This mid-term has 6 questions worth a total of 40 points. Answer the questions in the spaces provided on the question sheets. If you run out of room for an answer, continue on the back of the page allotted to that question. Clearly indicate your derivations and circle your final answer.

Last Name: ______________________________

First Name: ______________________________

1. $\boxed{\text{7 points}}$ Deterministic Signals

(a) $\boxed{\text{3 points}}$ What is the average power of $x(t) = 5\sin(5t + 21°)$?

$$P_x = \frac{1}{T_0}\int_0^{T_0} x^2(t)\,dt = \frac{1}{T_0}\int_0^{T_0} V_p^2 \sin^2(\omega_0 t + \phi)\,dt = \frac{V_p^2}{2}$$

$$= \frac{5^2}{2} = \frac{25}{2} = \boxed{12.5}$$

(b) $\boxed{\text{1 point}}$ What kind of signal is $x(t)$ above? Choose one of: energy signal, stochastic signal, power signal, stationary signal.

power signal

(c) $\boxed{\text{3 points}}$ What are the complex Fourier series coefficients of $x(t)$ above?

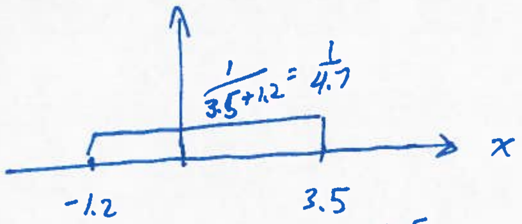
$$\sin a = \frac{e^{ja} - e^{-ja}}{2j} = \frac{-j}{2} e^{ja} + \frac{j}{2} e^{-ja}$$

$$\sin(5t + 21°) = \frac{-j}{2} e^{j5t} e^{j21°} + \frac{j}{2} e^{-j5t} e^{-j21°}$$

$$\therefore \boxed{c_1 = \frac{-j5}{2} e^{j21°} \qquad c_{-1} = \frac{j5}{2} e^{-j21°}}$$

2. $\boxed{\text{7 points}}$ Random Signals

(a) $\boxed{\text{5 points}}$ What is the average power of a signal modelled as a uniformly distributed random variable $-1.2 \leq X \leq 3.5$?



$$E[X^2] = \int_{-1.2}^{3.5} x^2 p(x) dx$$

$$= \frac{1}{4.7} \cdot \frac{x^3}{3} \Big|_{-1.2}^{3.5}$$

$$= \frac{1}{4.7} \cdot \frac{1}{3} (3.5^3 - (-1.2)^3)$$

$$= \frac{1}{4.7} \cdot \frac{1}{3} \cdot 44.6 = \frac{1}{14.1} \cdot 44.6$$

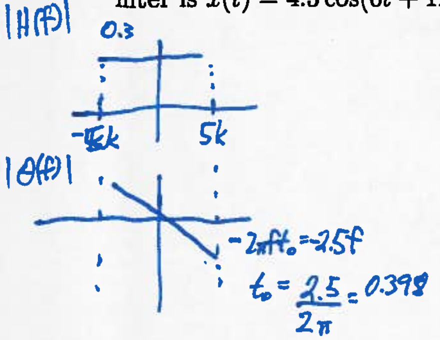
$$= 3.163$$

(b) $\boxed{\text{2 points}}$ You have a sequence $\{x_n\}$ of N random values. Complete the expression below for its autocorrelation (i.e. something is missing from the equation, complete it):

$$R_X[m] = \frac{1}{N-m} \sum_{n=1}^{N-m} x[n] \; x[n+m] \tag{1}$$

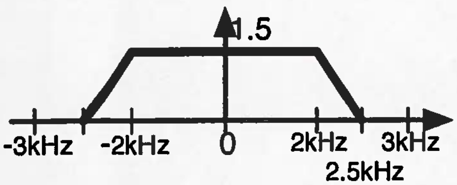
3. $\boxed{\text{9 points}}$ Systems and Signals

(a) $\boxed{\text{3 points}}$ The transfer function of an ideal low pass filter is $0.3e^{-jf2.5}$ below its bandwidth of 5-kHz. If the input to the filter is $x(t) = 4.3\cos(6t + 12°)$ what is the output $y(t)$?



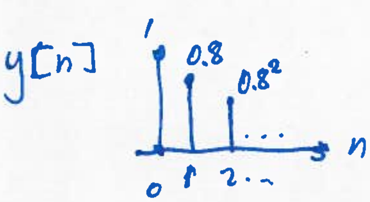
$$\therefore y(t) = 0.3 \times 4.3 \cos(6[t - 0.398] + 12°)$$
$$= 1.29 \cos(6[t - 0.398] + 12°)$$
$$= 1.29 \cos(6t - 124.78°)$$

(b) $\boxed{\text{4 points}}$ What is the noise equivalent bandwidth of the signal with PSD as drawn below?



$$W_n \times G_x(0) = P_x$$
$$W_x \times \cancel{1.5} = \cancel{1.5} \times 2 + \frac{\cancel{1.5} \times 0.5}{2}$$
$$W_n = 2 + \frac{1}{4} \doteq 2.25\,kHz$$

(c) $\boxed{\text{2 points}}$ For the filter with relation: $y[n] = 0.8y[n-1] + x[n]$ for $n \geq 0$ and $y[-1] = 0$ write a compact expression for the impulse response $h[n]$.



$$h[n] = \begin{cases} 0.8^n & \text{for } n \geq 0 \\ 0 & \text{else} \end{cases}$$

4. $\boxed{\text{9 points}}$ Sampling and Quantization

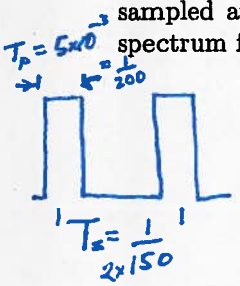
(a) $\boxed{\text{3 points}}$ Determine the minimum sampling rate necessary to sample and perfectly reconstruct the signal $x(t) = \sin(4321t)/(4321t)$

