1. The *satisfaction set* $Sat(f)$ is defined by

$$Sat(f) = \{\, s \in S \mid s \models f \,\}.$$

   (a) $Sat(a) =$
   (b) $Sat(f \wedge g) =$
   (c) $Sat(\neg f) =$
   (d) $Sat(\exists \bigcirc f) =$
   (e) $Sat(\forall \bigcirc f)) =$

2. Let the set S be finite and the function $G : 2^S \to 2^S$ be monotone, that is, for all $T, U \in 2^S$,

$$\text{if } T \subseteq U \text{ then } G(T) \subseteq G(U).$$

   For each $n \in \mathbb{N}$, the set G_n is defined by

$$G_n = \begin{cases} \emptyset & \text{if } n = 0 \\ G(G_{n-1}) & \text{otherwise} \end{cases}$$

   Prove that for all $n \in \mathbb{N}$, $G_n \subseteq G_{n+1}$.

3. Prove that $G_n = G_{n+1}$ for some $n \in \mathbb{N}$. (Hint: the set S is finite.)

4. We denote the G_n with $G_n = G_{n+1}$ by $\mathit{fix}(G)$. Prove that for all $T \subseteq S$, if $G(T) = T$ then $\mathit{fix}(G) \subseteq T$.

5. The function $F : 2^S \to 2^S$ is defined by

$$F(T) = Sat(g) \cup \{\, s \in Sat(f) \mid succ(s) \cap T \neq \emptyset \,\}$$

Prove that F is monotone.