L14: Bandpass Modulation

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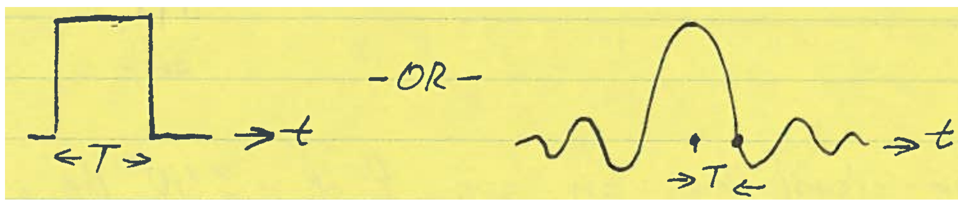
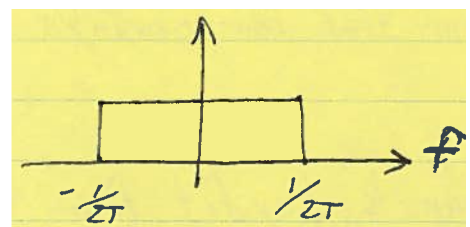
Outline

- 14.1 Basics
- 14.2 Bandpass Signaling
- 14.3 PSK: Phase Shift Keying
- 14.4 FSK: Frequency Shift Keying
- 14.5 ASK: Amplitude Shift Keying
- 14.6 APK: Amplitude Phase Keying
- 14.7 Signal Space Concepts
- 14.8 Signal Space Examples

Basics: Baseband

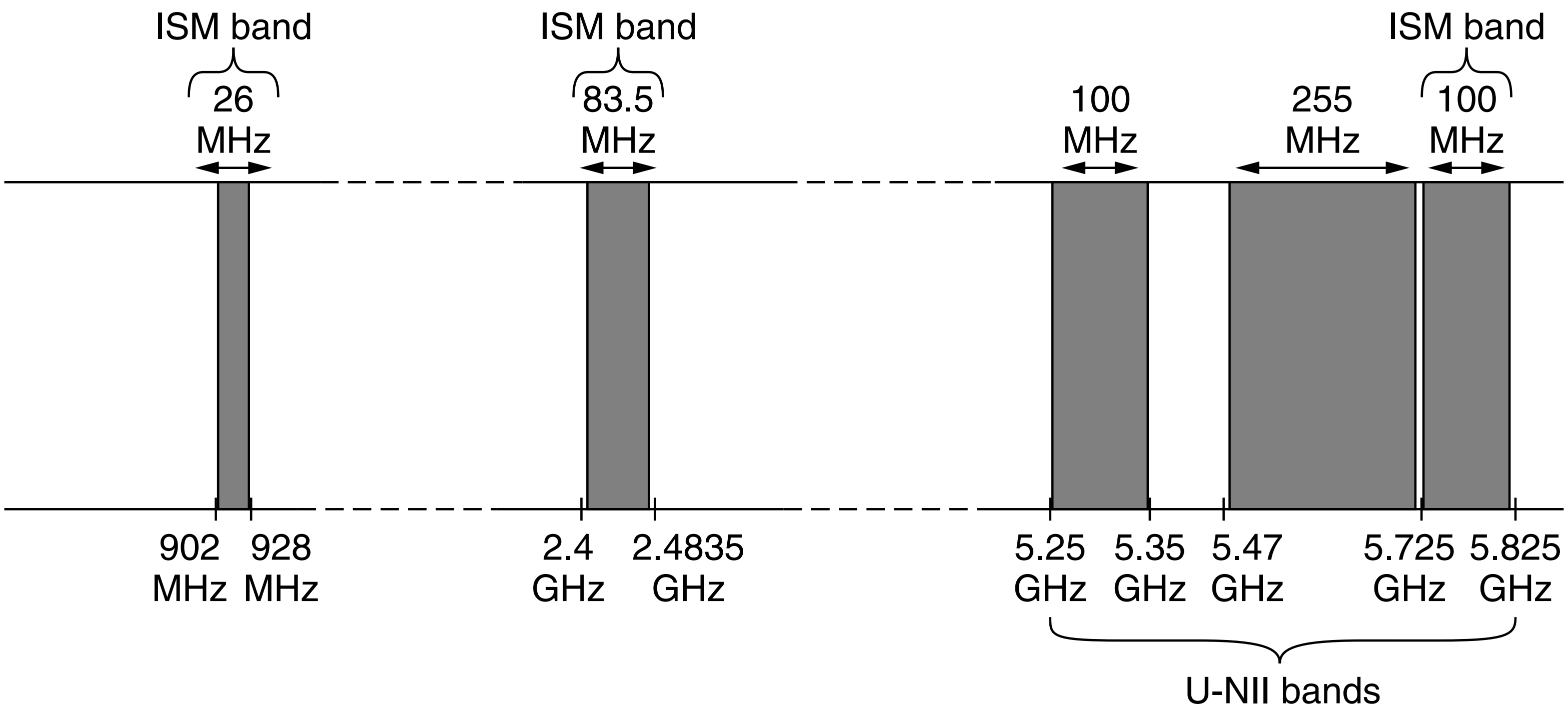
- So far we have discussed communication pulses with spectra centred at DC
  - Basband signalling





