Concurrent Depth-First Search Algorithms

Mohamad Alsabbagh

Department of Electrical Engineering and Computer Science
York University, Toronto

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Problems Tarjan’s algorithms solve

Tarjan's Algorithms solve three related problems relevant to model checking. Given a state graph;

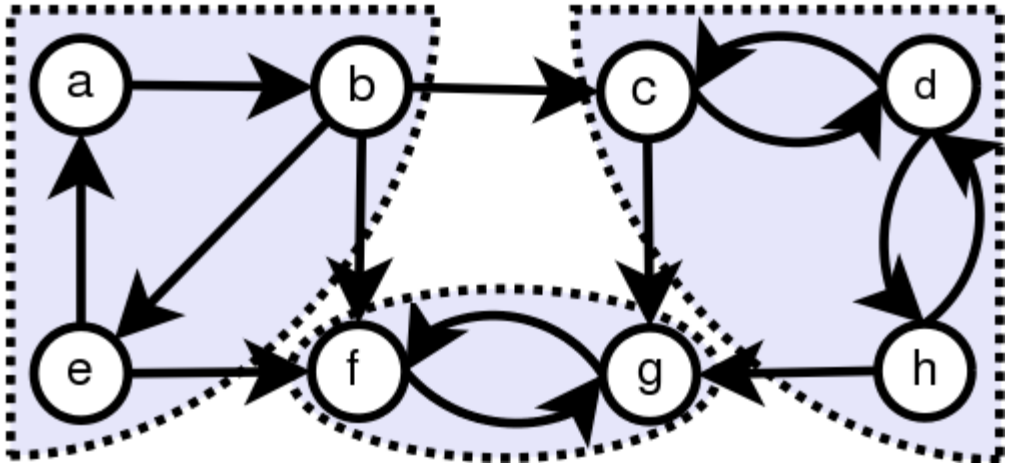
1. Find its Strongly Connected Components (SCCs)
2. Identify which nodes are in a loop
3. Locate which nodes are in a lasso

Why are these problems important?

- Lassos: FDR or failure-divergence refinement.
- SCCs: useful for performing compression on the transition graphs.
- Loops: important in linear temporal logic (LTL) model checking.

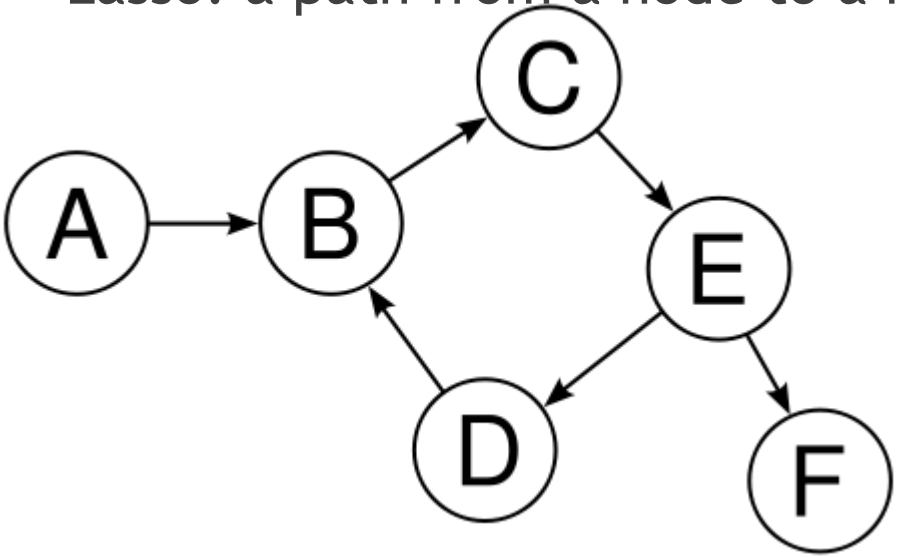
Strongly Connected Components

A directed subgraph that satisfy Strongly Connected attribute.



