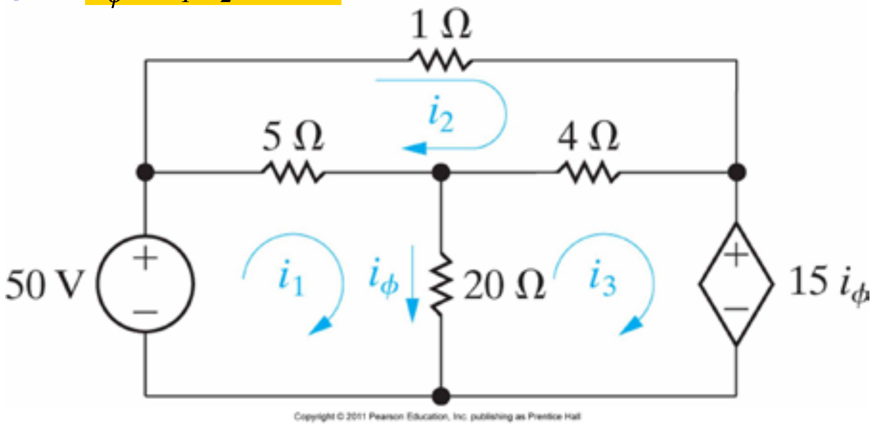
- Mesh-Current with Dependent Sources



Special Case 1

- You can easily write 3 mesh-current equations. But what about i_ϕ

$i_\phi = i_1 - i_2$

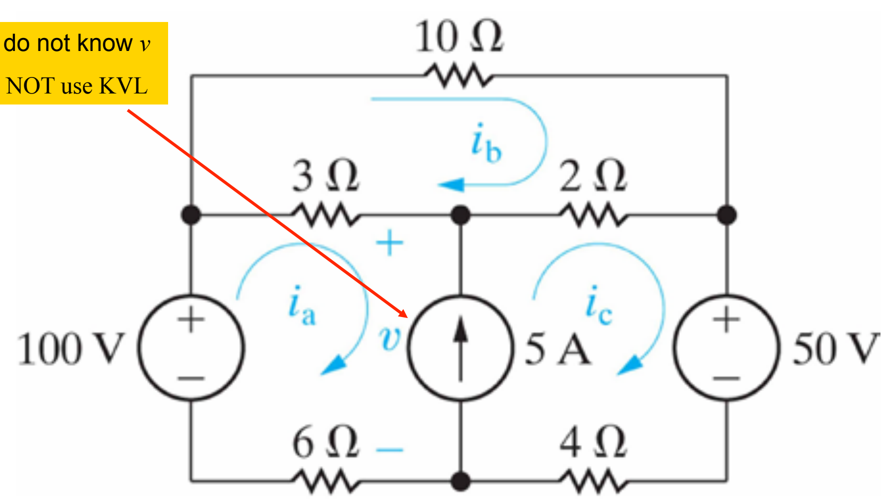


Special Case 2

- Current Source between two nodes

We do not know v

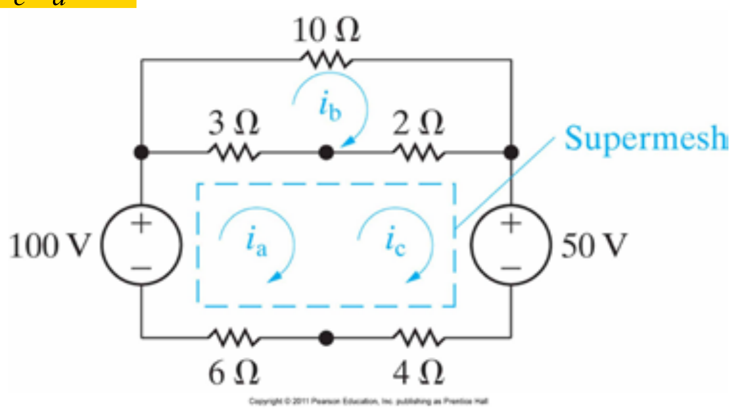
Can NOT use KVL



Supermesh

- 2 equations in 3 unknown, we still have:

  $i_c - i_a = 5$



Supermesh

$$\begin{cases} 10i_b + 2(i_b - i_c) + 3(i_b - i_a) = 0 \\ -100 + 3(i_a - i_b) + 2\left(i_c - i_b\right) + 50 + 4i_c + 6i_a = 0 \\ i_c - i_a = 5 \end{cases}$$

$$\begin{cases} -3i_a + 15i_b - 2i_c = 0 \\ 9i_a - 5i_b + 6i_c = 50 \\ -9i_a + 9i_c = 45 \end{cases} \Rightarrow \begin{cases} -9i_a + 45i_b - 6i_c = 0 \\ 9i_a - 5i_b + 6i_c = 50 \\ -5i_b + 15i_c = 95 \end{cases}$$

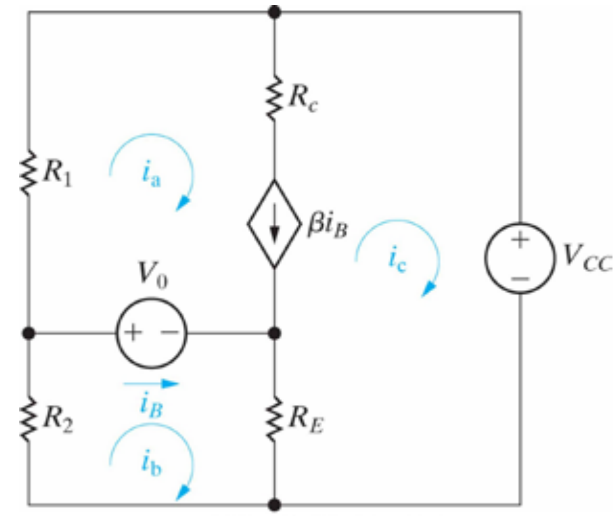
$$\Rightarrow 40i_b = 50, i_b = 1.25A$$

$$\Rightarrow 15i_c = 101.25, i_c = 6.75A$$

$$\Rightarrow i_a = 6.75 - 5 = 1.75A$$

Activity 7

- Find i_B.



EECS2200 Electric Circuits

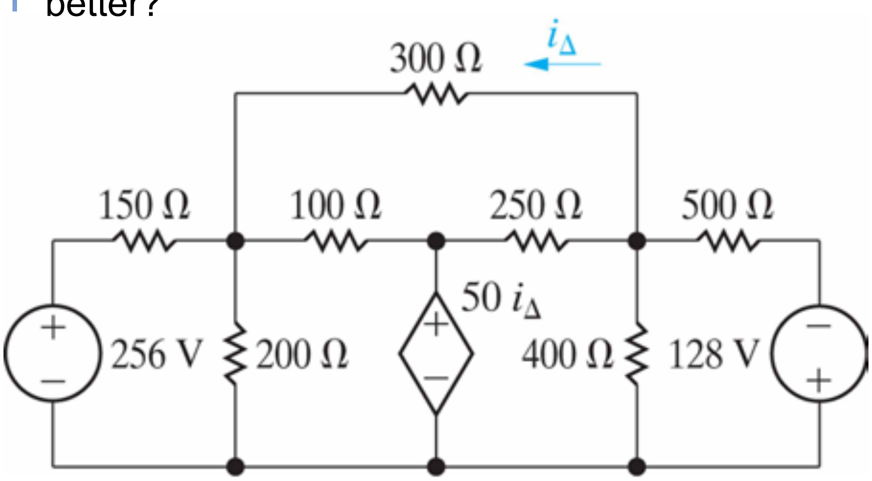
Node Voltage vs. Mesh Current

Node-Voltage vs. Mesh Current

- Which is better (less equations)
- Depends on:
  - Number of equations formed for each method
  - Any supernode (node voltage)
  - Any supermesh (mesh-current)
  - Do we have to solve for the entire circuit?

Activity 8

Find power in the 300 Ω resistor, which method is better?

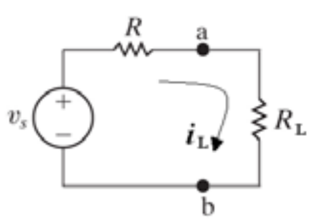


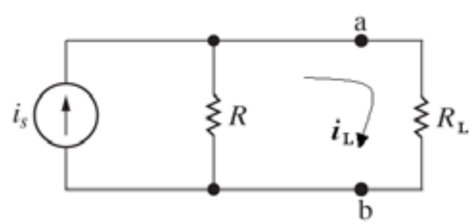
EECS2200 Electric Circuits

Source Transformation

Source Transformation

Under what condition(s) are the two circuits below "equivalent"? "Equivalent" means that if you attached any resistor to both circuits, it would have the same current, and therefore the same voltage and power.