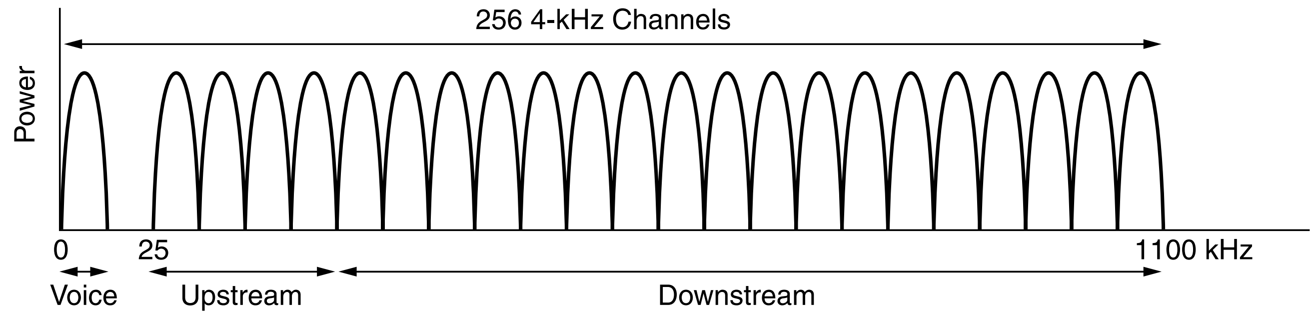
Bandpass Channels

- But there are channels not conducive to this
  - Wireless

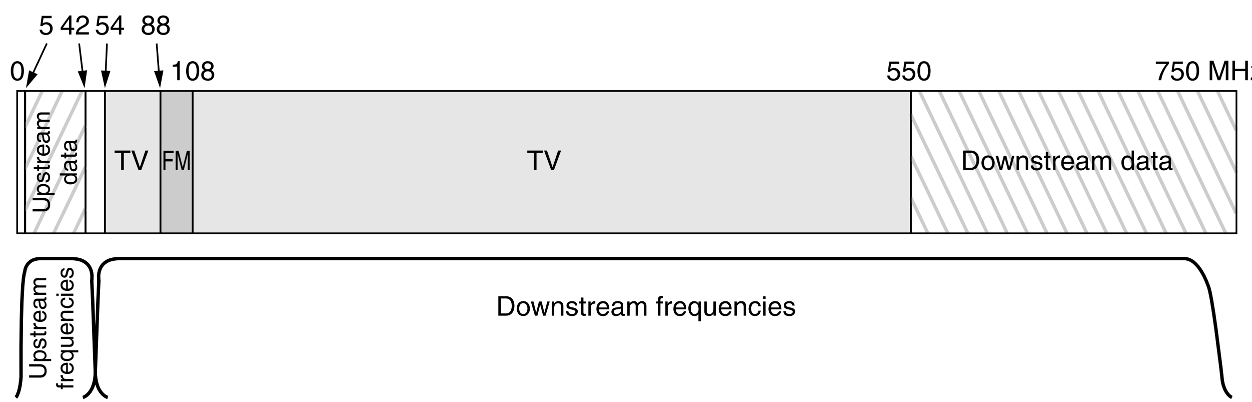


Bandpass Channels

- Wired can also be bandpass
  - ADSL

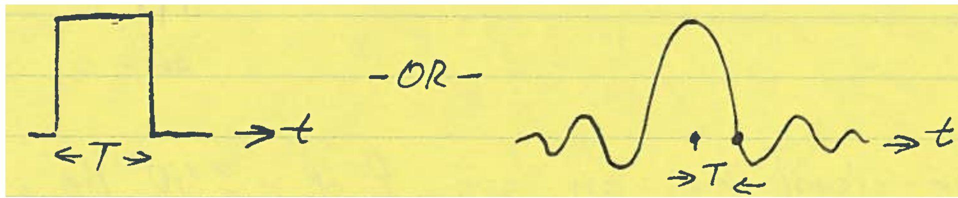


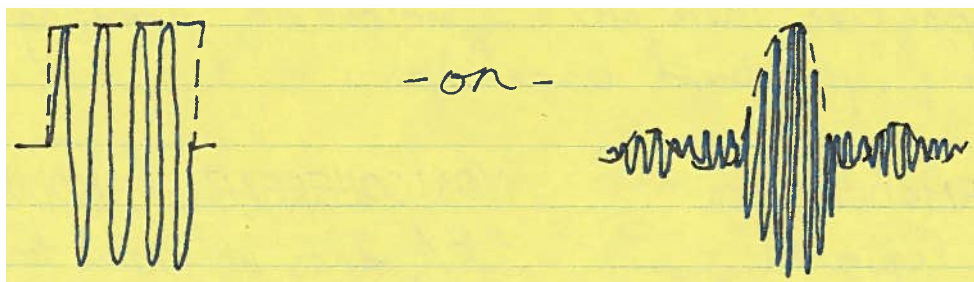
  - Cable



Bandpass Signalling

- Instead of pulse you send modulated sine waves





$$s(t) = A(t)\cos[\omega_o t + \phi(t)]$$

- Which result in bandpass channels (in frequency)
  - With less spectral efficiency!
    - At first look

