LE./EECS 4214 Digital Communications
Formula & Data Sheet...FINAL EXAM F14

1 Constant and Units

$c = 3 \times 10^8$ m/s (in free space), $c = 2 \times 10^8$ m/s (in media)
1 Å $= 10^{-10}$ m
1 μm $= 10^{-6}$ m
1 m $= 10^{-3}$ km
1 s $= 10^3$ ms
kbps $= 10^3$ bps
Mbps $= 10^6$ bps
KBps $= 10^3$ Bps (the simplifying relation we will use unless otherwise specified)
MBps $= 10^6$ Bps (the simplifying relation we will use unless otherwise specified)

2 Equations

2.1 Fourier Stuff

$$x(t) = \int_{-\infty}^{\infty} X(f) e^{j2\pi ft} \mathrm{d}f, X(f) = \int_{-\infty}^{\infty} x(t) e^{-j2\pi ft} \mathrm{d}t$$
$$\mathcal{F}\{\mathrm{rect}(t/T)\} = T\mathrm{sinc}(fT) = T\sin(\pi fT)/\pi fT$$
$$\mathcal{F}\{\mathrm{sinc}(t/T)\} = T\mathrm{rect}(fT)$$
$$\mathcal{F}\{1 - |\tau|/T\} = T\mathrm{sinc}^2(fT)$$
$$x(t) = \sum_{n=-\infty}^{\infty} c_n e^{j2\pi n f_o t}, c_n = \int_{-\infty}^{\infty} x(t) \exp(-j2\pi n f_o t) \mathrm{d}t$$

2.2 Basic Signals and Systems Stuff

$$\int_{-\infty}^{\infty} x^2(t) \mathrm{d}t = \int_{-\infty}^{\infty} |X(f)|^2 \mathrm{d}f$$
$$\psi_x(f) = |X(f)|^2, G_x(f) = \sum |c_n|^2 \delta(f - nf_o), G_x(f) = \lim_{T\to\infty} \tfrac{1}{T}|X_T(f)|^2$$
$$R_x(\tau) = \int_{-\infty}^{\infty} x(t)x(t+\tau)\mathrm{d}t, R_x(\tau) = \lim_{T\to\infty} \tfrac{1}{T} \int_{-\infty}^{\infty} x(t)x(t+\tau)\mathrm{d}t$$
$$\mathrm{sinc}(1.4/\pi) \approx 0.707, \int_{-\infty}^{\infty} \mathrm{sinc}(x)\mathrm{d}x = 1$$
$$y(t) = x(t) * h(t) = \int_{-\infty}^{\infty} x(t-\tau)h(\tau)\mathrm{d}\tau$$
$$H_{RC}(f) = \frac{1}{1 + j2\pi fRC}, f_{3dB} = 1/2\pi RC$$
$$z(mT) = \sum_{n=-N}^{n=N} c_n x(mT - nT)$$

2.3 Trig.

$$\sin(a+b) = \sin a \cos b + \cos a \sin b, \cos(a+b) = \cos a \cos b - \sin a \sin b$$
$$\sin(a \pm b) = \sin a \cos b \pm \cos a \sin b, \cos(a \pm b) = \cos a \cos b \mp \sin a \sin b$$
$$\sin 2a = 2 \sin a \cos a, \cos 2a = \cos^2 a - \sin^2 a = 2\cos^2 a - 1, \cos^2 a = 0.5 + 0.5\cos(2a)$$
$$\cos a = (e^{ja} + e^{-ja})/2, \sin a = (e^{ja} - e^{-ja})/j2, \tan a = \sin a / \cos a$$
$$\cos(\omega t) \cdot \cos(\omega t + \phi) = 0.5[\cos\phi + \cos(2\omega t + \phi)]$$

2.4 Analog-to-Digital

$$\text{SNR [dB]} = 10\log(\text{SNR})$$
$$SNR_q = \sigma_x^2/(q^2/12), SNR_{q,dB} = 6.02b + 10.8 + 10\log(\sigma_x^2/V_{pp}^2), SNR_j = 3/(\sigma_t^2 + f_H^2)$$

