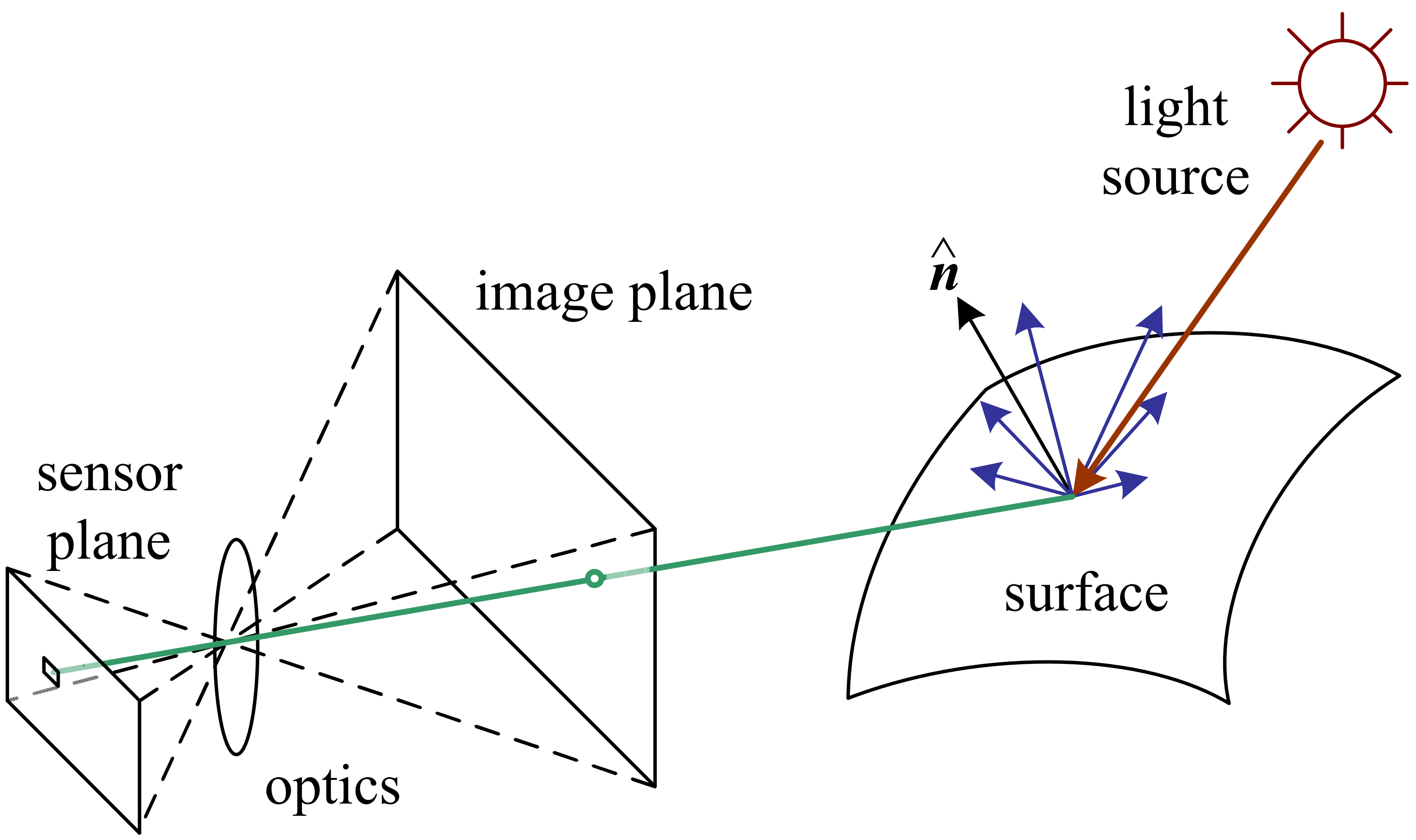
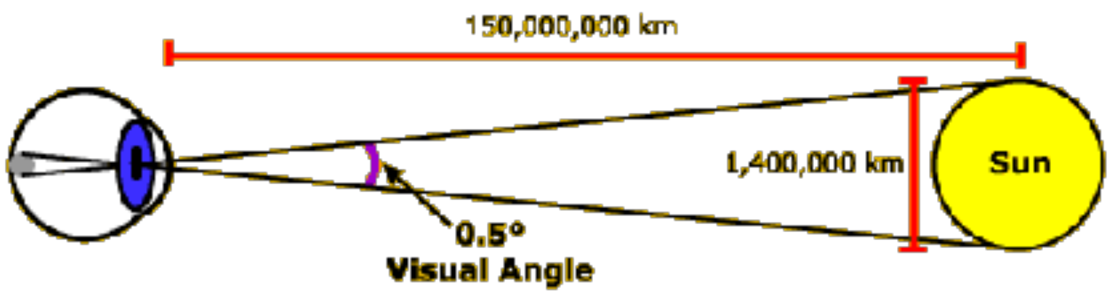
2.2 Photometric Image Formation