Bandpass Signal Parameters

- Three main parameters to control

$$s(t) = A(t)\cos[\omega_o t + \phi(t)]$$

  - amplitude: $A(t)$
  - frequency: ω_o
  - phase: $\phi(t)$

- These extra "levers" allow you to squeeze in more data per symbol
  - We'll see this more clearly soon

A Note on Signal Settings

- Common to designate amplitude in terms of signal energy per symbol

$$A_{rms} = \left\{ \frac{1}{T} \int_0^T A^2 \cos^2(\omega_o t) dt \right\}^{\frac{1}{2}} = \frac{A}{\sqrt{2}}$$

$$A = \sqrt{2} A_{rms} = \sqrt{2A_{rms}^2} = \sqrt{2P} = \sqrt{\frac{2E}{T}}$$

  - P = avg. power per symbol

A Peak at Demodulation

- Coherent Demodulation
  - RX has to know the phase of your sine wave to pick out the data
    - Carrier recover needed
- Non-Coherent Demodulation
  - RX does not need to know the phase of sine wave

14.3 PSK: Phase Shift Keying

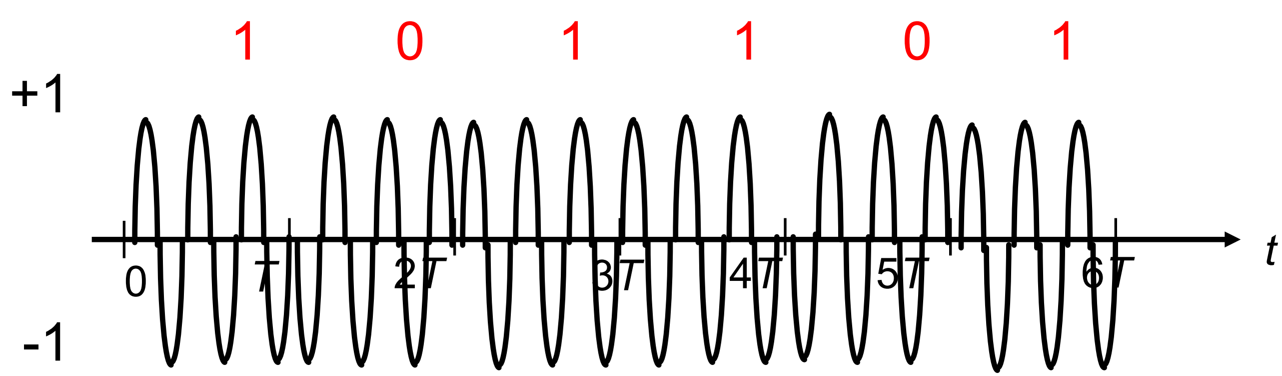
- Symbols have different phase

$$s_i(t) = \sqrt{\frac{2E}{T}} \cos[\omega_o t + \phi_i(t)], \quad 0 \leq t \leq T, i = 1, \ldots, M$$

$$\phi_i(t) = \frac{2\pi i}{M}$$

- BPSK: Binary Phase Shift Keying $\phi_1(t) = \pi \quad \phi_2(t) = 0$

Binary Phase Shift Keying (BPSK)

