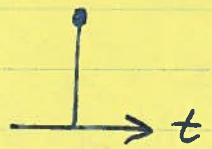
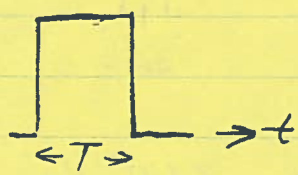
L14 Bandpass Modulation

14.1 Basics

- So far we have represented ~~of~~ our digital messages as impulses...

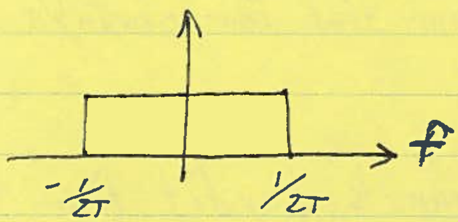


... of discrete amplitudes that are represented as analog waveforms (symbols)...



-OR-

... with spectra that encompass DC...



... and hence are referred to as baseband

- Unfortunately many physical channels have trouble with such low frequencies

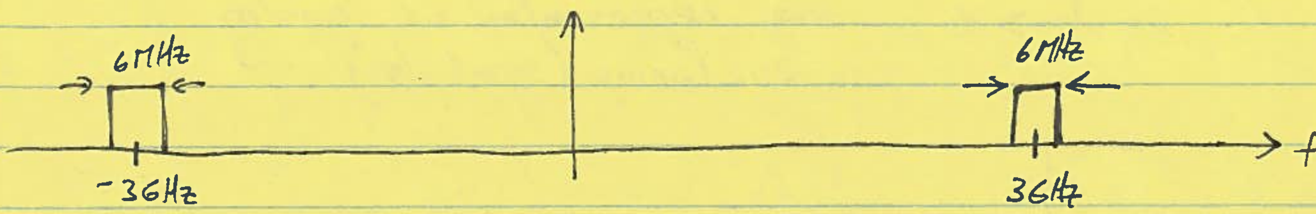
- A prominent example is wireless which require antennas whose size (length) is $\sim\lambda$ (avg. wavelength of the signal being transmitted/received)

... but recall $\lambda = \frac{c}{f}$

- So if you want to send a 10 Mbps signal that fits within ~~z~~ a bandwidth of say 3 MHz

$$\lambda = \frac{3\times10^{8}}{3\times10^{6}} = 100\,m$$ ← the size of an efficient antenna

- In this case you'd rather send



- now your signal has an avg. f of 3×10^{9} Hz

$$\lambda = \frac{3\times10^{8}}{3\times10^{9}} = 10\,cm$$ ... a much more manageable antenna size for portable apps.

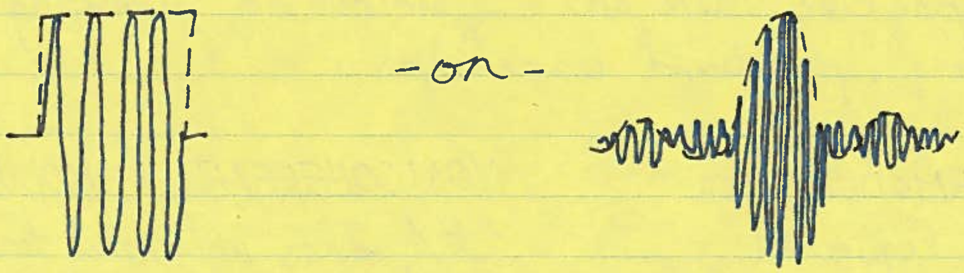
- Of course not only wireless channels benefit from such bandpass signalling

- The ability to shift your signal in frequency sets up opportunities for advanced modulation ~~sc~~ schemes that not only allow you to send signals better matched to the characteristics of your channel, but to share that channel among multiple users very efficiently (channelization)

- For example, ADSL uses a sophisticated bandpass modulation ~~sk~~ scheme to get high bandwidth internet over telephone lines

14.2 Bandpass Signalling

- Instead of just pulses you are sending modulated sine waves



- In general you have...

$$s(t) = A(t)\cos[\omega_0 t + \phi(t)]$$

... and therefore 3 parameters which can be used to represent data

$$\begin{aligned} &\text{amplitude} : A(t) \\ &\left.\begin{aligned} &\text{frequency} : \omega_0 \\ &\text{phase} : \phi(t) \end{aligned}\right\} \text{more generally the angle } \theta(t) \end{aligned}$$

