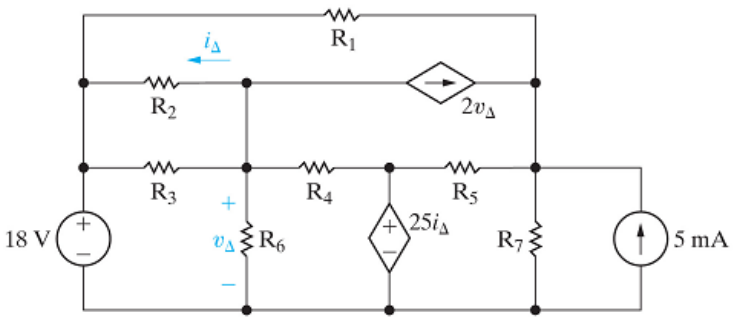
EECS2200 Electric Circuits

Chapter 4

Activities

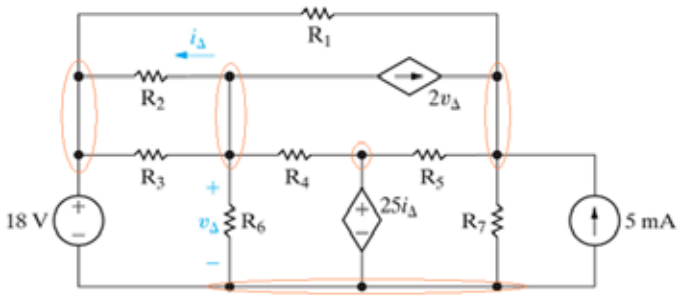
Activity 1

How many essential nodes does this circuit have?



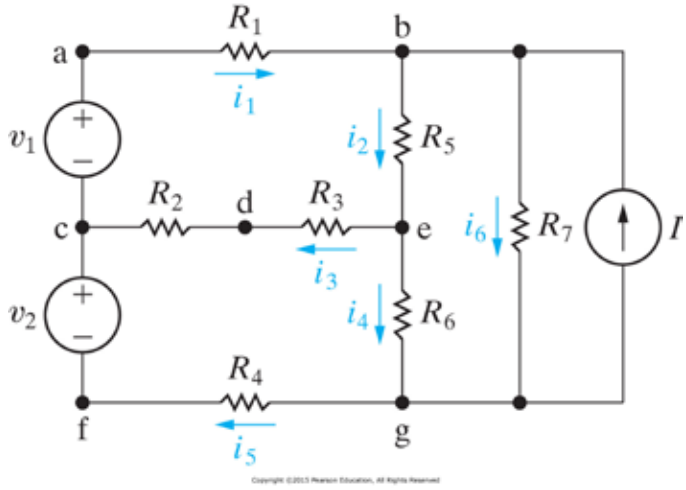
Solution

- 5 essential nodes



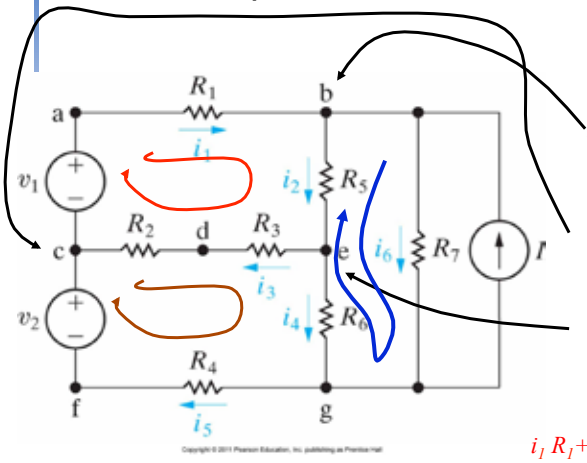
Activity 2

Write simultaneous equations for below circuit.



Solution

- Need 6 equations for 6 unknown



$-i_1 + i_2 + i_6 - I = 0$

$i_1 - i_5 - i_3 = 0$

$i_4 - i_2 + i_3 = 0$

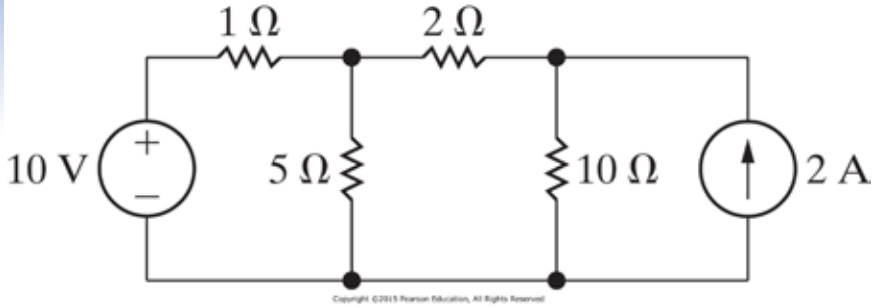
$i_1 R_1 + i_2 R_2 + i_3 (R_3 + R_4) - v_1 = 0$

$-i_3 (R_2 + R_3) + i_4 R_6 + i_5 R_4 - v_2 = 0$

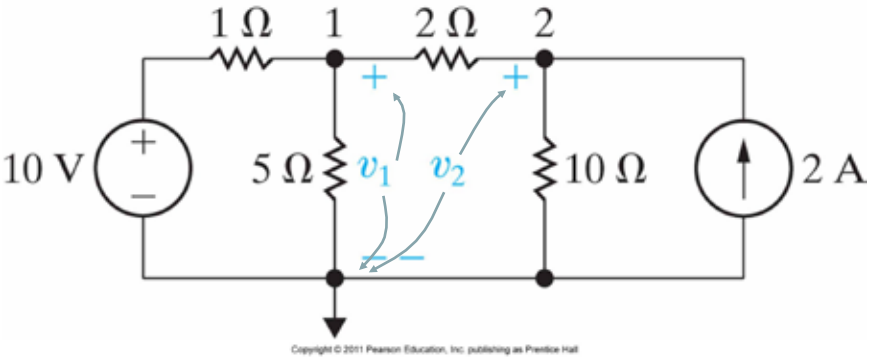
$i_2 R_5 + i_4 R_6 - i_6 R_7 = 0$

Activity 3

Find voltages and currents for each resistor.