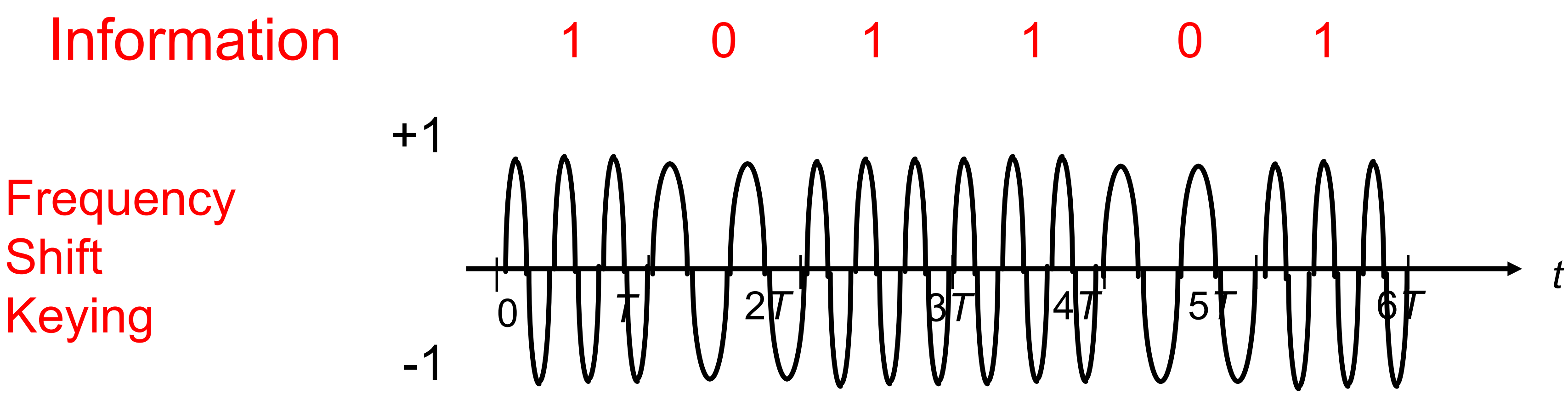


14.4 FSK: Frequency Shift Keying

- Symbols have different frequency
  - ω_i has M discrete values

$$s_i(t) = \sqrt{\frac{2E}{T}} \cos[\omega_i t + \phi], \quad 0 \leq t \leq T$$

  - Binary example:



14.5 ASK: Amplitude Shift Keying

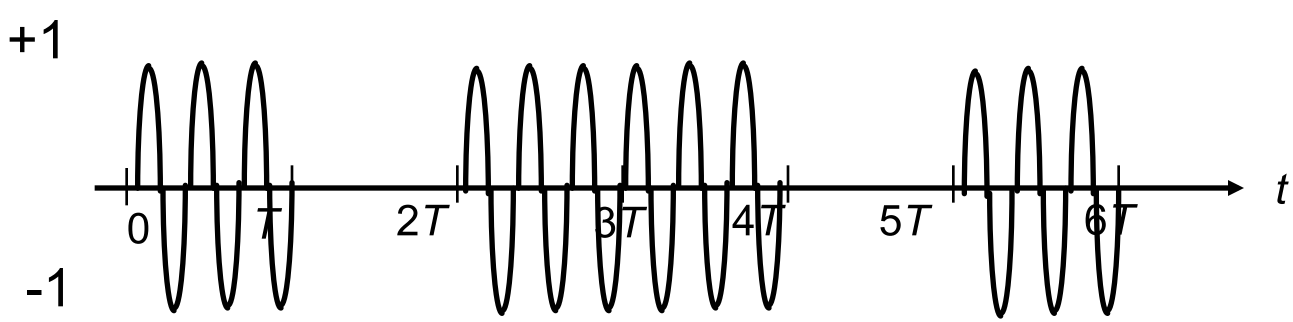
- Amplitude takes on different value for each symbol

$$s_i(t) = \sqrt{\frac{2E_i}{T}} \cos[\omega_o t + \phi], \quad 0 \le t \le T$$

- Binary example
  - OOK (On-Off Keying)

Information 1 0 1 1 0 1

Amplitude Shift Keying



14.6 APK: Amplitude-Phase Keying

- A combination of ASK & PSK

$$s_i(t) = \sqrt{\frac{2E_i}{T}} \cos[\omega_o t + \phi_i], \quad 0 \leq t \leq T$$