Note: $A_{rms} = \left\{ \frac{1}{T} \int_0^T (A\cos(\omega t))^2 dt \right\}^{\frac{1}{2}} = \frac{A}{\sqrt{2}}$

$$\therefore A = \sqrt{2}\, A_{rms} = \sqrt{2A_{rms}^2} = \sqrt{2\underset{\substack{\uparrow \\ \text{avg. power} \\ \text{per } T}}{P}} = \sqrt{\frac{2E}{T}}$$

- common to designate carrier amplitude in terms of signal energy per symbol

- the high freq. waveform that you are modulating is generally referred to as the carrier

- broadly speaking there are 2 methods of handling (demodulating) passband waveforms

COHERENT DEMODULATION

- RX has to figure out the phase of the carrier (even if that's not the part that your data has modulated)

- more complex
- better P_b

NONCOHERENT DEMODULATION

- RX does not need to figure out phase of carrier

- less complex
- worse P_b

- some bandpass modulation examples

14.3 PSK: Phase Shift Keying

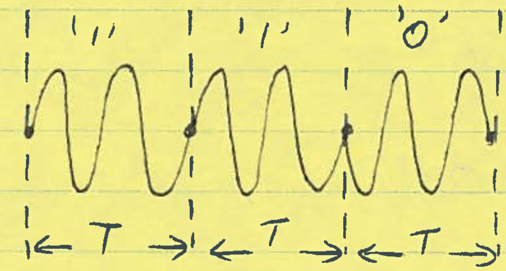
$$s_i(t) = \sqrt{\frac{2E}{T}} \cos[\omega_0 t + \phi_i(t)] \qquad 0 \le t \le T, \quad i = 1, \ldots, M$$

$$\phi_i(t) = \frac{2\pi i}{M}$$

e.g. in the binary case, $M=2$: $\underset{\text{'1'}}{\phi_1(t) = 0}$ $\quad \underset{\text{'0'}}{\phi_2(t) = \pi}$ (or vice-versa)

BPSK

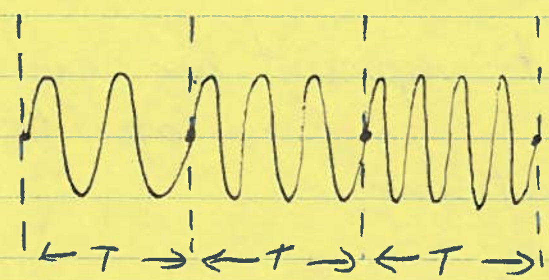
and in time the signal looks like



14.4 FSK: Frequency Shift Keying

$$s_i(t) = \sqrt{\frac{2E}{T}} \cos(\omega_i t + \phi) \qquad 0 \le t \le T$$

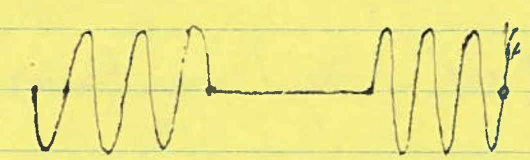
ω_i has M discrete values



14.5 ASK: Amplitude Shift Keying

$$s_i(t) = \sqrt{\frac{2E_i(t)}{T}} \cos(\omega_o t + \phi)$$

- simple 2-level case: ON-OFF KEYING (OOK)



14.6 APK: Amplitude Phase Keying

- A combination of ASK & PSK

$$s_i(t) = \sqrt{\frac{2E_i(t)}{T}} \cos(\omega_0 t + \phi_i(t))$$

- e.g. m-ary QAM

14.7 Signal Space Concepts

- A powerful way to view our symbol waveforms follows a geometric perspective

- In particular, it is possible to represent our signal by an orthonormal function series expansion analogous to classical Fourier series representation

- These functions effectively define the coordinates of some multidimensional space and hence allow us to view our signal (symbol) as some vector in this multidimensional space

- Formally we refer to these coordinate defining functions as basis functions

$$\psi_m(t) \quad m = 0, 1, \ldots \quad (\text{set of real waveforms})$$

- defined over some interval $T_i \rightarrow T_f$

initial final

- number of functions and their interval does not have to be finite

- the functions that define the coordinates of our space must be perpendicular to each other... ORTHONORMAL

$$\int_{T_i}^{T_f} \psi_i(t)\,\psi_j(t)\,dt = \delta_{ij} = \begin{cases} 1 & \text{if } i=j \\ 0 & \text{if } i\neq j \end{cases}$$