Solution



$$-\frac{10-v_1}{1}+\frac{v_1}{5}+\frac{v_1-v_2}{2}=0 \qquad -\frac{v_1-v_2}{2}+\frac{v_2}{10}-2=0$$

Solution

$$(1)\ -\frac{10-v_1}{1}+\frac{v_1}{5}+\frac{v_1-v_2}{2}=0 \Rightarrow 10v_1-100+2v_1+5v_1-5v_2=0$$

$$(2)\ -\frac{v_1-v_2}{2}+\frac{v_2}{10}-2=0 \Rightarrow 5v_2-5v_1+v_2-20=0$$

$$\begin{cases}17v_1-5v_2=100\\-5v_1+6v_2=20\end{cases}$$

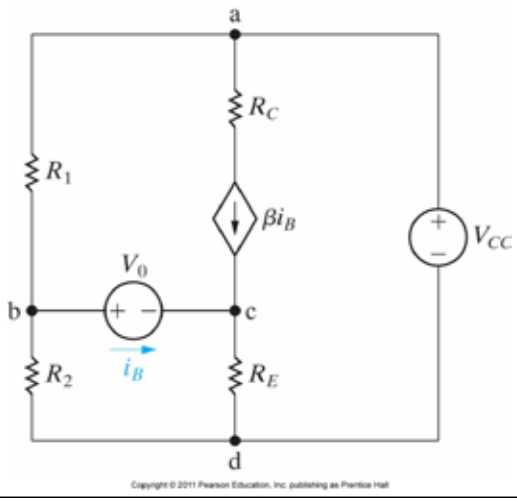
$$\begin{cases}102v_1-30v_2=600\\-25v_1+30v_2=100\end{cases} \Rightarrow 77v_1=700$$

$$\therefore v_1=9.09V, v_2=10.91V$$

Activity 4

Use supernode concept to find i_B.

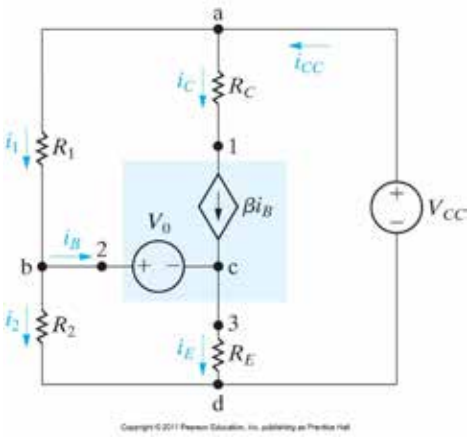
(Same circuit as in Chapter 2 Activity 10)



Chapter 2 Activity 10

The circuit represents a common configuration encountered in the analysis and design of transistor amplifiers. Assume that the values of R_1, R_2, R_C, R_E, V_{cc} and V_0 are known.

Find i_B in terms of the circuit element values.



Chapter 2 Solution

Apply KCL to nodes a, b, c, and 1, we have:

$$(1)\ i_1 + i_C - i_{CC} = 0$$
$$(2)\ i_B + i_2 - i_1 = 0$$
$$(3)\ i_E - i_B - i_C = 0$$
$$(4)\ i_C = \beta i_B$$

Apply KVL to 2 loops bcdb and badb, we have:

$$(5)\ V_0 + i_E R_E - i_2 R_2 = 0$$
$$(6)\ -i_1 R_1 + V_{CC} - i_2 R_2 = 0$$

Chapter 2 Solution

Solve Eq.(6) for i_1 and substitute i_1 into Eq. (2)

$$i_1 = \frac{V_{CC} - i_2 R_2}{R_1}$$

$$\frac{V_{CC} - i_2 R_2}{R_1} = i_B + i_2 \Rightarrow i_2 = \frac{V_{CC} - i_B R_1}{R_1 + R_2}$$

Substitute i_2 to Eq.(5), solve for i_E

$$\frac{V_0 + i_E R_E}{R_2} = \frac{V_{CC} - i_B R_1}{R_1 + R_2} \Rightarrow i_E = \left(\frac{(V_{CC} - i_B R_1) R_2}{(R_1 + R_2) R_E} - \frac{V_0}{R_E} \right)$$

Chapter 2 Solution

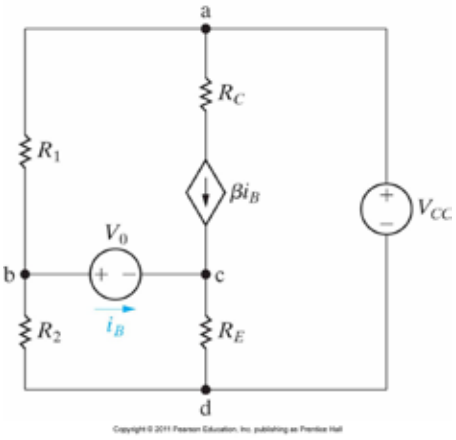
Substitute i_E into Eq. (3), and use Eq.(4) to eliminate i_c in Eq.(3), we have:

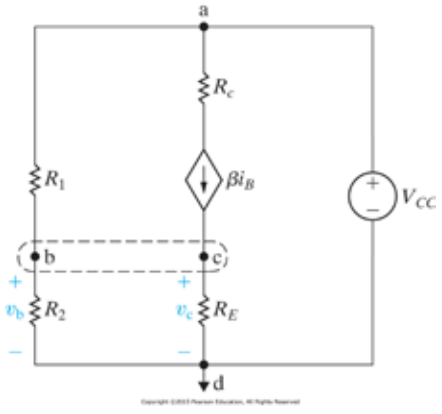
$$\frac{(V_{CC} - i_B R_1) R_2}{(R_1 + R_2) R_E} - \frac{V_0}{R_E} = i_B (1 + \beta)$$

$$\therefore i_B = \frac{V_{CC} R_2 / (R_1 + R_2) - V_0}{R_1 R_2 / (R_1 + R_2) + (1 + \beta) R_E}$$

Solution

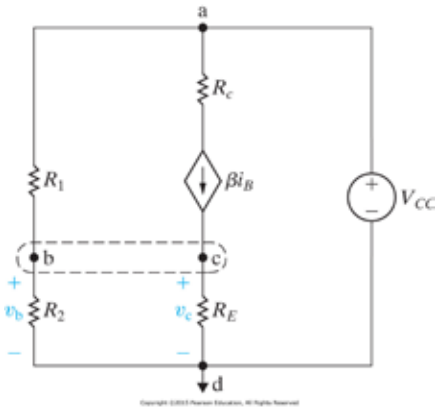
- Apply supernode concept to nodes b and c.