Illumination

- Computer vision theory is often developed with the assumption of a point light source at infinity.
- But even the sun has a finite extent (about 0.5 deg visual angle)
- Typical visual environments have more complex illumination



Measuring the Light Field

- The light field at a point can be measured by
  - Taking calibrated photos of a spherical mirror
  - Using a spherical camera

e.g., Southampton-York Natural Scenes Dataset

Spheron HDR Spherical Camera

The BRDF

- The bidirectional reflectance distribution function (BRDF) describes the proportion of light coming from each incident direction that is redirected to each reflected direction, as a function of wavelength.

- the BRDF is reciprocal (can exchange the incident and reflected directions).

$$f_r(\theta_i, \phi_i, \theta_r, \phi_r; \lambda)$$

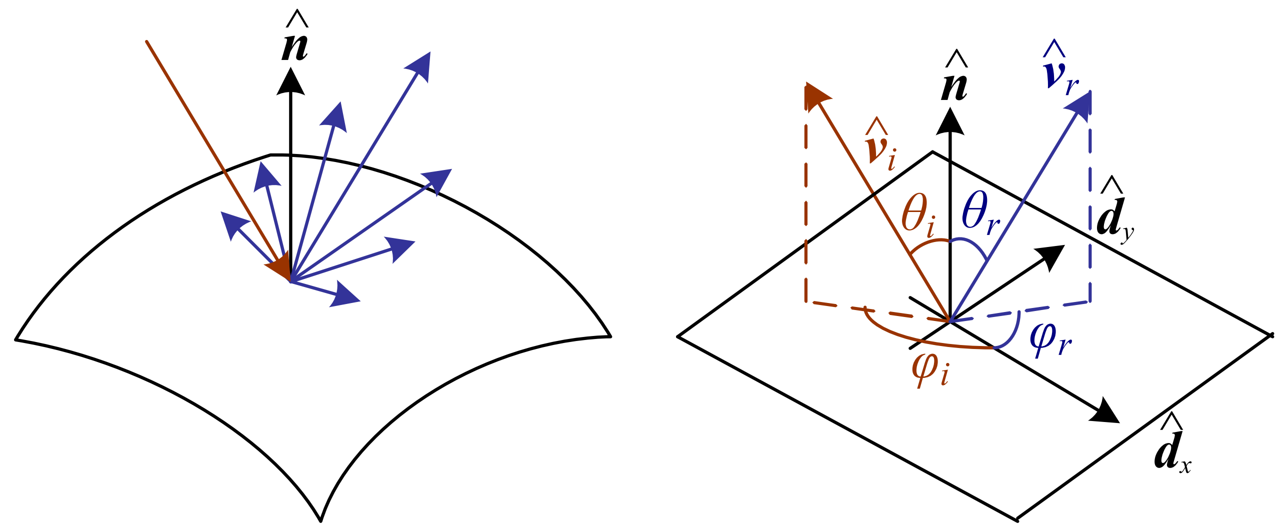
θ_i = elevation of incident light

ϕ_i = azimuth of incident light

θ_r = elevation of reflected light

ϕ_r = azimuth of reflected light

λ = wavelength



The BRDF

- For isotropic surfaces:

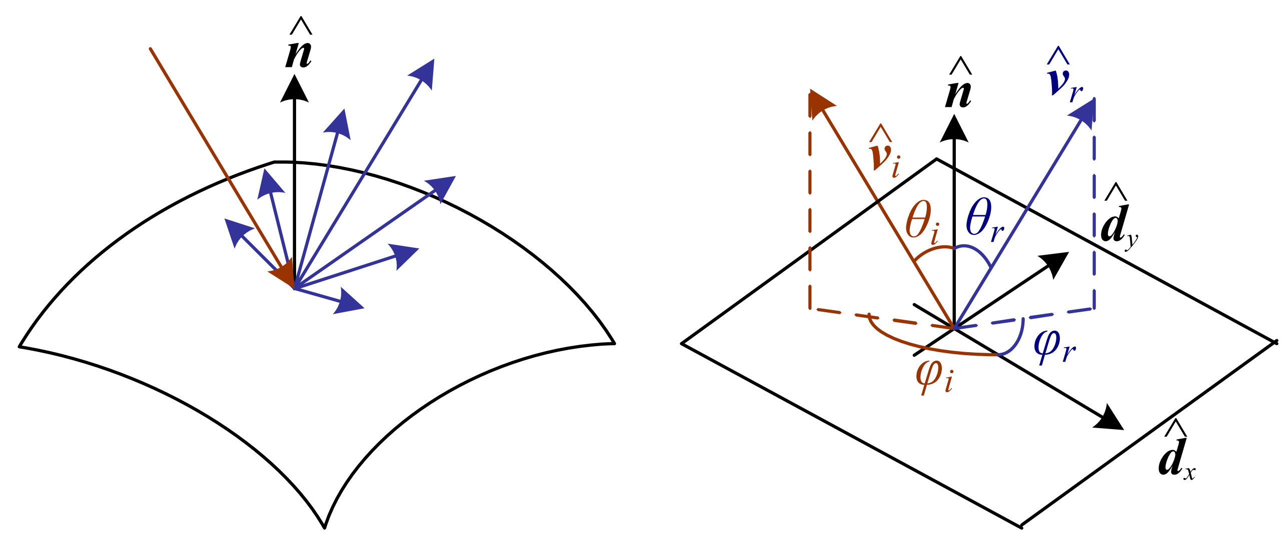
$$f_r(\theta_i, \theta_r, |\phi_r - \phi_i|; \lambda) \text{ or } f_r(\hat{\boldsymbol{v}}_i, \hat{\boldsymbol{v}}_r, \hat{\boldsymbol{n}}; \lambda)$$

To calculate amount of light exiting a surface point $\boldsymbol{p}$ in direction $\hat{\boldsymbol{v}}_r$, integrate product of incoming light $L_i(\hat{\boldsymbol{v}}_i; \lambda)$ with the BRDF, taking into account the foreshortening of the illuminant:

$$L_r(\hat{\boldsymbol{v}}_r; \lambda) = \int L_i(\hat{\boldsymbol{v}}_i; \lambda) f_r(\hat{\boldsymbol{v}}_i, \hat{\boldsymbol{v}}_r, \hat{\boldsymbol{n}}; \lambda) \cos^+ \theta_i \, d\hat{\boldsymbol{v}}_i,$$

where

$$\cos^+ \theta_i = \max(0, \cos \theta_i).$$



Diffuse (Lambertian, Matte) Reflection

- The diffuse component of the BRDF scatters light uniformly, giving rise to Lambertian shading.

$$f_d(\hat{\boldsymbol{v}}_i, \hat{\boldsymbol{v}}_r, \hat{\boldsymbol{n}}; \lambda) = f_d(\lambda)$$