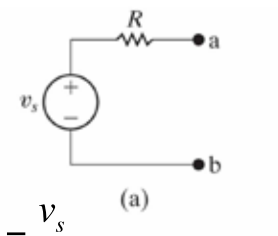


$$i_L = \frac{v_S}{R + R_L} \qquad i_L = \frac{R \parallel R_L}{R_L} i_S = \frac{RR_L / R + R_L}{R_L} i_S = \frac{Ri_S}{R + R_L}$$

$\Rightarrow \quad v_S = Ri_S$ -- the condition for source transformation

Source Transformation

- A voltage source in series with a resistor can be replaced by a current source in parallel with the same resistor or vice versa.
- Some times it helps in reducing the circuit complexity



$$i_s = \frac{v_s}{R}$$

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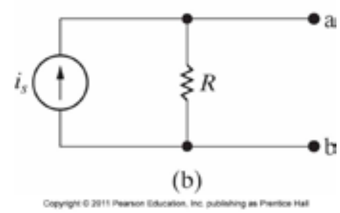
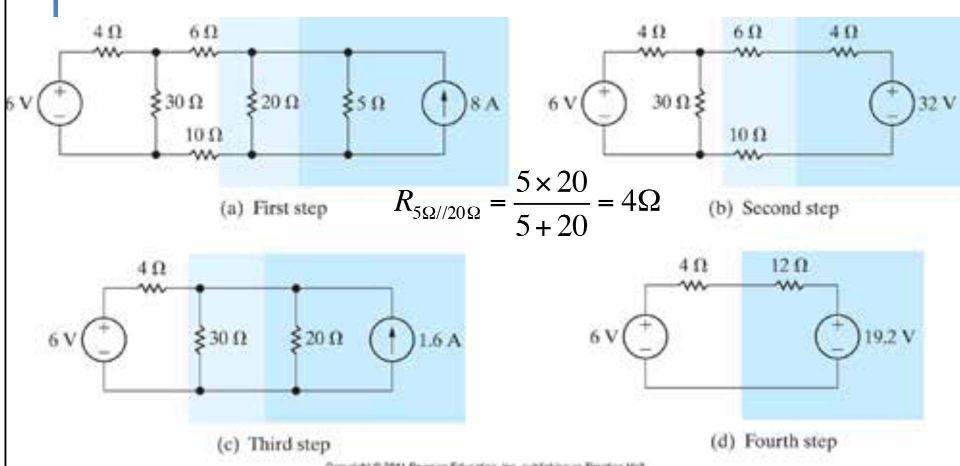


An Example

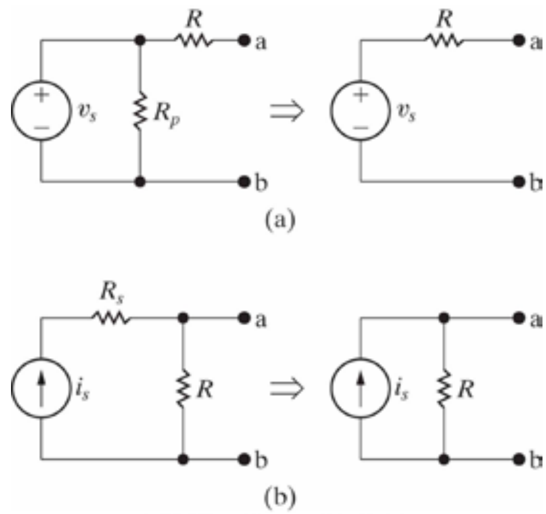


$$R_{5\Omega//20\Omega} = \frac{5\times 20}{5+20} = 4\Omega$$

$$R_{30\Omega//20\Omega} = \frac{30\times 20}{30+20} = 12\Omega$$

Activity 9

Are these 2 circuits equivalent?

