LASSONDE SCHOOL OF ENGINEERING | YORK UNIVERSITÉ UNIVERSITY

No. 7

Graphical Models

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Outline

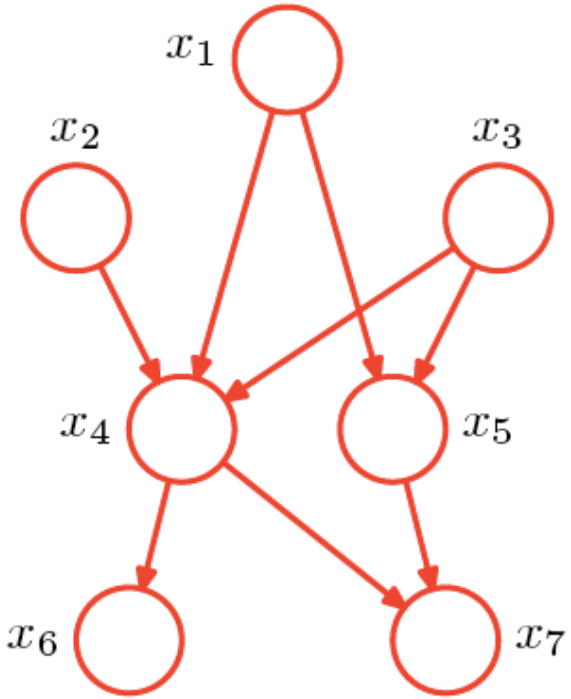
- Graphical Model: concepts
- Graphical Model: types
  - Directed Graphical Models (aka. Bayesian Networks)
  - Indirected Graphical Models (aka. Markov Random Fields)
- Exact Inference
    - Example: a chain model
    - Sum-product algorithm
    - Max-sum algorithm
- Approximate Inference
    - Loopy Belief Propagation
    - Variational Inference
    - Expectation Propagation
    - Monte Carlo Sampling

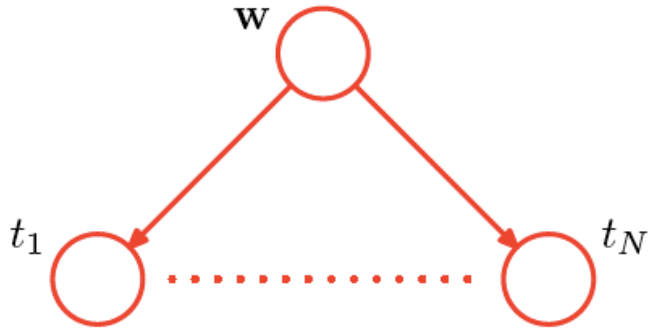
Graphical Model

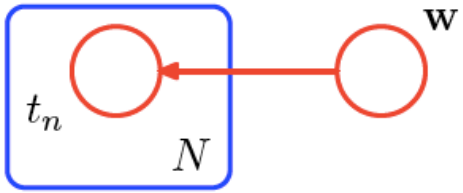
- **Use a graph to represent joint distributions of random variables**
  - **Nodes —> random variables (RV)**
  - **Linking —> dependency among RVs**
- **Graphical Models may imply conditional independence among RVs.**

Bayesian Networks (1)

- **Use a directed graph to represent joint distributions of random variables**
  - **Nodes —> random variables (RV)**
  - **Linking —> conditional distribution of children given the parents**

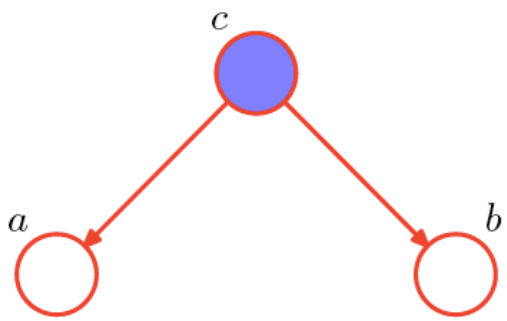


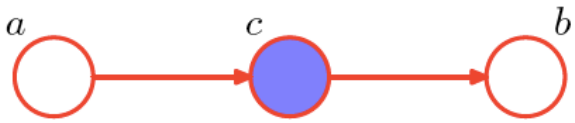


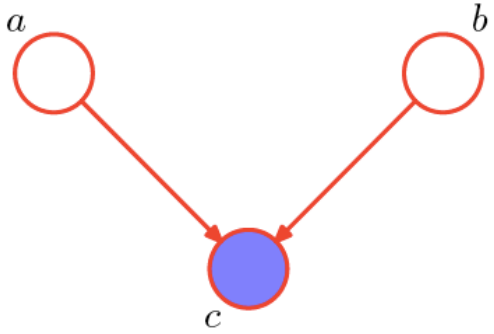


Bayesian Networks (2)