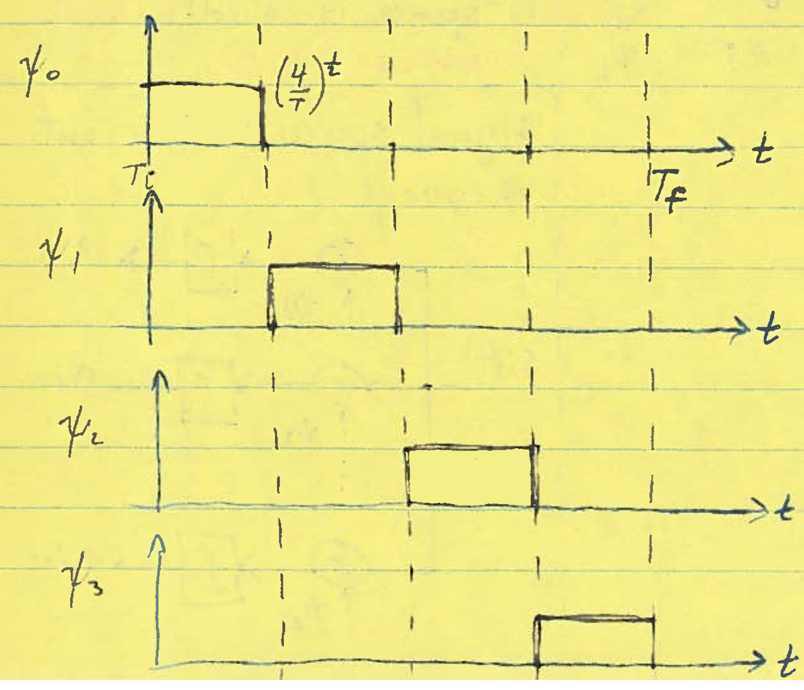
Kronecker delta function

(basis fns. look like they are unit energy)

- e.g. in the case of a fn. described by a Fourier Series (F.S.) the F.S. coefficients define an infinite dimensional space over $T = T_f - T_i$ where $T = \frac{2\pi}{\omega_0}$ and...

$$\ldots\ \psi_m(t) = \left\{ \left(\frac{1}{T}\right)^{\frac{1}{2}}, \left(\frac{2}{T}\right)^{\frac{1}{2}}\cos(m\omega_0 t), \left(\frac{2}{T}\right)^{\frac{1}{2}}\sin(m\omega_0 t), \ldots \right\} \quad m=1,2,\ldots$$

- another example is the finite set of N nonoverlapping pulses



- suppose a class of deterministic signals $s_i(t)$ $i = 1, 2, \ldots M$ (an M-ary set) ~~est~~ each has a finite energy

- we can (it can be shown) express these as an N-term series expansion of orthonormal basis functions

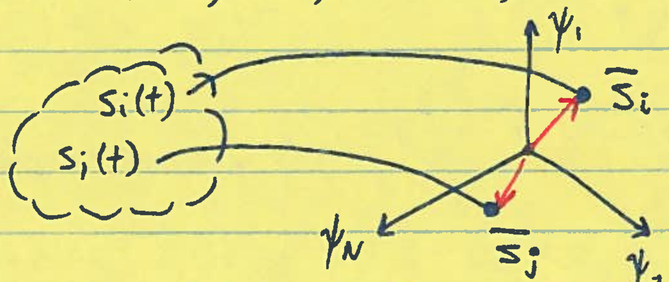
$$s_i(t) = \sum_{j=1}^{N} a_{ij}\,\psi_j(t)$$

- where the expansion coefficients, a_{ij}, are found with

$$a_{ij} = \int_{T_i}^{T_f} s_i(t)\,\underbrace{\psi_j(t)}_{\text{basis fn.}}\,dt$$

