Structured Proofs

EECS 4315

www.cse.yorku.ca/course/4315/

Equivalence Relation

Definition

Let X be a set. A relation $R \subseteq X \times X$ is an equivalence relation if for all $x, y, z \in X$

(reflexive) $x \, R \, x$,

(symmetric) if $x \, R \, y$ then $y \, R \, x$, and

(transitive) if $x \, R \, y$ and $y \, R \, z$ then $x \, R \, z$.

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Proposition

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Task

Prove the above proposition.

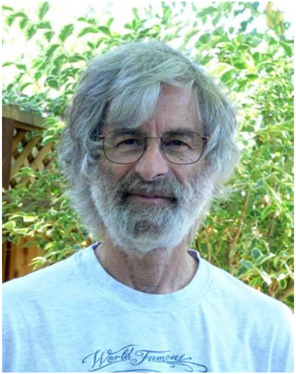
A Proof

Proof

For any $x \in X$, by the symmetry we have $x\,R\,y$ implies $y\,R\,x$. By transitivity we have $x\,R\,y$ and $y\,R\,x$ imply $x\,R\,x$. Therefore, using only symmetry and transitivity, we obtain reflexivity.

Leslie Lamport

- Recipient of the Turing Award (2013)
- Recipient of the Dijkstra Prize (2005)
- Recipient of the IEEE John von Neumann Medal (2008)
- Member of the American Academy of Arts and Sciences (2011)



Source: wikipedia

Proofs

Leslie Lamport. How to write a 21st century proof. *Journal of Fixed Point Theory and Applications*, 11(1):43–63, April 2012.

Today's "proofs are still written in prose pretty much the way they were in the 17th century." ... These "proofs are hard to understand, and they encourage sloppiness that leads to errors."

Proofs

Two principles to make proofs easier to understand:

- structure and
- naming.

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Two principles to make proofs easier to understand:

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- Structure: sequence of named statements, each with a proof.
- Naming: sequential numbers.

Quiz on Thursday

Material covered in Monday's lecture.