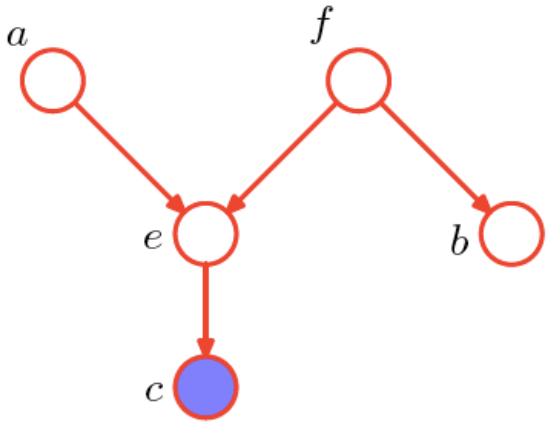
- **Conditional independence in Bayesian Networks**
  - **tail-to-tail, head-to-tail, head-to-head (explain-away)**

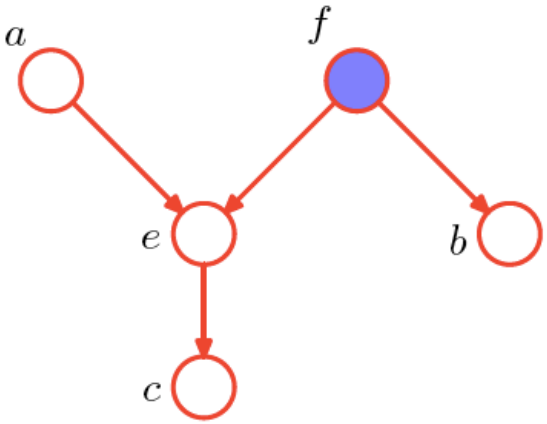






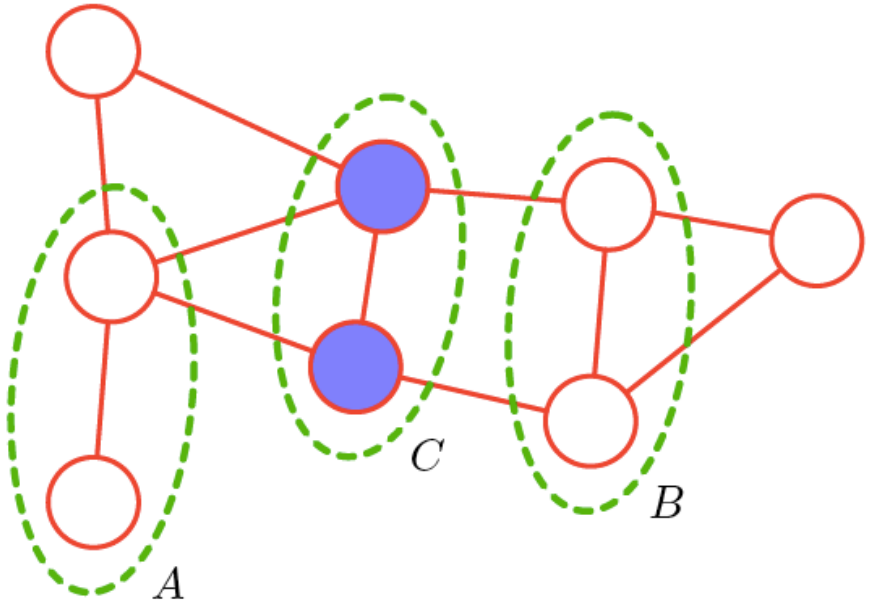
  - **D-separation rule**





Markov Random Fields (1)

- Use an undirected graph to represent joint distributions of random variables
  - Nodes —> random variables (RV)
  - Linking —> conditional dependency
- Conditional independence == simple graph separation