Solution

Write KCL at supernode:



$$\frac{v_b - V_{CC}}{R_1} + \frac{v_b}{R_2} + \frac{v_c}{R_E} - \beta i_B = 0$$

Need 2 more equations to get rid of v_c and v_b

$$v_b - v_c = V_0$$

$$v_c = \left(i_B + \beta i_B\right) R_E$$

Solution

Solve 3 equations for i_B:
(compared to 6 equations in Chapter 2)

$$\frac{v_c + V_o - V_{CC}}{R_1} + \frac{v_c + V_o}{R_2} + \frac{v_c}{R_E} - \beta i_B = 0$$

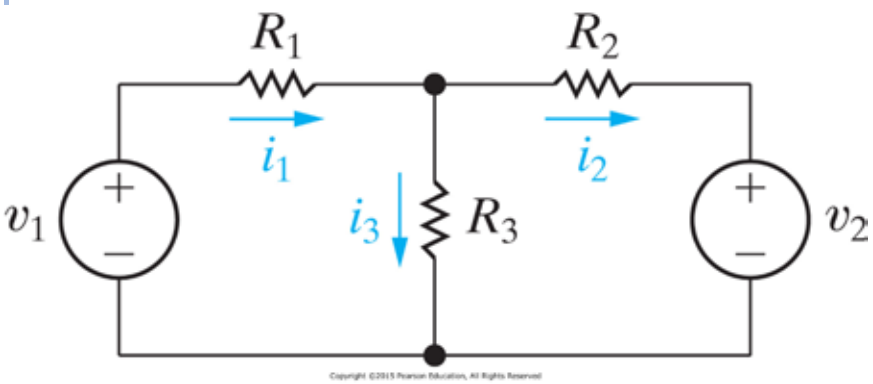
$$\Rightarrow \frac{\left(R_1 + R_2\right) v_c + \left(R_1 + R_2\right) V_o - R_2 V_{CC}}{R_1 R_2} + i_B + \beta i_B - \beta i_B = 0$$

$$\Rightarrow \left(R_1 + R_2\right)(1+\beta) i_B R_E + R_1 R_2 i_B = R_2 V_{CC} - \left(R_1 + R_2\right) V_o$$

$$\Rightarrow i_B = \frac{R_2 V_{CC} - \left(R_1 + R_2\right) V_o}{R_1 R_2 + \left(R_1 + R_2\right)(1+\beta) R_E} = \frac{\left(R_2 V_{CC}\right)/\left(R_1 + R_2\right) - V_o}{R_1 R_2 / \left(R_1 + R_2\right) + \left(1+\beta\right) R_E}$$

Activity 5

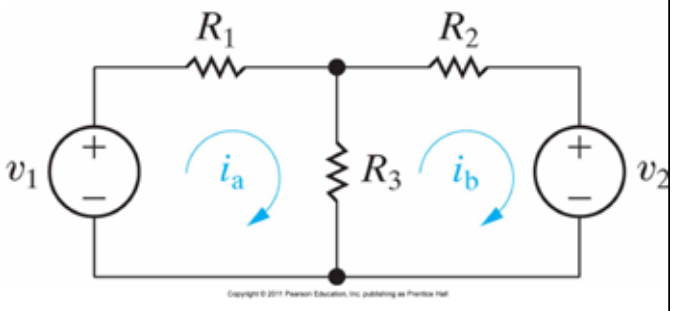
- Find currents in i_1, i_2, and i_3.



Solution

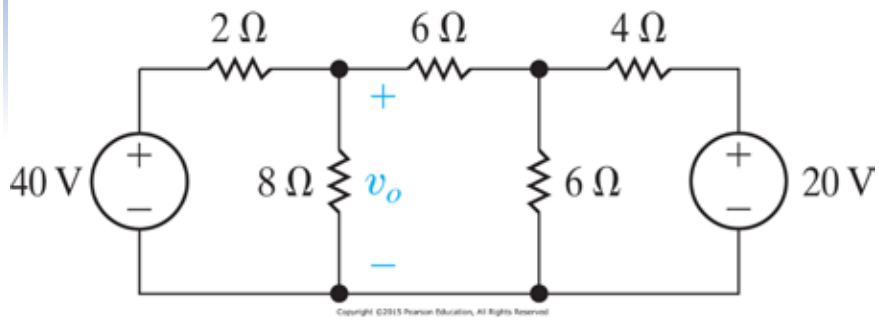
- Use mesh-current method, there are two meshes for two KVL equations

$$i_a R_1 + R_3(i_a - i_b) = v_1$$
$$R_3(i_b - i_a) + i_b R_2 = -v_2$$

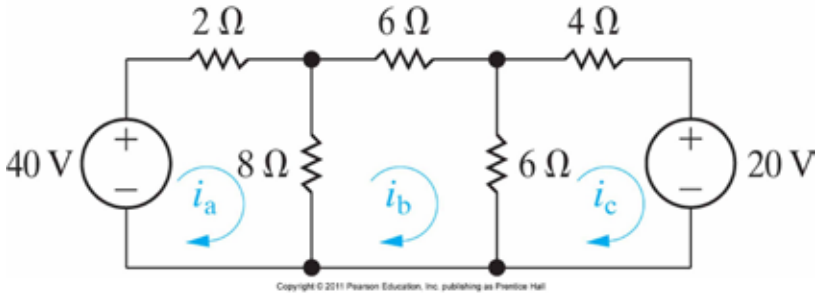


Activity 6

- Use the mesh-current method to determine the power associated with each voltage source in the circuit.