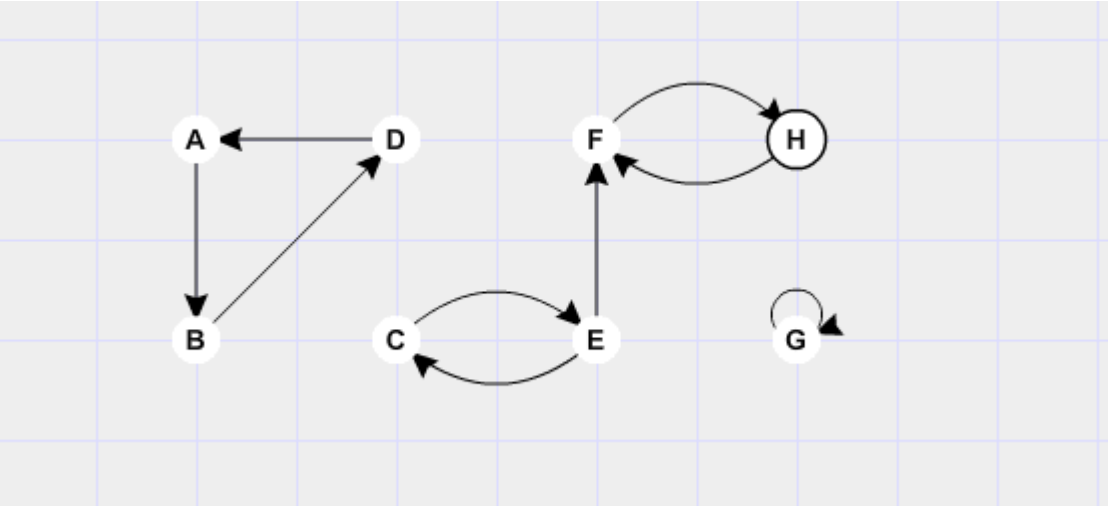
Loops & Lassos

Loop: a node is part of a direct cycle
Lasso: a path from a node to a node on a cycle



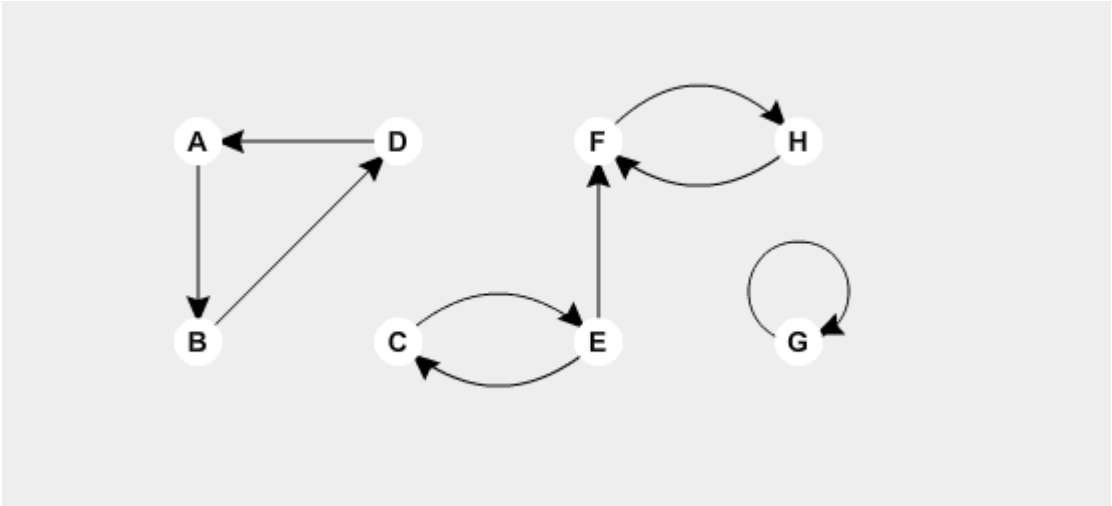
Sequential Tarjan's Algorithm

Depth-First Search to identify SCCs.