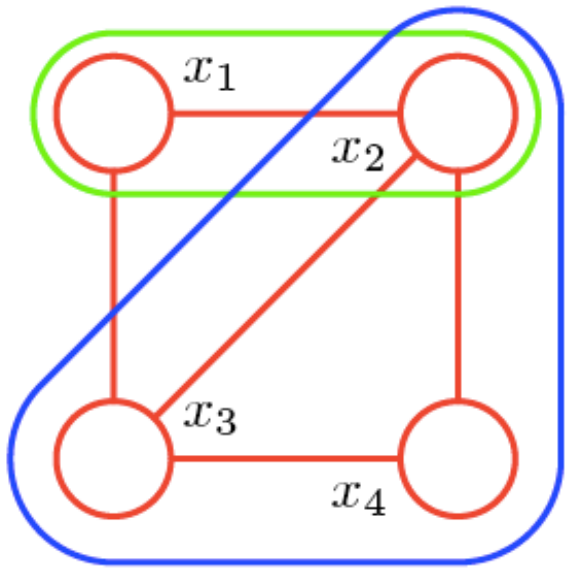
Markov Random Fields (2)

- **How to form the joint probability distribution?**
  - **Potential functions: defined over maximal cliques**

$$p(\mathbf{x}) = \frac{1}{Z} \prod_C \psi_C(\mathbf{x}_C).$$

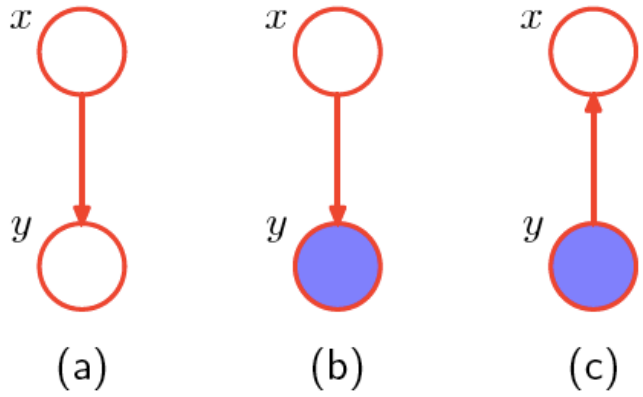
  - **Partition function: normalization constant**

$$Z = \sum_{\mathbf{x}} \prod_C \psi_C(\mathbf{x}_C)$$

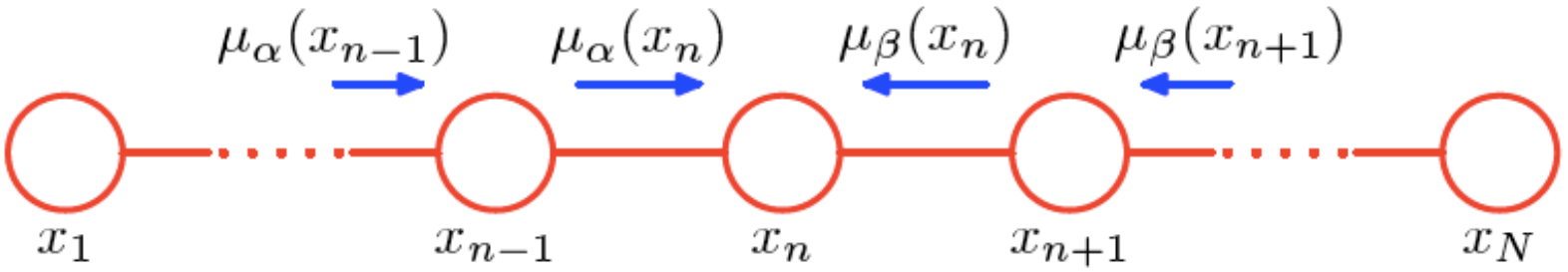


Exact Inference In Graphical Models

- **What is Inference?**
- **Message Propagation for Inference**
  - **Two nodes —> Bayes' theorem**



  - **Inference on a chain**



Exact Inference In Graphical Models

- **Tree-structured Graphical models**
  - **Sum-Product (Max-sum) algorithm**
- **General Graphs**
  - **Junction tree algorithms**
  - **Computationally expensive**

Approximate Inference Methods

- **Loopy Belief Propagation**
- **Variational Inference**
- **Expectation Propagation**
- **Mento Carlo Sampling**