2.5 Probability

$$p_X(x) = \mathrm{d}F_X(x)/\mathrm{d}x$$
$$p_n(x) = \frac{\exp\{[-(x-\mu)^2]/2\sigma^2\}}{\sqrt{2\pi\sigma^2}}, F_n(X > a) = Q((a-\mu)/\sigma)$$
$$E[X^n] = \int_{-\infty}^{\infty} x^n p(x)\mathrm{d}x, \sigma_x^2 = E[(X-m_x)^2] = E[X^2] - m_x^2$$
$$G_x(f) = \int_{-\infty}^{\infty} R_x(\tau)\exp[-j2\pi f\tau]\mathrm{d}\tau, \sigma_x^2 = R_x(0) - m_x^2$$
$$-(z-a_1)^2/2\sigma_o^2 + \ln P(s_1) \underset{H_2}{\overset{H_1}{\gtrless}} -(z-a_2)^2/2\sigma_o^2 + \ln P(s_2)$$
$$P_B = P(H_2|s_1)P(s_1) + P(H_1|s_2)P(s_2), P_B = Q(\frac{a_1-a_2}{2\sigma_o}), P_B = Q\left(\frac{A}{\sigma_o}\right)$$
$$P_B = Q\left(\sqrt{\frac{E_d}{2N_0}}\right), P_E \approx (\log_2 M)P_B$$
$$P_B = 0.5e^{-E_b/2N_o}, P_B \approx \frac{M/2}{M-1}\mathrm{Q}\left(\sqrt{\frac{2E_b}{N_o}}\right)$$
$$P_B = Q\left\{\sqrt{E_T}\left[\int_{-\infty}^{\infty}\frac{G_n^{0.5}|P_r|}{|H_c|}\mathrm{d}f\right]^{-1}\right\}, E_T = \int_{-\infty}^{\infty}|H_t|^2\mathrm{d}f$$

2.6 And All the Rest

$$SNR_T = \frac{2}{N_0}\int_{-\infty}^{\infty}|S_f|^2\mathrm{d}f, SNR_T = \frac{(a_1-a_2)^2}{\sigma_0^2}$$
$$\sum_{k=-\infty}^{k=\infty} P_r\left(f + \frac{k}{T}\right) = T, P_{RC} = T \text{ for} 0 \le |f| < (1-r)/2T$$
$$P_{RC} = \frac{T}{2}\left\{1 + \cos\left[\frac{\pi T}{r}\left(|f| + \frac{1-r}{2T}\right)\right]\right\} \text{ for } (1-r)/2T < |f| \le (1+r)/2T$$
$$P_{RC} = 0 \text{ for } (1+r)/2T < |f|, p_{RC}(t) = \mathrm{sinc}(t/T)\frac{\cos(\pi rt/T)}{1-(2rt/T)^2}$$
$$|H_r| = \frac{k|P_r|^{0.5}}{G_n^{0.25}|H_c|^{0.5}}, |H_t| = \frac{(A/k)|P_r|^{0.5}G_n^{0.25}}{|H_c|^{0.5}}$$
$$1.21 \approx \int_0^1 [1+\cos(\pi x)][1+(2x)^2]^{0.5}\mathrm{d}x$$
$$c = (X^TX)^{-1}X^Tz, c = \left(\frac{I}{SNR} + X^TX^{-1}\right)X^Tz$$
$$B_k(i,j) = |z_k - s^{(i,j)}|^2$$
$$s_i(t) = \sum_{j=1}^{N} a_{ij}\psi_j(t), a_{ij} = \int_{T_i}^{T_f} s_i\psi_j\mathrm{d}t, \int_{T_i}^{T_f}\psi_i\psi_j\mathrm{d}t = \delta_{ij}$$
$$Q(x) \approx \frac{1}{x\sqrt{2\pi}}\exp(-x^2/2) \text{ for } x > 3, \int_0^{\infty}\frac{\sin^2 px}{x^2}\mathrm{d}x = \frac{\pi p}{2}$$

Q-Function Table

z	$Q(z)$	z	$Q(z)$
0.0	0.50000	2.0	0.02275
0.1	0.46017	2.1	0.01786
0.2	0.42074	2.2	0.01390
0.3	0.38209	2.3	0.01072
0.4	0.34458	2.4	0.00820
0.5	0.30854	2.5	0.00621
0.6	0.27425	2.6	0.00466
0.7	0.24196	2.7	0.00347
0.8	0.21186	2.8	0.00256
0.9	0.18406	2.9	0.00187
1.0	0.15866	3.0	0.00135
1.1	0.13567	3.1	0.00097
1.2	0.11507	3.2	0.00069
1.3	0.09680	3.3	0.00048
1.4	0.08076	3.4	0.00034
1.5	0.06681	3.5	0.00023
1.6	0.05480	3.6	0.00016
1.7	0.04457	3.7	0.00011
1.8	0.03593	3.8	0.00007
1.9	0.02872	3.9	0.00005