Concurrent Tarjan's Algorithm

A single concurrent version of Tarjan’s algorithm to identify SCCs

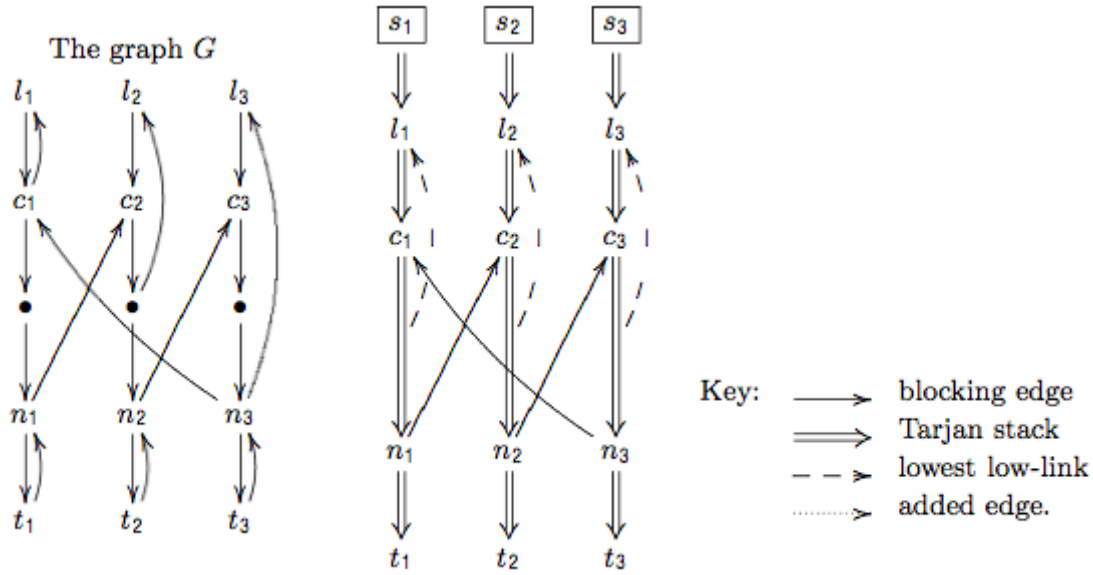


Tarjan's Node Structure

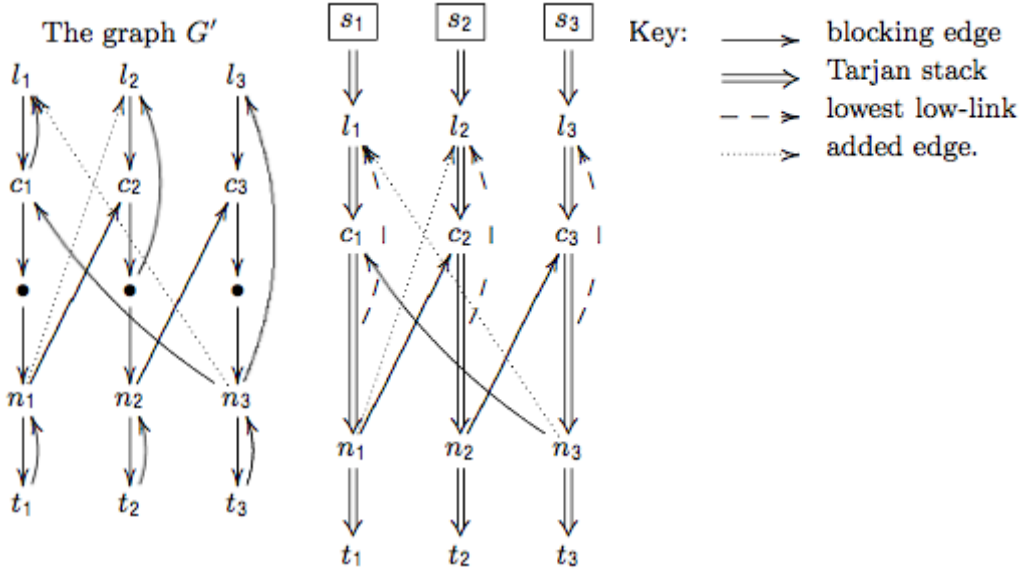
Each node in the graph G has the following attributes:

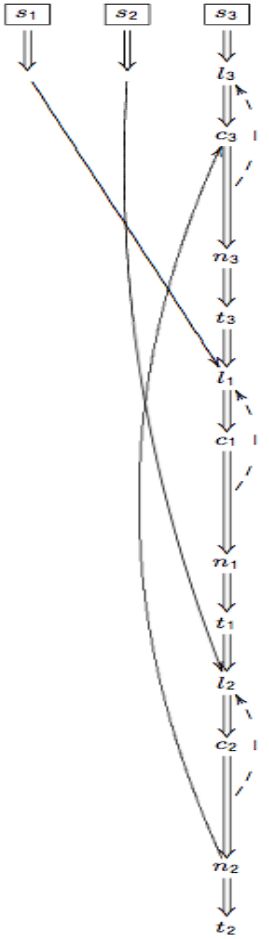
- index (sequential and concurrent):
  - which is a sequence counter, corresponding to the order in which nodes were encountered
- lowlink (sequential and concurrent):
  - which records the smallest index of a node n′ in the stack that is reachable via the descendents of n fully considered so far
- search (concurrent):
  - identifying which search a node belongs to

Circular Dependency



Circular Dependency Node Transfer





Circular Dependency Node Transfer

When transferring a node from s_1 to s_3 we will need to recalculate its index amd lowlink values:

- $\delta_1 = (s_3.\text{index} - l_1.\text{index})$
- we add δ_1 onto the index and lowlink of each node transferred from s_1 and update.

Next Steps

Plan:

- implement all three algorithms
- compare their performance

Q&A

Thanks