Solution



$$2i_a + 8(i_a - i_b) - 40 = 0$$

$$6i_b + 6(i_b - i_c) + 8(i_b - i_a) = 0$$

$$4i_c + 20 + 6(i_c - i_b) = 0$$

Solution

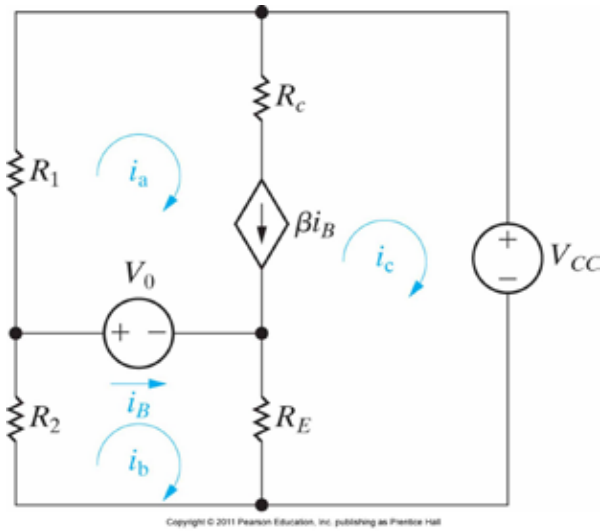
$$\begin{cases} 2i_a + 8(i_a - i_b) - 40 = 0 \\ 6i_b + 6(i_b - i_c) + 8(i_b - i_a) = 0 \\ 4i_c + 20 + 6(i_c - i_b) = 0 \end{cases}$$

$$\begin{cases} 10i_a - 8i_b = 40 \\ -8i_a + 20i_b - 6i_c = 0 \\ -6i_b + 10i_c = -20 \end{cases} \Rightarrow \begin{cases} i_a = 5.6A \\ i_b = 2.0A \\ i_c = -0.8A \end{cases}$$

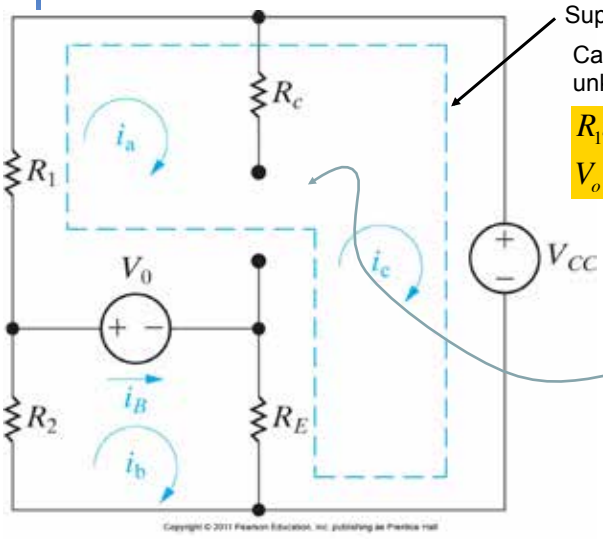
$$p_{40V} = -40i_a = -224W, p_{20V} = 20i_c = -16W$$

Activity 7

- Find i_B.



Solution



Supermesh

Can write 2 equations in 3 unknown, need one more

$$R_1 i_a + V_{CC} + R_E(i_c - i_b) - V_o = 0$$
$$V_o + R_E(i_b - i_c) + R_2 i_b = 0$$

$$\beta i_B = i_a - i_c$$
$$i_B = i_b - i_a$$

Solution

$$\begin{cases} \beta i_B = i_a - i_c \\ i_B = i_b - i_a \end{cases} \Rightarrow (1+\beta) i_B = i_b - i_c$$

$$\begin{cases} R_1 i_a + V_{CC} + R_E(i_c - i_b) - V_o = 0 \\ V_o + R_E(i_b - i_c) + R_2 i_b = 0 \end{cases} \Rightarrow \begin{cases} -R_1 R_2 i_a + R_2 R_E(i_b - i_c) = R_2 V_{CC} - R_2 V_o \\ R_1 R_2 i_b + R_1 R_E(i_b - i_c) = -R_1 V_o \end{cases}$$

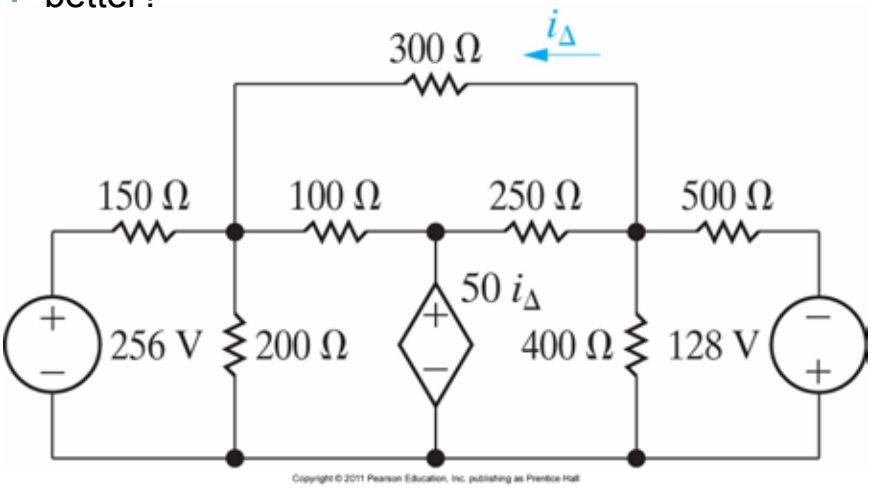
$$\Rightarrow R_1 R_2 (i_b - i_a) + (R_1 + R_2) R_E (1+\beta) i_B = R_2 V_{CC} - (R_1 + R_2) V_o$$

$$\Rightarrow \frac{R_1 R_2 i_B}{(R_1 + R_2)} + R_E (1+\beta) i_B = \frac{R_2 V_{CC}}{(R_1 + R_2)} - V_o$$

$$\therefore i_B = \frac{R_2 V_{CC} / (R_1 + R_2) - V_o}{R_1 R_2 / (R_1 + R_2) + R_E (1+\beta)}$$