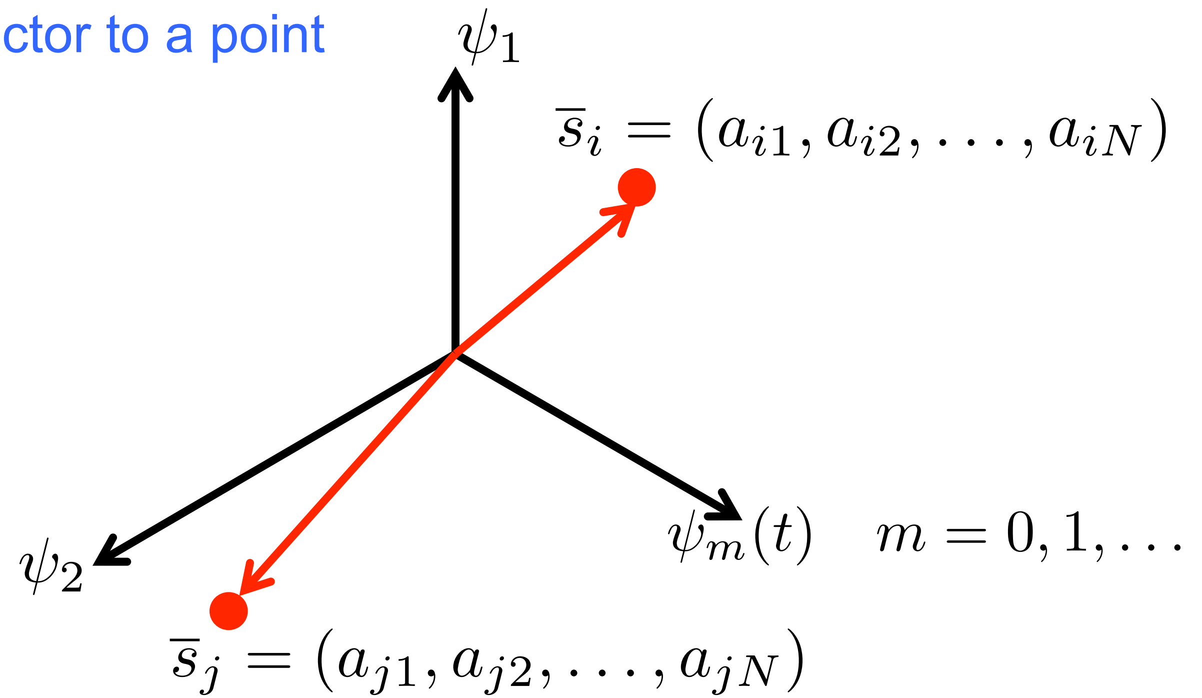
- QAM: Quadrature Amplitude Modulation

14.7 Signal-Space Concepts

- A more general description of signals is possible
  - Not just bandpass signals
  - But also signal sequences!
- The description consists of…
  - …sequence of orthogonal signals
- These descriptions allow more efficient organization of receiver structures for sophisticated signals

Signal Space

- Your signal can be represented as a point in an abstract space
  - Rather a vector to a point



- The axes represent orthogonal signals
  - Which constitute any signal you may wish to define in this space

Basis Functions

- Basis Functions
  - The functions that define our signal space coordinates

$$\psi_m(t) \quad m = 0, 1, \ldots$$

  - Defined over some time interval T_i to T_f
    - T_i: initial time, T_f: final time
  - Make sure the functions are PERPENDICULAR to each other
    - Orthonormal

$$\int_{T_i}^{T_f} \psi_i(t)\psi_j(t)dt = \delta_{ij} = \begin{cases} 0, & \text{if } i \neq j \\ 1, & \text{if } i = j \end{cases}$$

Basis Function Example

- Fourier series coefficients

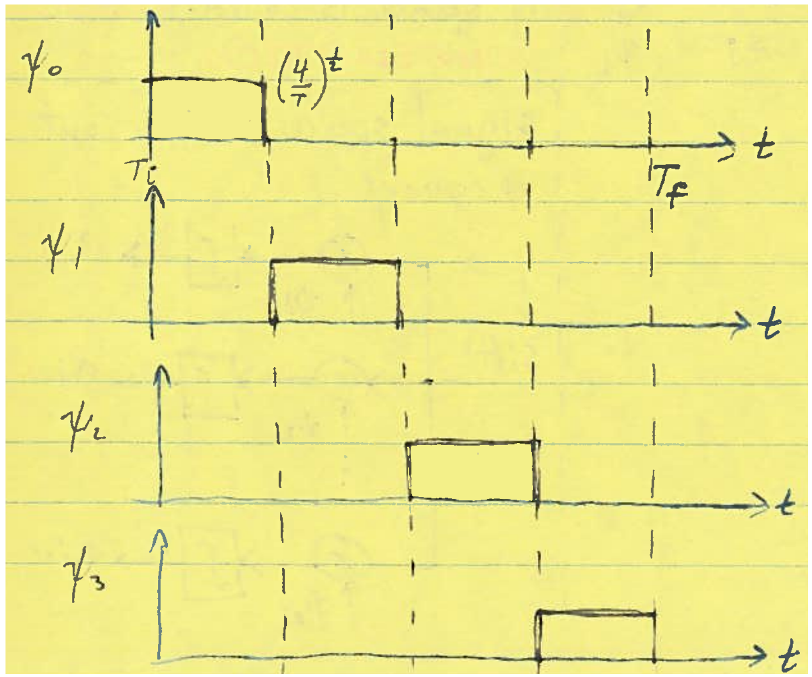
$$\psi_m(t) = \left\{ \left(\frac{1}{T}\right)^{\frac{1}{2}}, \left(\frac{2}{T}\right)^{\frac{1}{2}} \cos(m\omega_o t), \left(\frac{2}{T}\right)^{\frac{1}{2}} \sin(m\omega_o t), \ldots \right\} \; m = 1, 2, \ldots$$

$$T = T_f - T_i$$

$$T = \frac{2\pi}{\omega_o}$$

Basis Function Example

- Finite set of N non-overlapping pulses



Signals in Space

- Any of *M* symbols: $s_i(t),\ i = 1, 2, \ldots, M$
- Can be expressed as a *N*-term series expansion
  - of orthonormal basis functions