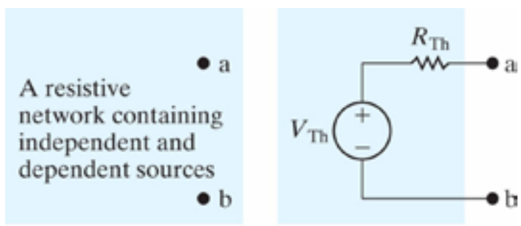


EECS2200 Electric Circuits

Thevenin and Norton Equivalents

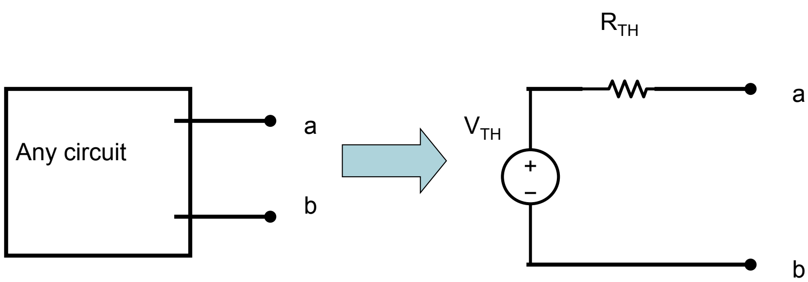
Thevenin Equivalent Circuit

- Any resistive network with dependent or independent sources could be represented as an independent voltage source V_{TH} in series with a resistance R_{TH}.
- Especially useful when we are interested in the behavior of the circuit between two terminals.



(a) (b)

Thevenin Equivalent Circuit

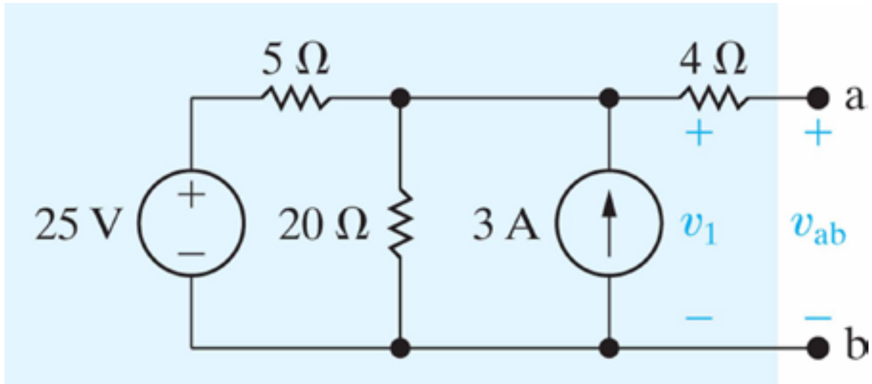


V_{TH} = the open circuit voltage between a and b.

$R_{TH} = V_{TH}/I_{sc}$ where I_{sc} is the short circuit current from a to b, **EXCEPT for an ideal current source.**

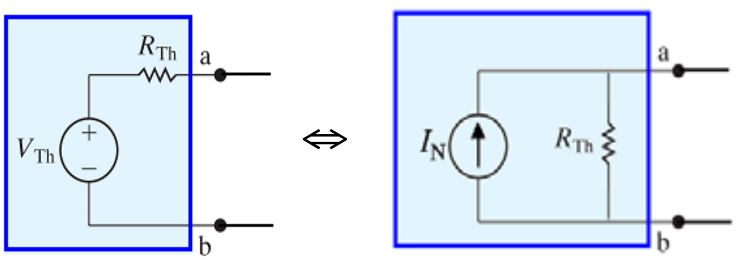
Activity 10

- Find Thevenin equivalent



Norton Equivalent Circuit

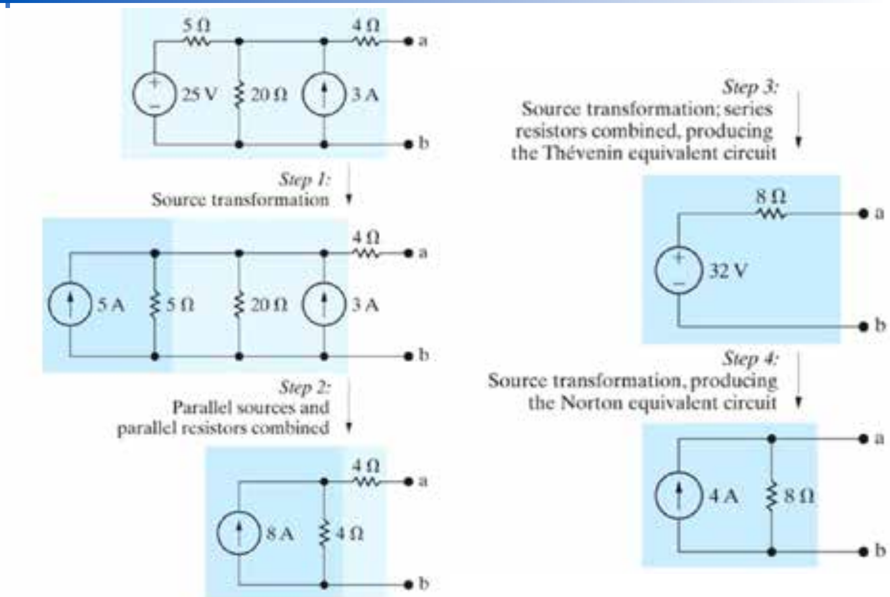
- The Norton equivalent is just the source-transform of the Thevenin equivalent.