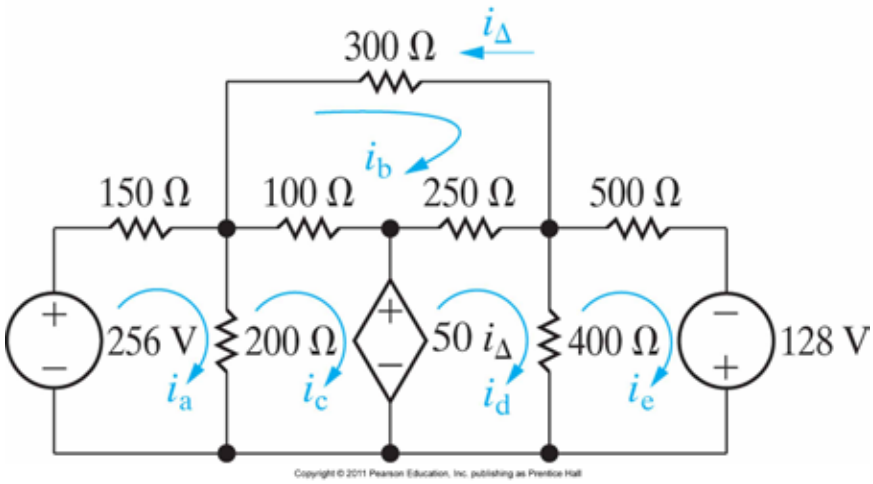
Activity 8

Find power in the 300 Ω resistor, which method is better?



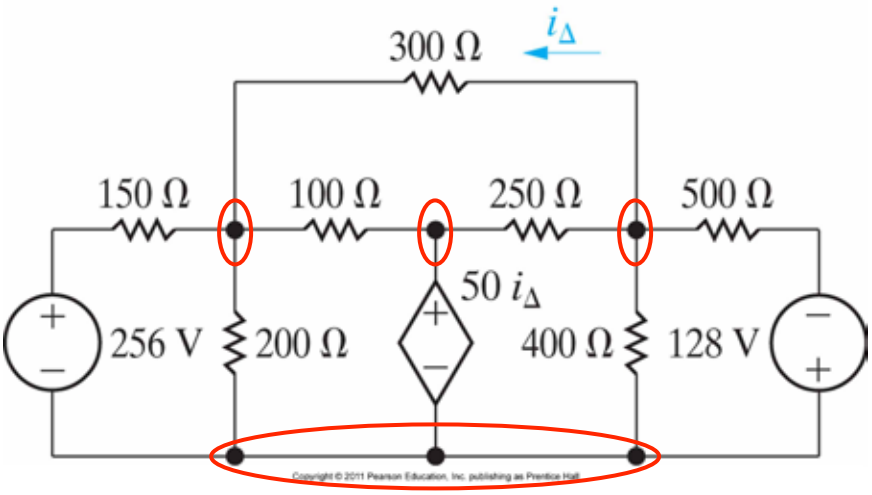
Solution

- Mesh-current: 5 equations (note that i_Δ=-i_b)



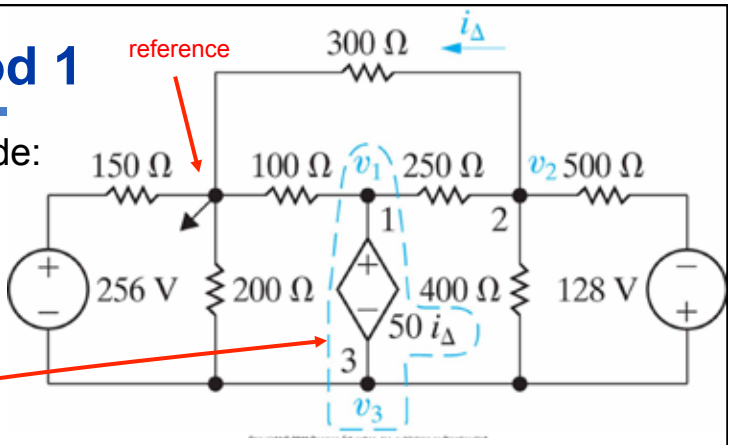
Solution

- Node voltages: 4 nodes, need only 3 equations



Method 1

Supernode:



$$\frac{v_1}{100}+\frac{v_3}{200}+\frac{v_3+256}{150}+\frac{v_1-v_2}{250}+\frac{v_3-v_2}{400}+\frac{v_3-128-v_2}{500}=0$$

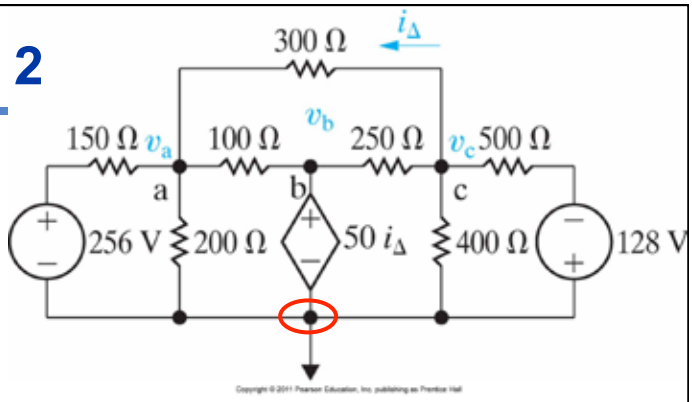
$$\frac{v_2-v_1}{250}+\frac{v_2+128-v_3}{500}+\frac{v_2-v_3}{400}+\frac{v_2}{300}=0$$

At node 2

$$v_1-v_3=50i_\Delta=50\frac{v_2}{300}$$

Constraint at supernode

Method 2



$$\frac{v_a}{200}+\frac{v_a-256}{150}+\frac{v_a-v_b}{100}+\frac{v_a-v_c}{300}=0$$ Node a

$$\frac{v_c-v_b}{250}+\frac{v_c}{400}+\frac{v_c+128}{500}+\frac{v_c-v_a}{300}=0$$ Node c

