Concurrent Depth-First Search Algorithms

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Problems Tarjan’s algorithms solve

Tarjan's Algorithms solve three related problems relevant to model checking. Given a state graph:
1. Find its Strongly Connected Components (SCCs)
2. Identify which nodes are in a loop
3. Locate which nodes are in a lasso
Why are these problems important?

- **Lassos**: FDR or failure-divergence refinement.
- **SCCs**: useful for performing compression on the transition graphs.
- **Loops**: important in linear temporal logic (LTL) model checking.
Strongly Connected Components

A directed subgraph that satisfy Strongly Connected attribute.
Loops & Lassos

Loop: a node is part of a direct cycle
Lasso: a path from a node to a node on a cycle
Sequential Tarjan's Algorithm

Depth-First Search to identify SCCs.
Concurrent Tarjan's Algorithm

A single concurrent version of Tarjan’s algorithm to identify SCCs
Each node in the graph \( G \) has the following attributes:

- **index (sequential and concurrent):**
  - which is a sequence counter, corresponding to the order in which nodes were encountered

- **lowlink (sequential and concurrent):**
  - which records the smallest index of a node \( n' \) in the stack that is reachable via the descendents of \( n \) fully considered so far

- **search (concurrent):**
  - identifying which search a node belongs to
Circular Dependency

The graph $G$

Key:
- blocking edge
- Tarjan stack
- lowest low-link
- added edge.
Circular Dependency Node Transfer

The graph $G'$

Key:
- blocking edge
- Tarjan stack
- lowest low-link
- added edge.
Circular Dependency Node Transfer

When transferring a node from $s_1$ to $s_3$ we will need to recalculate its index and lowlink values:

- $\delta_1 = (s_3.\text{index} - l_1.\text{index})$
- we add $\delta_1$ onto the index and lowlink of each node transferred from $s_1$ and update.
Next Steps

Plan:
- implement all three algorithms
- compare their performance
Q&A

Thanks