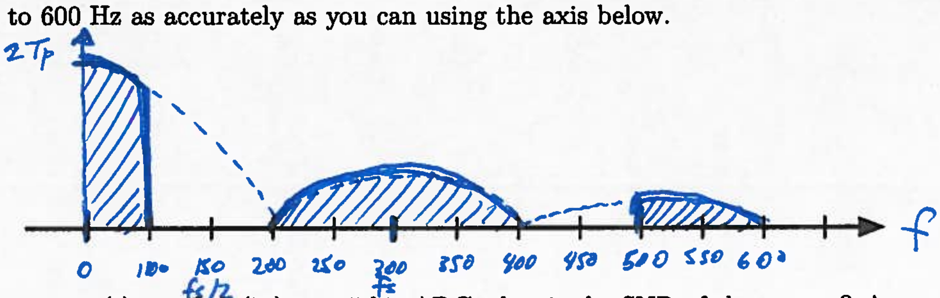
$$\frac{\sin(4321t)}{4321t} = \frac{\sin(\pi t/T)}{\pi t/T} = \mathrm{sinc}(t/T) \qquad \frac{\pi}{T} = 4321 \qquad \frac{1}{T} = \frac{4321}{\pi}$$

$$f_m = \frac{1}{2T} = \frac{4321}{2\pi} = 687.7$$

$$\therefore f_s = 2\times 687.7 = 1.375\times 10^3$$

(b) $\boxed{\text{3 points}}$ A signal's spectral magnitude is equal to 2 for $|f| \leq 100$ Hz and 0 for $|f| > 100$ Hz. The signal is flat-top sampled at 1.5 times the Nyquist rate with pulses of 5-millisecond duration. Sketch the magnitude of the resulting spectrum from 0 to 600 Hz as accurately as you can using the axis below.





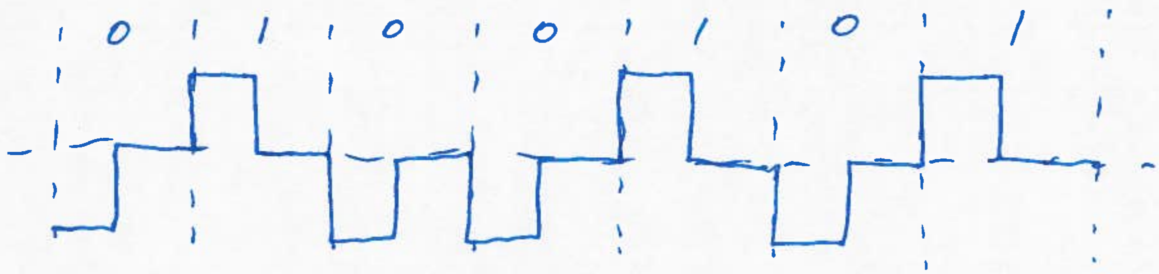
(c) $\boxed{\text{3 points}}$ For an input $x(t) = 0.5\sin(7t)$ to a 5-bit ADC what is the SNR of the output? Assume the ADC's maximum peak-to-peak input range is 1.

$$SNR = \frac{\sigma_x^2}{\sigma_q^2} = \frac{V_p^2}{2} \times \frac{12}{\Delta^2} = \frac{V_p^2}{2} \times \frac{12}{(2\cdot V_p/2^b)^2}$$

$$= \frac{1}{2} \times \frac{12}{4/2^{2b}} = \frac{12}{8} \times 2^{2b} = \frac{3}{2} \times 2^{10} = 6144$$

5. $\boxed{\text{5 points}}$ Line Coding

(a) $\boxed{\text{2 points}}$ Sketch a bipolar RZ signal conveying the bit pattern 0100101.



(b) $\boxed{\text{3 points}}$ What is the average power of a bipolar RZ signal switching between +2 V and -1.5 V. The data rate (that is bits per second) being conveyed is 2.25 Gbps.

$$P_x = \frac{1}{2}(2)^2 \times \frac{1}{2} + \frac{1}{2}(1.5)^2 \times \frac{1}{2}$$

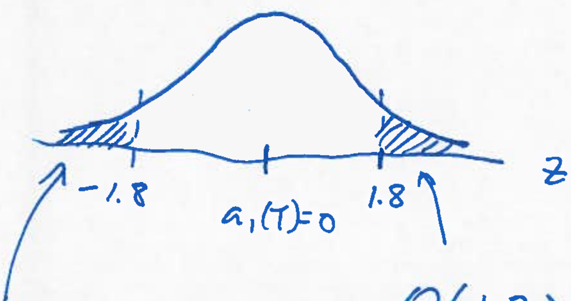
$$= 1 + \frac{2.25}{4}$$

$$= 1 + 0.5625$$

$$= 1.5625$$

6. $\boxed{\text{3 points}}$ Digital detection

(a) $\boxed{\text{3 points}}$ A detector receives a signal $z(T) = a_1(T) + n_o(T)$. If $a_1(T) = 0$ and $n_o(T)$ is noise with variance of 0.25 what is the probability that $z(T)$ is not between -1.8 and 1.8?



$\sigma^2 = 0.25$

$\sigma = 0.5$

$1 - Q\left(\frac{-1.8}{0.5}\right)$

$= Q\left(\frac{1.8}{0.5}\right)$

$Q\left(\frac{1.8}{0.5}\right)$

$P = 2Q\left(\frac{1.8}{0.5}\right) = 2Q(3.6) = 2 \times 1.6 \times 10^{-4}$

$= 3.2 \times 10^{-4}$