$V_{Th} = R_{Th}\, I_N$

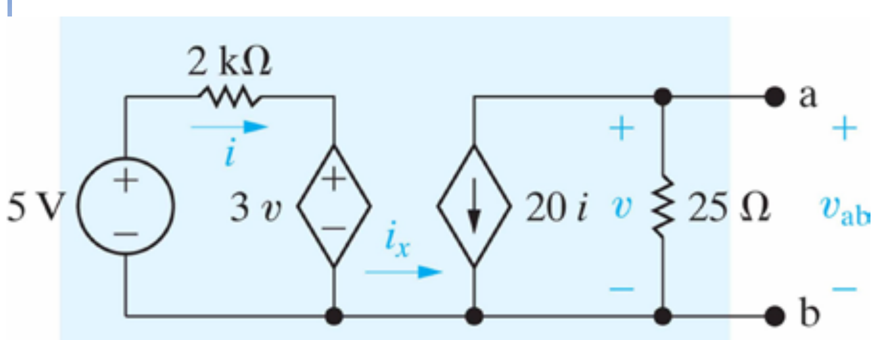
$I_N = V_{Th} / R_{Th}$

Example: Norton Equivalent



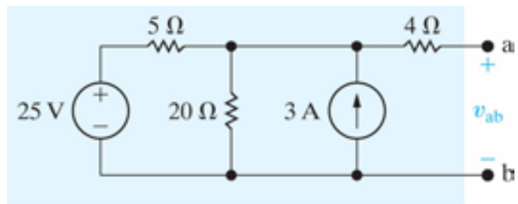
Activity 11

Find the Thevenin equivalent.



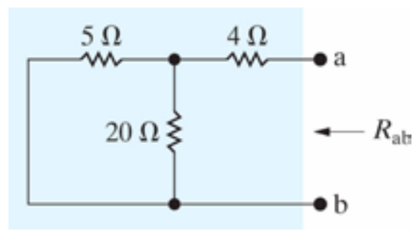
More on Deriving Thevenin Eqv.

- Determine R_{TH} – deactivate all independent sources and calculate R_{ab}.
- Voltage source
  - Short circuit
- Current source
  - Open circuits



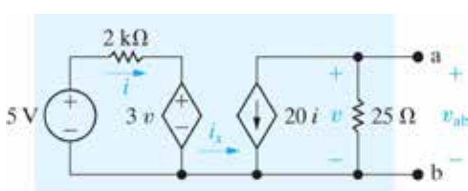
$$R_{ab} = R_{TH} = 5\Omega // 20\Omega + 4\Omega$$

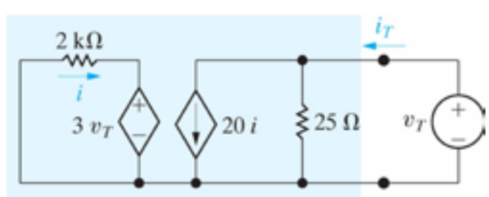
$$R_{TH} = \frac{5 \times 20}{5 + 20} + 4 = 8\Omega$$



More on R_{TH}

- Deactivate all independent sources
- Apply a test voltage (or current) from ab
- Calculate the current i_T, $R_{TH} = v_T / i_T$