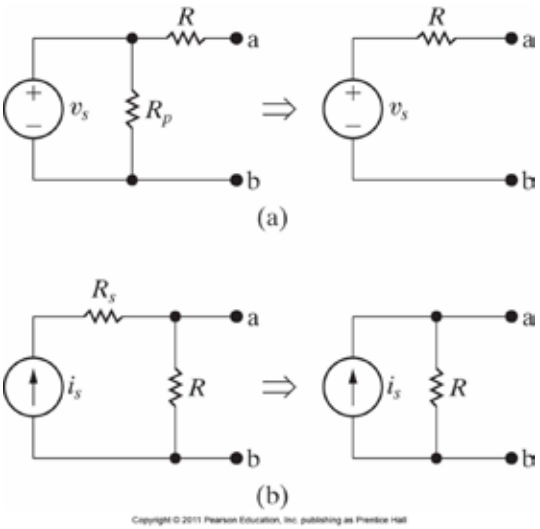
$$v_b=50i_\Delta=50\frac{v_c-v_a}{300}$$ Constraint at supernode

Solution

- For both methods, the power dissipated by 300Ω resister, is 16.57W.

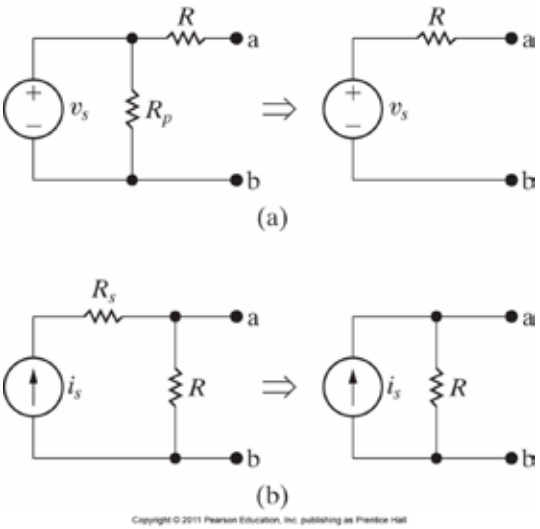
Activity 9

Are these 2 circuits equivalent?



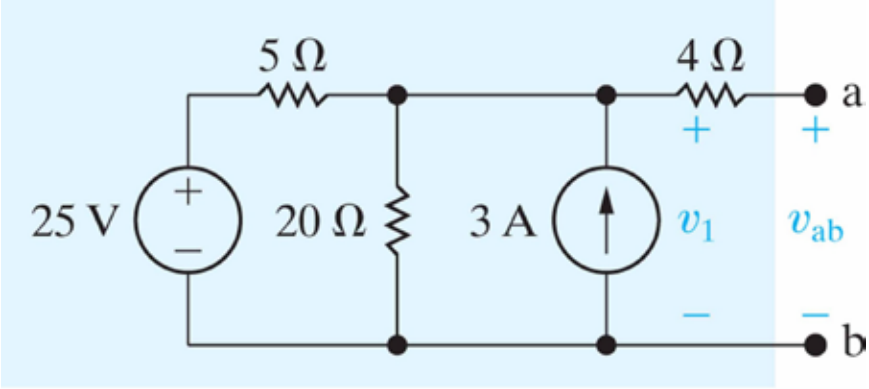
Solution

They are equivalent with respect to terminals a and b.



Activity 10

- Find Thevenin equivalent

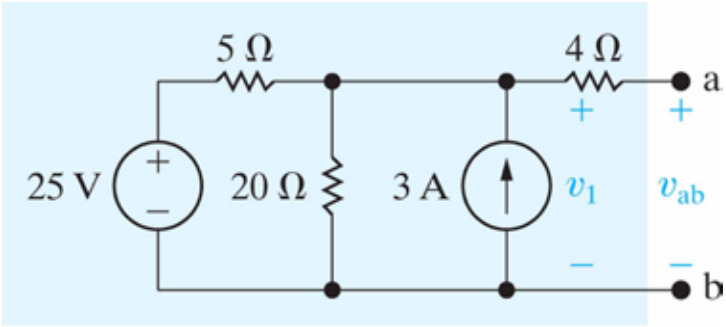


Solution

1. Find v_{ab} using node voltage.

$$3 + \frac{25 - v_1}{5} = \frac{v_1}{20} \Rightarrow v_1 = 32V$$

$$\therefore v_{th} = v_1 = 32V$$

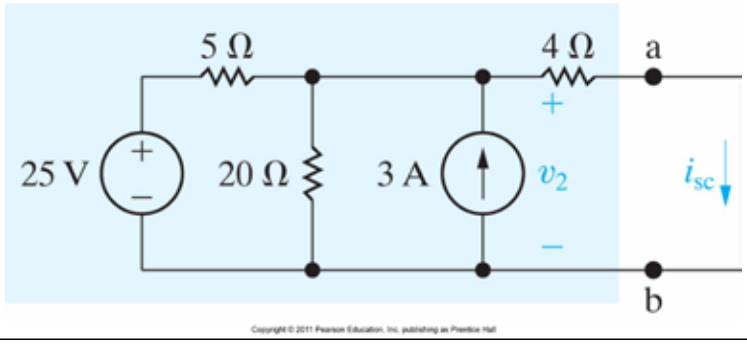


Solution

2. find the short circuit current.