} INTERPRET as projection of ith symbol on jth basis function

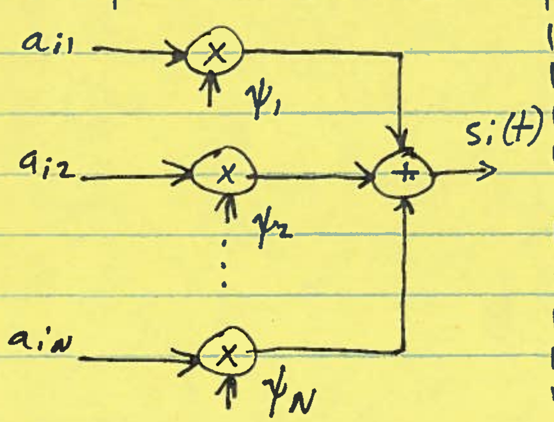
- thus every symbol $s_i(t)$ of M-ary set is mapped onto some N-dimensional space at coordinate $\bar{s}_i = (a_{i1}, a_{i2}, \ldots a_{iN})$... which defines a vector in the...



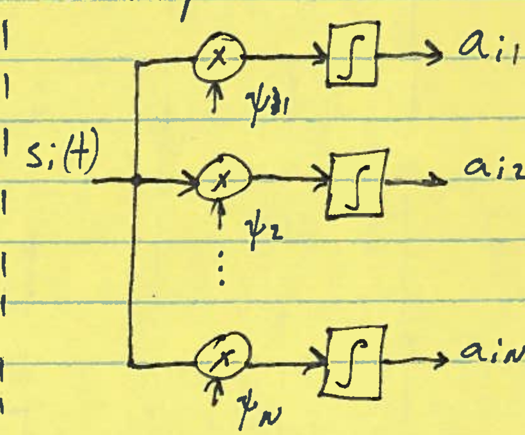
... SIGNAL SPACE

collection of M points in N-space is called: SIGNAL CONSTELLATION

signal generation from signal space coefficients

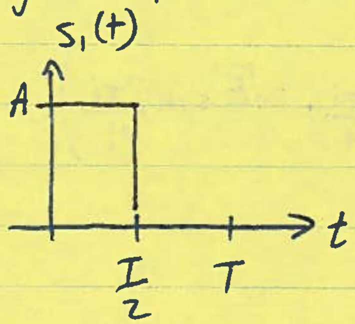


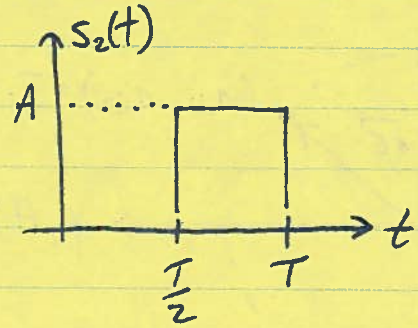
signal space coefficient recovery



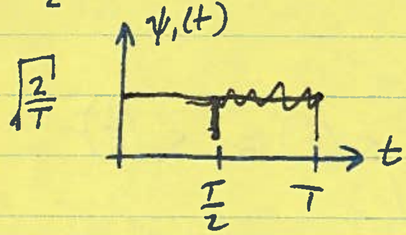
14.8 Signal Space Examples

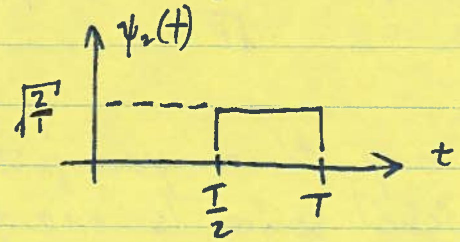
Say



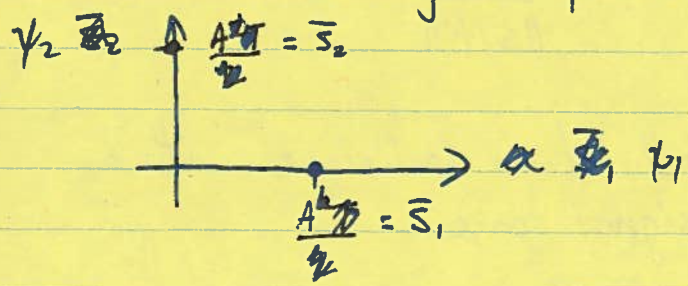


- the original symbols are already orthogonal
- just need to scale these to make orthonormal basis functions
  ($\frac{T}{2} \cdot y^2 = 1 \Leftrightarrow y = \sqrt{2/T}$)





- two-dimensional signal-space representation



try M-ary PSK...