$$s_i(t) = \sum_{j=1}^{N} a_{ij}\psi_j(t)$$

  - expansion coefficients a_{ij}

$$a_{ij} = \int_{T_i}^{T_f} s_i(t)\psi_j(t)dt$$

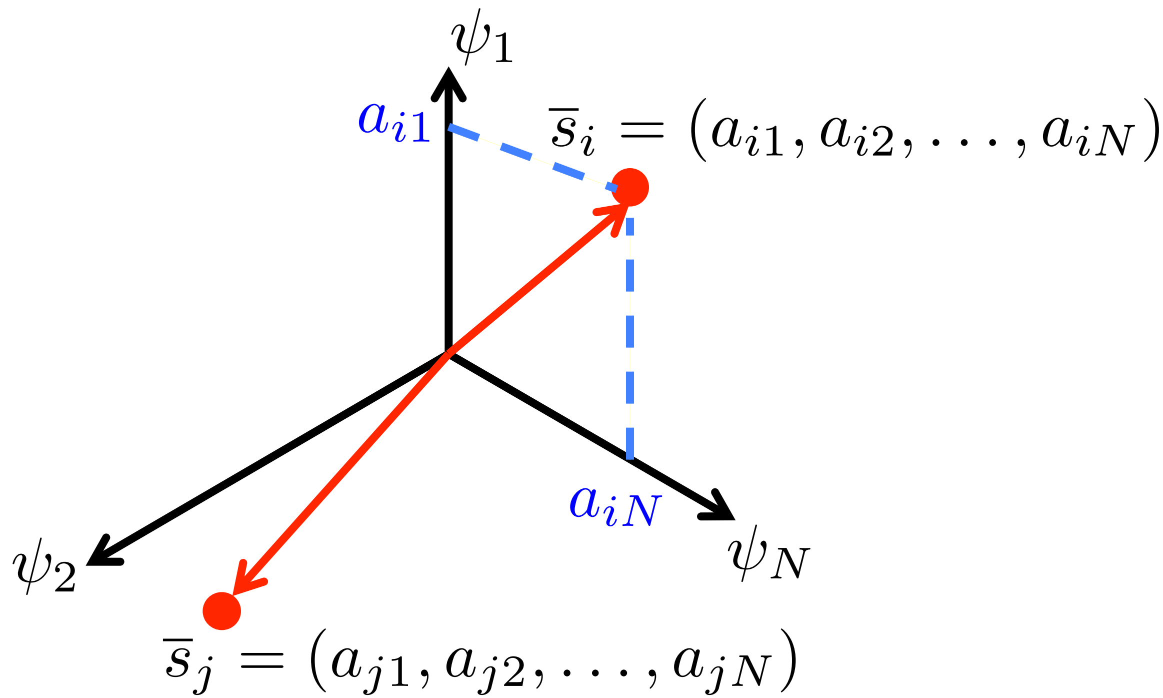
    - projection of *i*th symbol on *j*th basis function

Signal Space Pictorially

- Collection of M points in N-space
  - Signal Constellation

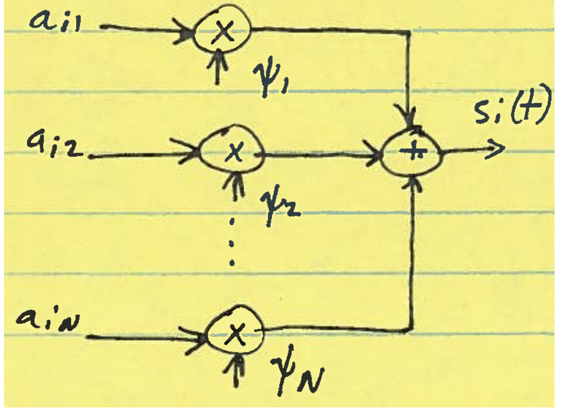
$$s_i(t) = \sum_{j=1}^{N} a_{ij}\psi_j(t)$$