- Colour of reflected light greatly influenced by material

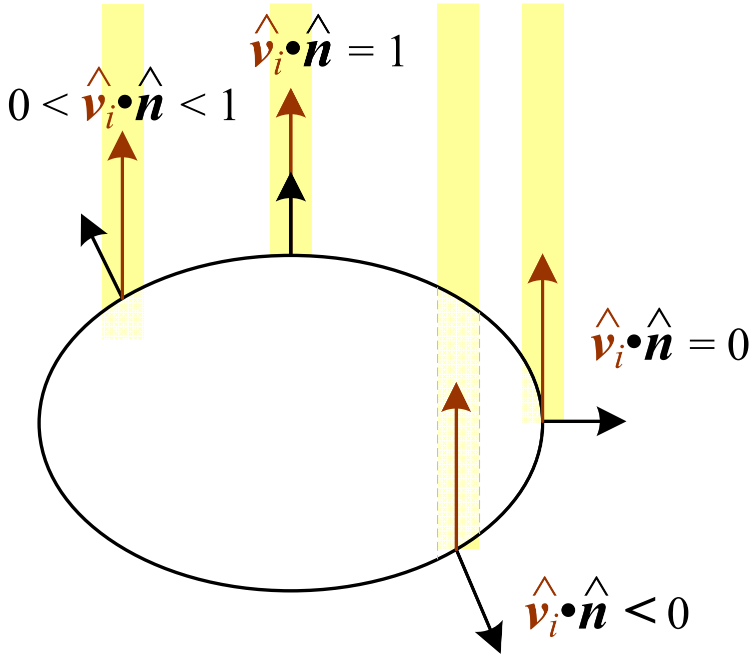
- The amount of light reflected still depends upon the incident elevation angle due to the foreshortening factor

$$L_d(\hat{\boldsymbol{v}}_r; \lambda) = \sum_i L_i(\lambda) f_d(\lambda) \cos^+ \theta_i = \sum_i L_i(\lambda) f_d(\lambda) [\hat{\boldsymbol{v}}_i \cdot \hat{\boldsymbol{n}}]^+,$$

where

$$[\hat{\boldsymbol{v}}_i \cdot \hat{\boldsymbol{n}}]^+ = \max(0, \hat{\boldsymbol{v}}_i \cdot \hat{\boldsymbol{n}})$$

Johann Heinrich Lambert (1728–1777)



Specular (Mirror) Reflection

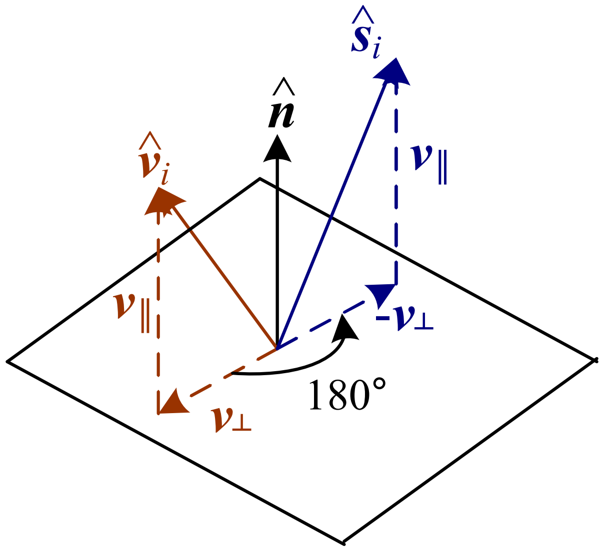
- Specular reflection direction: 180 deg rotation around surface normal.

$$\hat{\boldsymbol{s}}_i = \boldsymbol{v}_{\|} - \boldsymbol{v}_{\perp}$$

- Recall:

$$\boldsymbol{v}_{\|} = \hat{\boldsymbol{n}}(\hat{\boldsymbol{n}} \cdot \boldsymbol{v}) = (\hat{\boldsymbol{n}}\hat{\boldsymbol{n}}^T)\boldsymbol{v}$$

$$\boldsymbol{v}_{\perp} = \boldsymbol{v} - \boldsymbol{v}_{\|} = (\boldsymbol{I} - \hat{\boldsymbol{n}}\hat{\boldsymbol{n}}^T)\boldsymbol{v}$$



- Thus

$$\hat{\boldsymbol{s}}_i = \boldsymbol{v}_{\|} - \boldsymbol{v}_{\perp} = (2\hat{\boldsymbol{n}}\hat{\boldsymbol{n}}^T - \boldsymbol{I})\boldsymbol{v}_i$$