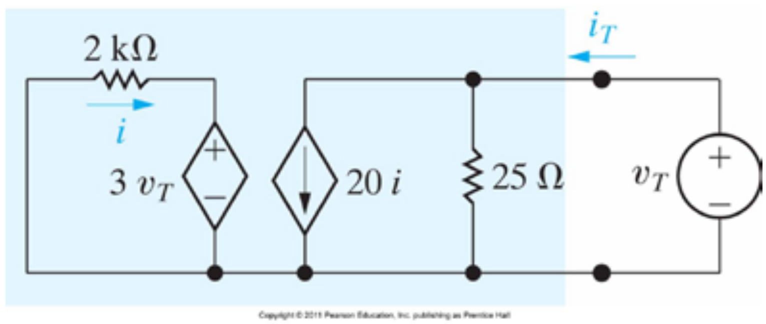


More on R_{TH}

$$KCL: i_T = \frac{v_T}{25} + 20i$$

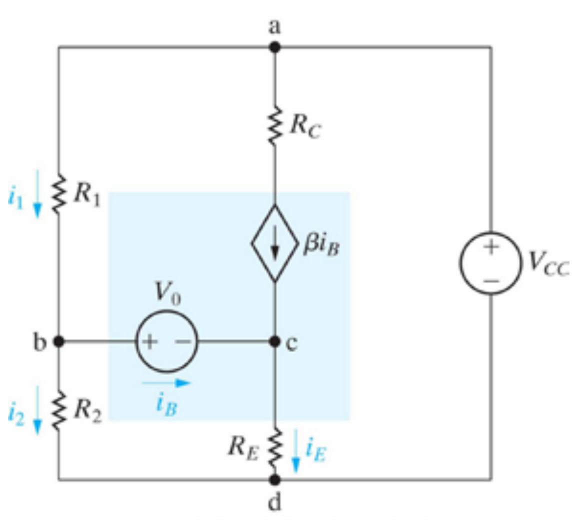
$$KVL: 2000i + 3v_T = 0, i = -3v_T/2000$$

$$i_T = \frac{v_T}{25} - \frac{60v_T}{2000} = \frac{v_T}{100} \Rightarrow R_{TH} = 100\Omega$$



Activity 12

Find i_B



EECS2200 Electric Circuits

Maximum Power Transfer

Maximum Power Transfer

- Condition for maximum power transfer
  - Suppose we can vary the load resistance. For what value of load resistance will maximum power be absorbed by the load?

$$p_L = i^2 R_L; \qquad i = \frac{V_{Th}}{R_{Th} + R_L}$$

$$\therefore \quad p_L = \frac{R_L V_{Th}^2}{(R_{Th} + R_L)^2}$$

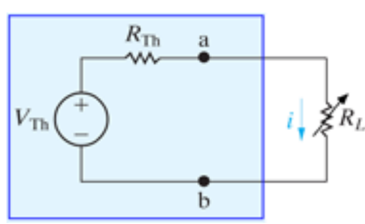
$$\text{For max. power, } \frac{dp_L}{dR_L} = 0$$

$$\frac{dp_L}{dR_L} = \frac{V_{Th}^2}{(R_{Th} + R_L)^2} - \frac{2R_L V_{Th}^2}{(R_{Th} + R_L)^2}$$

$$= \frac{V_{Th}^2 (R_{Th} + R_L) - 2R_L V_{Th}^2}{(R_{Th} + R_L)^3} = 0$$

$$\Rightarrow \quad V_{Th}^2 (R_{Th} + R_L) - 2R_L V_{Th}^2 = 0$$

$$\Rightarrow \quad (R_{Th} + R_L) = 2R_L \qquad \therefore \qquad R_L = R_{Th}$$



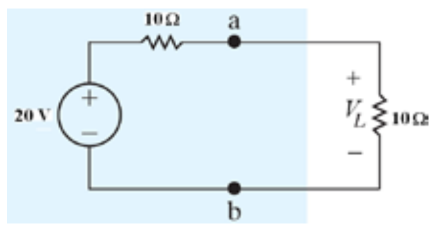
Represents the Thevenin equivalent of an arbitrary subcircuit that will not change

Represents a variable load

Activity 13

In the circuit below, the load resistor is matched to the Thevenin resistance, how much power is absorbed by the load resistor?

A. 100 W
B. 50 W
C. 20 W
D. 10 W



EECS2200 Electric Circuits

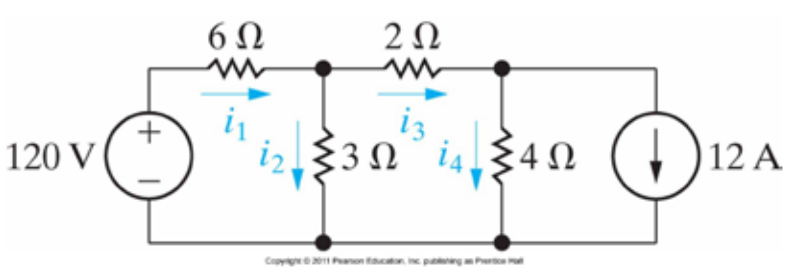
Superposition

Superposition

- When a linear system is excited by more than one independent source, the total response is the sum of the individual responses where each response is the result of one of the independent sources acting alone.

Activity 14

- Find i2



Activity 15

Find v_o using superposition