$$a_{ij} = \int_{T_i}^{T_f} s_i(t)\psi_j(t)dt$$



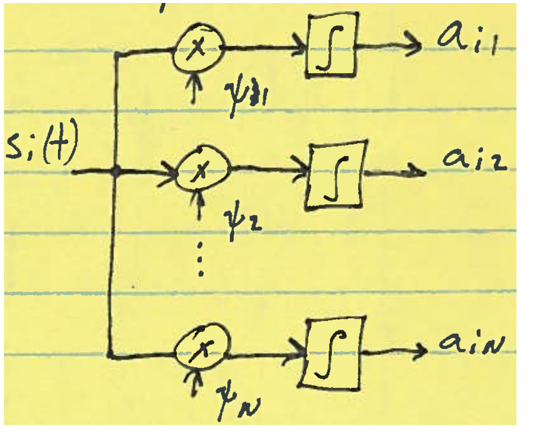
Signal Generation & Recovery

- Generation



$$s_i(t) = \sum_{j=1}^{N} a_{ij}\psi_j(t)$$

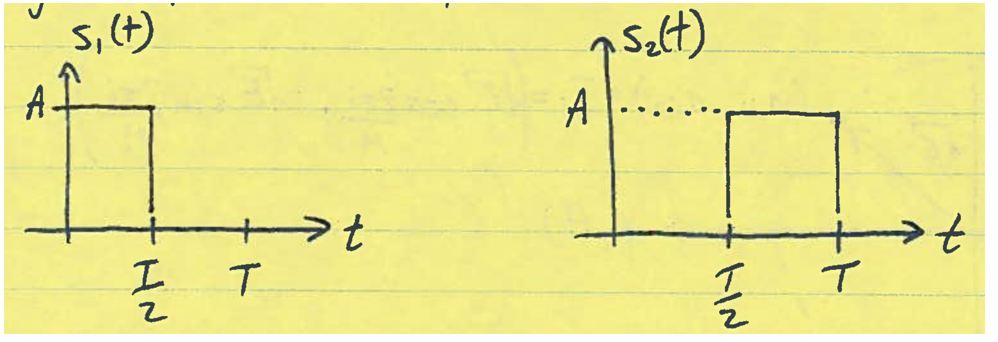
- Recovery



$$a_{ij} = \int_{T_i}^{T_f} s_i(t)\psi_j(t)dt$$

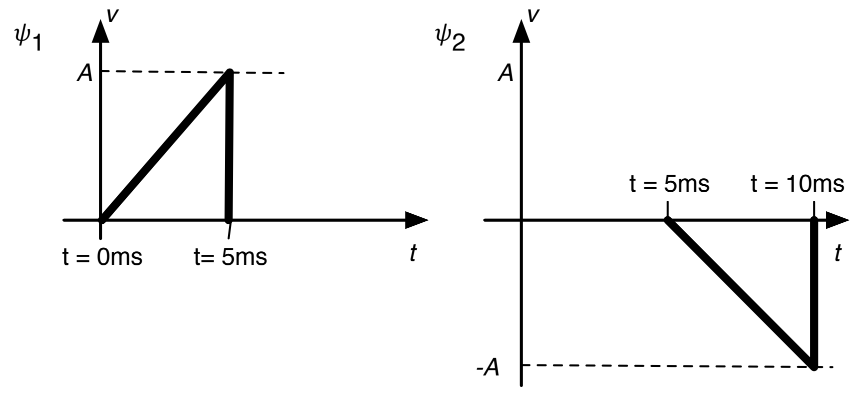
14.8 Signal Space Examples

- Draw the signal constellation



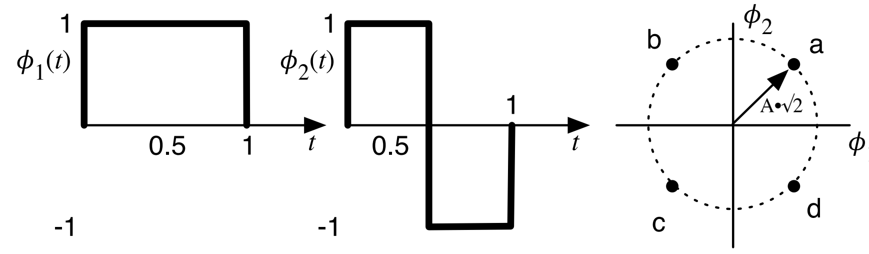
Signal Space Example

- What value of A is needed to make these basis functions orthonormal?



Signal Space Example

- For the basis functions shown, sketch the signal waveforms corresponding to the indicted constellation points



M-ary PSK in Signal Space

- The M-ary PSK

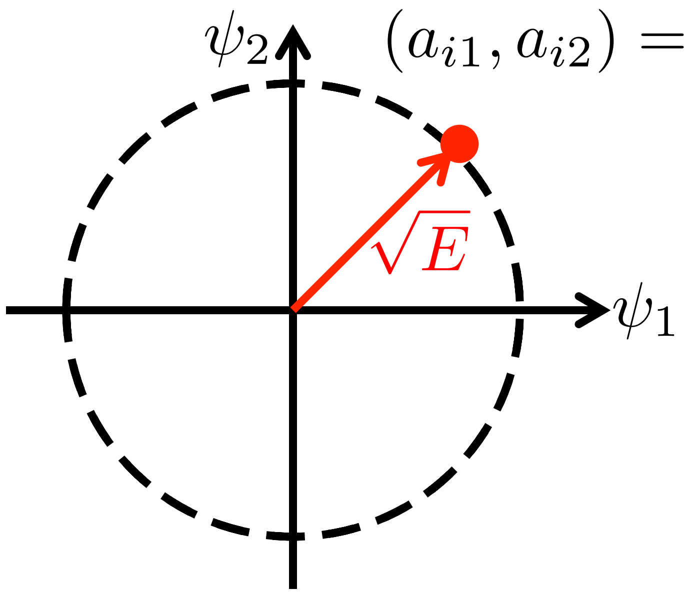
$$s_i(t) = \sqrt{\frac{2E}{T}} \cos\left(\omega_o t + \frac{2\pi i}{M}\right), \ \ i = 0, \ldots, M-1$$

- Just a harmonic signal with some phase
- Fourier series only needs 2 coefficients for this
  - Hence our basis functions can be…

$$\psi_1(t) = \sqrt{\frac{2}{T}} \cos(\omega_o t) \qquad \psi_2(t) = \sqrt{\frac{2}{T}} \sin(\omega_o t)$$

M-ary PSK Constellation

- The resulting constellation is



$$(a_{i1}, a_{i2}) = \bar{s}_i = \left(\sqrt{E} \cos \frac{2\pi i}{M}, \sqrt{E} \sin \frac{2\pi i}{M} \right)$$

