Linear Temporal Logic (LTL)

Franck van Breugel

March 25, 2019

1 LTL

LTL is defined by the grammar

$$f ::= a \mid f \wedge f \mid \neg f \mid \bigcirc f \mid f \,\mathsf{U}\, f$$

1. Which LTL formula expresses "initially the light is red and next it becomes green."

2. Which LTL formula expresses "the light becomes eventually amber."

3. Which LTL formula expresses "the light is infinitely often red."

4. What does the formula $\Box(\text{green} \Rightarrow \neg \bigcirc \text{red})$ express?

2 Transition systems

1. Draw the state space diagram of a model of a traffic light. Label (with colours) the states.

2. 2^L denotes the set of subsets of L. What is $2^{\{1,2,3\}}$?

3. A transition system is a tuple $< S, L, I, \rightarrow, \ell >$ consisting of
   - a set S of states,
   - a set L of labels,
   - a set $I \subseteq S$ of initial states,

- a transition relation $\rightarrow \subseteq S \times S$ such that for all $s \in S$ there exists $t \in S$ such that $s \rightarrow t$, and
- a labelling function $\ell : S \rightarrow 2^L$.

Formally define the transition system modelling a traffic light.

3 Semantics of LTL

1. How can we express $p \models \Diamond f$ in terms of $\cdots \models f$?

2. How can we express $p \models \Box f$ in terms of $\cdots \models f$?

3. The LTL formulas f and g are equivalent, denoted $f \equiv g$, if for all transition systems TS,

$$TS \models f \text{ iff } TS \models g.$$

   Are the following formulas equivalent? Either provide a proof or a counter example.

   (a) $\Diamond(f \wedge g) \equiv \Diamond f \wedge \Diamond g$?

   (b) $\Diamond \bigcirc f \equiv \bigcirc \Diamond f$?