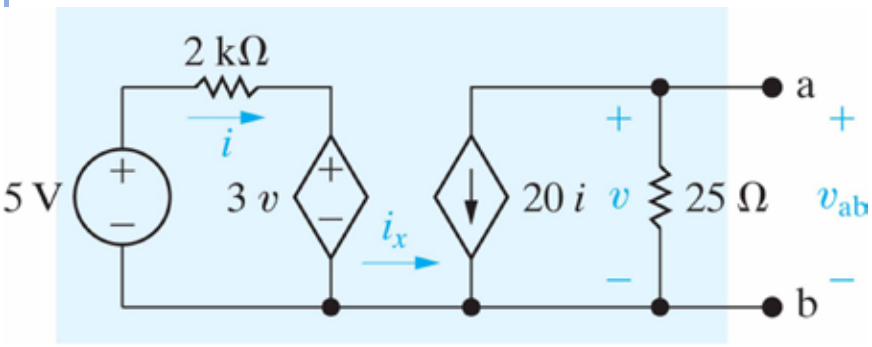
$$\frac{v_2 - 25}{5} + \frac{v_2}{20} + \frac{v_2}{4} - 3 = 0 \Rightarrow v_2 = 16V$$

$$i_{sc} = \frac{v_2}{4} = 4A \Rightarrow R_{th} = \frac{V_{th}}{i_{sc}} = \frac{32}{4} = 8\Omega$$



Activity 11

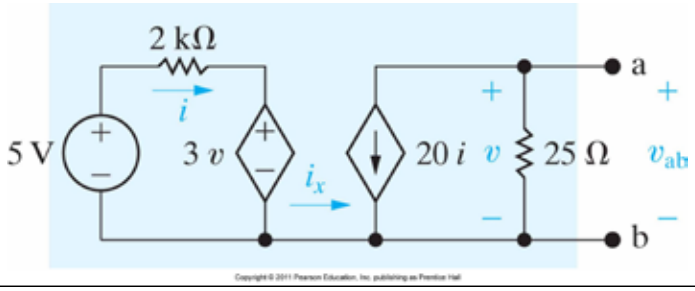
Find the Thevenin equivalent.



Solution

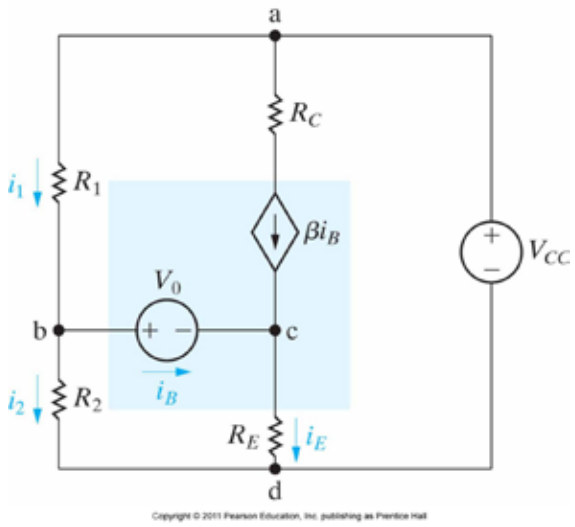
1. Note that ix = 0 (why?)
2. KCL at node "a": $20i + \frac{v}{25} = 0 \Rightarrow v = -500i$
3. KVL at left loop: $5 = 2000i + 3v = 500i \Rightarrow i = 10mA, v = V_{th} = -5V$
4. Short circuit a and b, $v = 0 \Rightarrow 3v = 0, i = 5/2000 = 2.5mA$

$$i_{sc} = -20i = -50mA \Rightarrow R_{th} = V_{th}/i_{sc} = 100\Omega$$



Activity 12

Find i_B



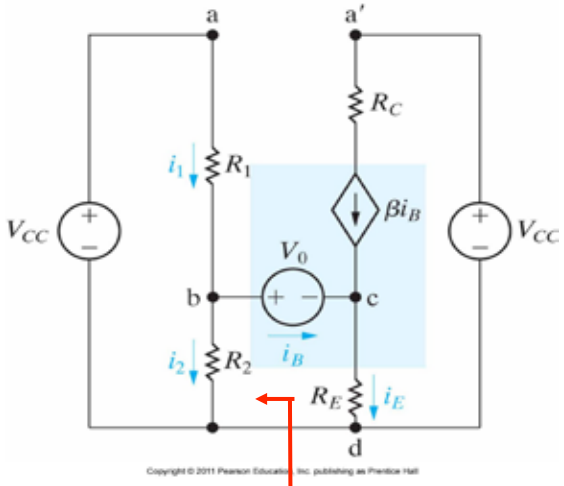
Solution

1.Redraw it.

2. Find V_{TH}, R_{TH}

$$V_{TH} = \frac{R_2}{R_1 + R_2} V_{CC}$$

$$R_{TH} = R_1 // R_2 = \frac{R_1 R_2}{R_1 + R_2}$$



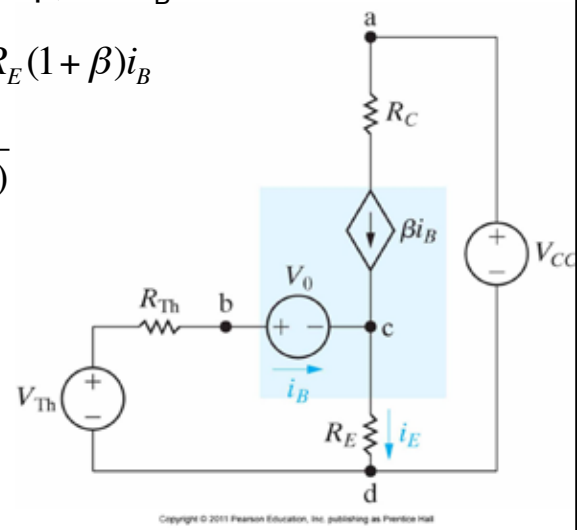
Find Thevenin Equivalent

Solution

3. KVL to the left loop, find i_B

$$V_{TH} = R_{TH} i_B + V_o + R_E(1+\beta) i_B$$

$$i_B = \frac{V_{TH} - V_o}{R_{TH} + R_E(1+\beta)}$$



Solution

4. Substitute V_{TH} and R_{TH} into i_B

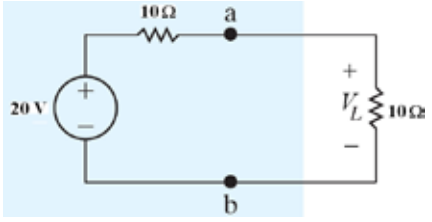
$$i_B = \frac{V_{TH} - V_o}{R_{TH} + R_E(1+\beta)}, V_{TH} = \frac{R_2 V_{CC}}{R_1 + R_2}, R_{TH} = \frac{R_1 R_2}{R_1 + R_2}$$

$$\therefore i_B = \frac{R_2 V_{CC}/(R_1 + R_2) - V_o}{R_1 R_2/(R_1 + R_2) + R_E(1+\beta)}$$