Amount of light reflected in direction $\hat{\boldsymbol{v}}_r$ depends on angle $\theta_s = \cos^{-1}\left(\hat{\boldsymbol{v}}_r \cdot \hat{\boldsymbol{s}}_i\right)$.

e.g., Phong model:

$$f_s(\theta_s; \lambda) = k_s(\lambda) \cos^{k_e} \theta_s$$

Colour — Sharpness

Phong Shading

- The full Phong model combines diffuse and specular components contributed by the main illuminant with an *ambient* term that attempts to account for all other light incident upon the surface from other parts of the scene (sky, walls, etc.)

$$L_r(\hat{\boldsymbol{v}}_r;\lambda) = k_a(\lambda)L_a(\lambda) + k_d(\lambda)\sum_i L_i(\lambda)[\hat{\boldsymbol{v}}_i \cdot \hat{\boldsymbol{n}}]^+ + k_s(\lambda)\sum_i L_i(\lambda)(\hat{\boldsymbol{v}}_r \cdot \hat{\boldsymbol{s}}_i)^{k_e}$$

Ambient Diffuse Specular

NB: I can't make sense of Fig. 2.18: please ignore.

- Typically:

$k_a(\lambda) \simeq k_d(\lambda)$ (both due to sub-surface scatter).

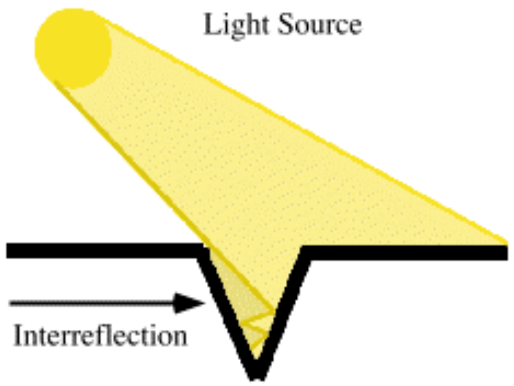
$k_s(\lambda) \simeq$ constant, thus specularity assumes colour of illuminant.

$L_a(\lambda) \neq L_i(\lambda)$

Bui Tuong Phong (1942-1975)