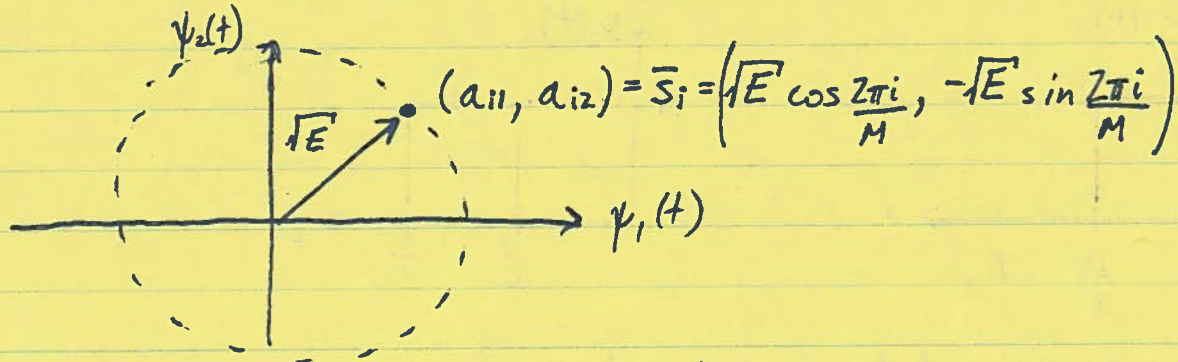
$$s_i(t) = \sqrt{\frac{2E}{T}} \cos\left(\omega_0 t + \frac{2\pi i}{M}\right) \qquad i = 0, 1, \ldots M-1$$

$$0 \le t \le T$$

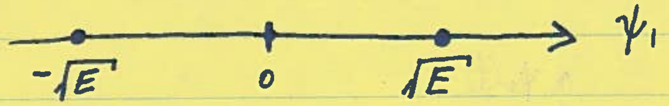
by trig identities obvious that

$$\psi_1(t) = \sqrt{\frac{2}{T}} \cos \omega_0 t \qquad \psi_2(t) = \sqrt{\frac{2}{T}} \sin \omega_0 t$$

so M-PSK sets have 2-dim. constellations



for 2-PSK = BPSK ... we only have a 1-dimensional constellation



- in general any time you just have $s_1(t) = -s_2(t)$
- then your signal space is one dimensional

$$\psi_1 = \frac{s_1(t)}{\|s_1(t)\|}$$

QAM ... a popular modulation technique with a 2-dimensional signal space

$$s_i(t) = a_i\sqrt{\frac{2}{T}}\cos w_0 t + b_i\sqrt{\frac{2}{T}}\sin w_0 t \qquad 0 \le t \le T \quad i = 0, 1, \ldots M-1$$

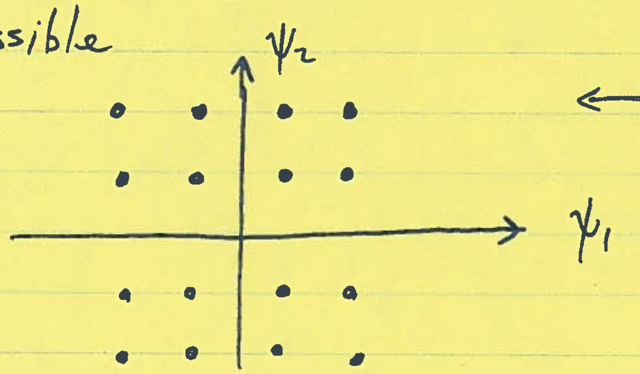
our standard PSK basis functions

if all signals have equal energy: $(a_i^2 + b_i^2)^{\frac{1}{2}} = E^{\frac{1}{2}}$

modulation is strictly through phase and QAM is just M-PSK

- otherwise with QAM all manner of 2-D constellation arrangements are possible

$M = 16$



← such rectangular constellation arrangements are very common

- a common variation is pulse shaped QAM

$$s_i(t) = h_T(t)\left[a_i \cos\omega_0 t + b_i \sin\omega_0 t\right] \quad -\infty < t < \infty \quad i = 0, 1, \ldots, M-1$$

↑ tx filter response

- basis set is modified by h_T ... but 2D signal constellation is unchanged