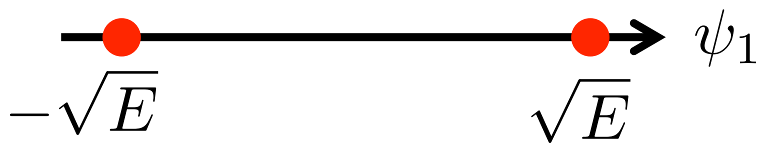
$$a_{ij} = \int_{T_i}^{T_f} s_i(t) \psi_j(t) dt$$

$$s_i(t) = \sqrt{\frac{2E}{T}} \cos\left(\omega_o t + \frac{2\pi i}{M} \right), \;\; i = 0, \ldots, M-1$$

2-ary PSK = BPSK

- Constellation is… $\phi_1(t) = 0 \quad \phi_2(t) = \pi$

$$-\sqrt{\frac{2E}{T}}\cos(\omega_o t) = s_2(t) \qquad s_1(t) = \sqrt{\frac{2E}{T}}\cos(\omega_o t)$$



$$\psi_1(t) = \sqrt{\frac{2}{T}}\cos(\omega_o t)$$

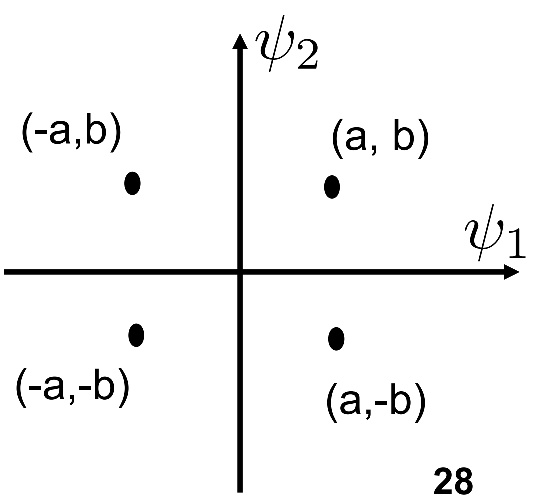
QAM

- Quadrature Amplitude Modulation

$$s_i(t) = a_{i1}\sqrt{\frac{2}{T}}\cos(\omega_o t) + a_{i2}\sqrt{\frac{2}{T}}\sin(\omega_o t)$$

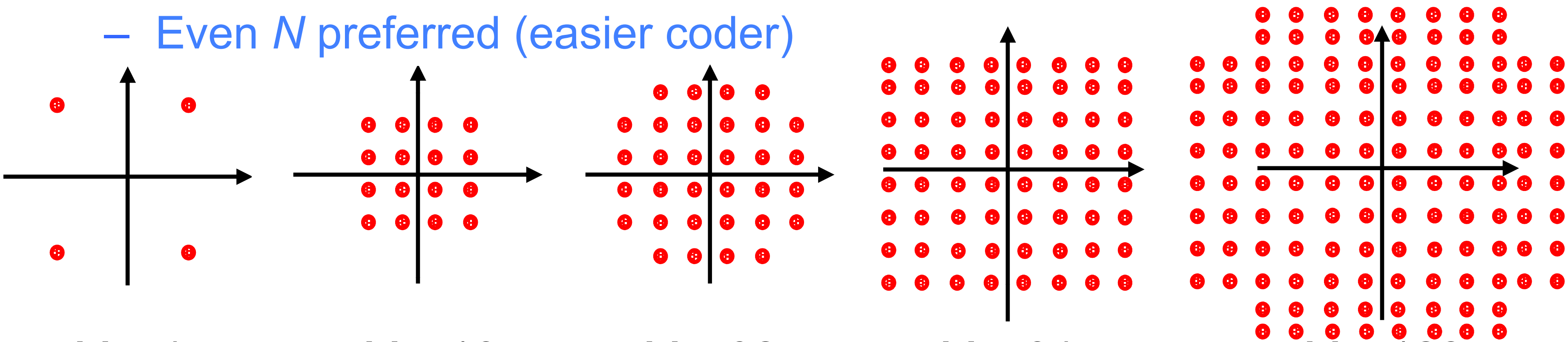
$$s_i(t) = a_i\sqrt{\frac{2}{T}}\cos(\omega_o t) + b_i\sqrt{\frac{2}{T}}\sin(\omega_o t)$$

- Signal space coordinates: (a_i, b_i)
  - Typically chosen from points on a 2D square grid
  - 4-QAM example



Larger QAM Constellations

- With 4-QAM
  - bits represented per symbol: $N = 2$
  - Constellation points: $M = 2^N = 4$
- Many other possibilities (rectangular constellation)
  - $N = 3,4,5,\ldots$
  - Even N preferred (easier coder)



$M = 4$, $L = 2$ | $M = 16$, $L = 4$ | $M = 32$, $L = 6$ | $M = 64$, $L = 8$ | $M = 128$, $L = 12$

- Increasing N requires more power