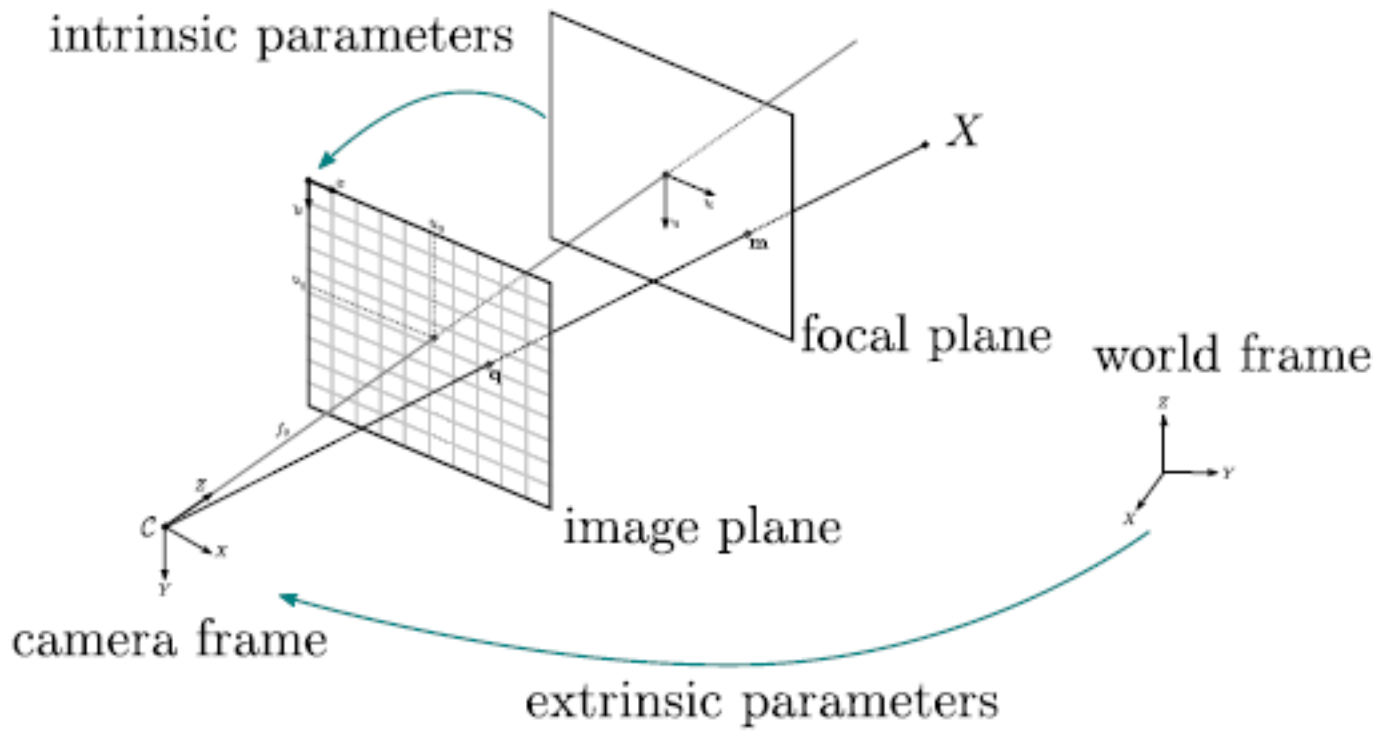
Ray Tracing

- The Phong model assumes a finite number of discrete light sources.
- Light emitted by these sources bounces off the surface and into the camera.
- In reality, some of these sources may be shadowed by other objects, and the surface is generally also illuminated by inter-reflections (multiple bounces)
- Two approaches, depending on nature of scene:
  - If mostly specular, use ray tracing:
    - Follow each ray from camera across multiple bounces toward light sources
  - If mostly matte, use radiosity:
    - Model light interchanged between all pairs of surface patches, and then solve as linear system with light sources as forcing function.



Optics

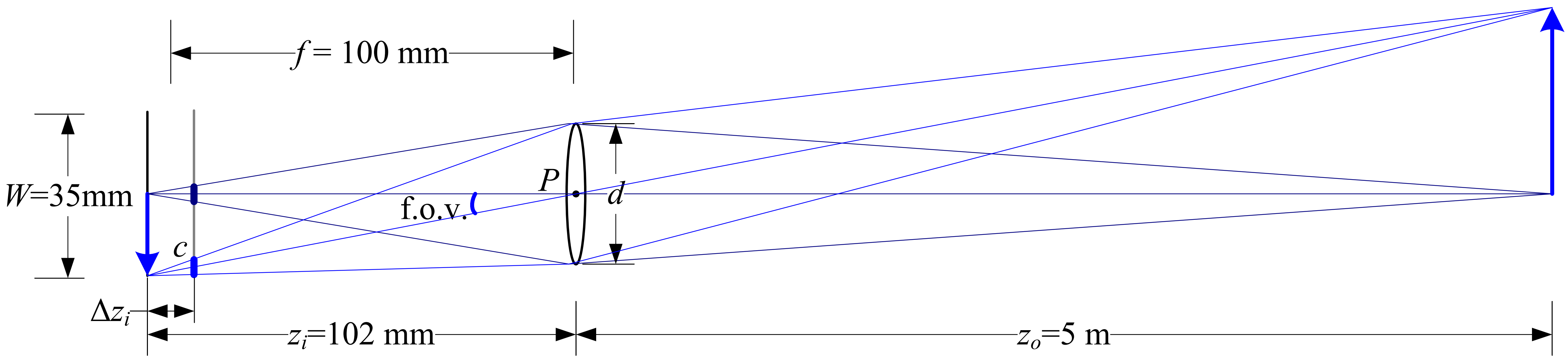
- In Lecture 2.1, we treated projection to the image using a pinhole camera model.