Activity 13

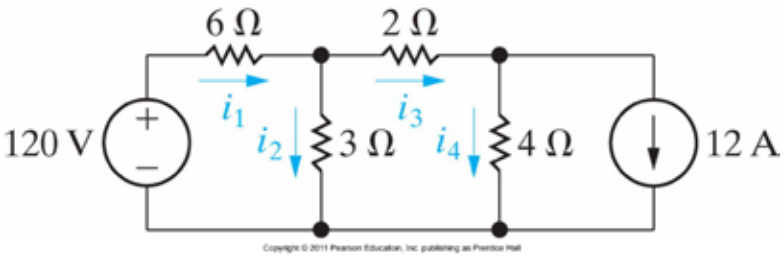
In the circuit below, the load resistor is matched to the Thevenin resistance, how much power is absorbed by the load resistor?

A. 100 W
B. 50 W
C. 20 W
D. 10 W



Activity 14

- Find i2



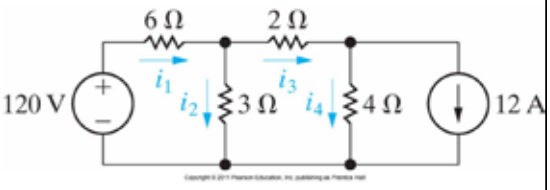
Solution

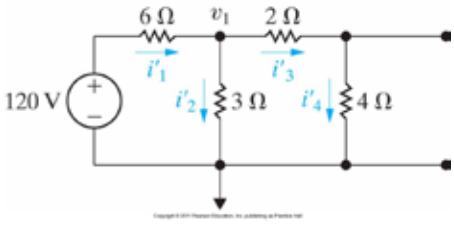
Deactivated current source

$$R_{total} = 6 + \frac{3 \times 6}{3+6} = 8\Omega$$

$$i_1' = \frac{120}{8} = 15A$$

$$i_2' = \frac{6}{3+6} \times 15 = 10A$$





Solution

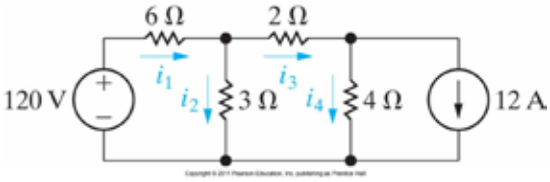
Deactivated voltage source

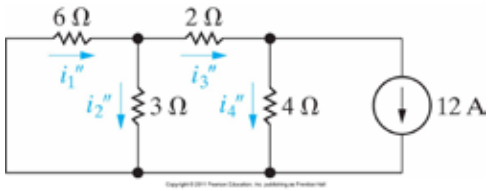
$$R_{6//3+2} = \frac{6\times 3}{6+3} + 2 = 4\Omega$$

$$i_3^{''} = \frac{12}{2} = 6A, i_4^{''} = -6A$$

$$i_2^{''} = -6 \times \frac{6}{3+6} = -4A$$

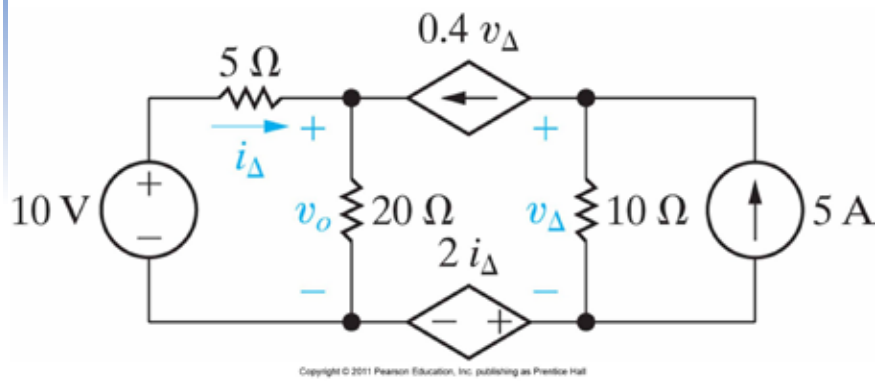
$$i_2 = i_2^{'} + i_2^{''} = 10 - 4 = 6A$$





Activity 15

Find v_o using superposition

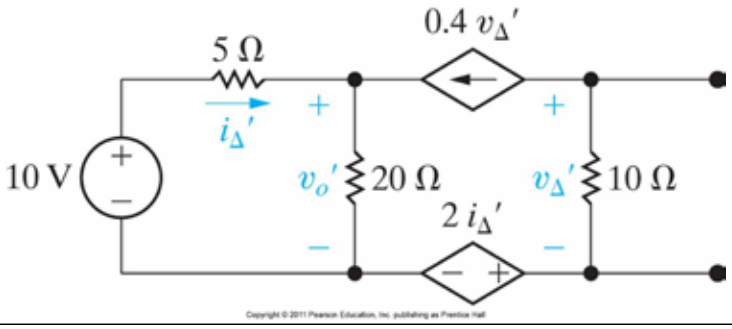


Solution

Deactivate current source

$$\text{KCL at right node}: \frac{v_\Delta'}{10} + 0.4v_\Delta' = 0 \Rightarrow v_\Delta' = 0$$

$$\text{Voltage divider}: v_o' = \frac{20}{5+20} \times 10 = 8V$$



Solution

Deactivate voltage source

$$\text{KCL at node a}: \frac{v_o''}{20} + \frac{v_o''}{5} - 0.4v_\Delta'' = 0 \Rightarrow 5v_o'' - 8v_\Delta'' = 0$$

$$\text{KCL at node b}: 0.4v_\Delta'' + \frac{v_b - 2i_\Delta''}{10} - 5 = 0 \Rightarrow 4v_\Delta'' + v_b - 2i_\Delta'' = 0$$

$$v_b = 2i_\Delta'' + v_\Delta'' \Rightarrow v_\Delta'' = 10V, v_o'' = 16V$$

$$v_o = v_o' + v_o'' = 8 + 16 = 24V$$