- To account for focus, aperture, aberrations etc. we need to elaborate this model.

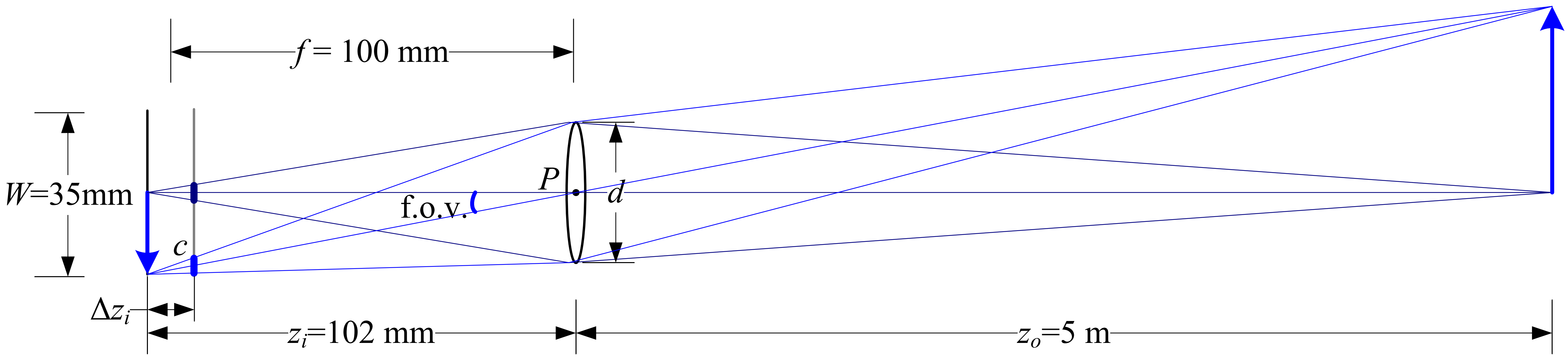
Thin Lens Model

- Assume low-curvature, symmetric, convex spherical lens



- f = focal length
- W = sensor width
- z_0 = distance from optical centre to object
- z_i = distance from optical centre to where focused image of object is formed
- d = aperture
- c = circle of confusion

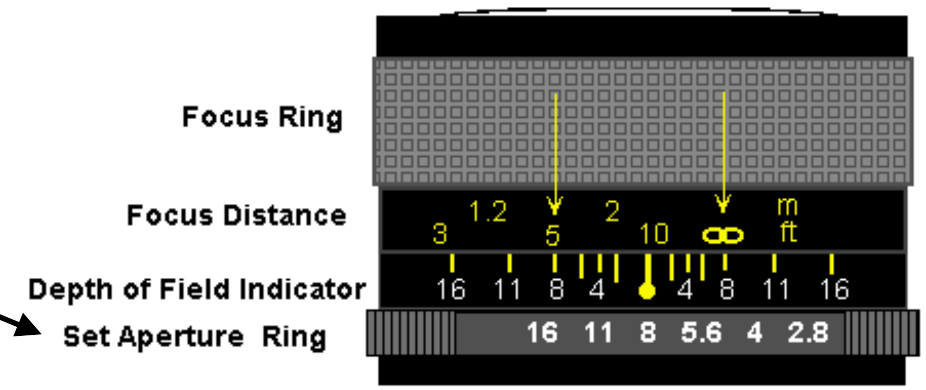
Lens Equation



$$\frac{1}{z_o} + \frac{1}{z_i} = \frac{1}{f}$$

f-number (f-stop) $= f \,/\, d$.

Note that $\lim_{z_0 \to \infty} z_i = f$.



- If the sensor plane does not lie at z_i, a point on the object will be imaged as a blurred disk (the circle of confusion *c*).
- Allowable depth variation that limits this blur to an acceptable level called the *depth of field*.
- Depth of field increases with larger apertures and longer viewing distances.

Chromatic Aberration

- Index of refraction of glass varies slightly as a function of wavelength.
- As a result, different wavelengths focus at slightly different distances.
- To reduce aberrations, most photographic lenses are compound lenses using multiple elements.