Basics of Antenna Power Transfer

Sebastian Magierowski

***Abstract*— A brief discussion on some details of antenna power transfer as they lead to Friis' free-space transmission formula.**

I. The Basics

No, you can't really do this in a short technical note, but there are a few basic concepts that can be covered. This paper attempts to do so in as physically motivated a fashion as possible.

The transmitted power density (in W/m^2) at a distance r from a transmitting antenna in empty (homogenous) space is

$$S_T(\theta, \phi, r) = \frac{U(\theta, \phi)}{r^2} \tag{1}$$

where $U(\theta, \phi)$ is the power-per-steradian (W/Ω_A) transmitted in the (θ, ϕ) direction. θ is take to be the elevation angle and ϕ the azimuth angle in this paper although the discussion is so general that such a distinction need not be made. In terms of the injected power (into the transmit antenna) we can also write the transmitted power density as

$$S_T(\theta, \phi, r) = \frac{\eta}{r^2} \cdot P_{inj} \cdot \Omega_T(\theta, \phi) \tag{2}$$

where $\Omega_T(\theta, \phi)$ is the effective solid-angle of the transmit antenna (in steradians) in the (θ, ϕ) direction; this variable indicates the density of the power in a particular direction at a unit distance from the antenna terminals. It is a sort of reference radiation pattern. Also, note that the efficiency, η at which injected power is converted to transmitted radiation power (and vice-versa for a receiving antenna) is assumed to be accounted for in the Ω_T term.

The distance at which an antenna "sees" (in receive-mode) or emits (in transmit-mode) far-field radiation is proportional to the wavelength, λ, of that radiation. For this reason, the above solid angle is normalized to an area removed by λ from the antenna terminals and the above equation is almost always written as

$$S_T(\theta, \phi, r) = \frac{1}{r^2} \cdot P_{inj} \cdot \frac{A_T(\theta, \phi)}{\lambda^2} \tag{3}$$

where $A_T(\theta, \phi)$ is the effective area of the transmit antenna (in m^2) λ meters away from the "center of the antenna" (admittedly a somewhat nebulous concept) in the (θ, ϕ) direction. So, A_T is a way for us to keep track of what effective surface area that the antenna can "suck in" or "blow out" far field radiation (assuming that it can only "interact" with far-field radiation one λ away from its "center" — no farther and no closer).

Shifting gears a bit, another way to express the transmitted power density is with

$$S_T(\theta, \phi, r) = \frac{1}{r^2} \cdot \frac{P_{inj}}{4\pi} \cdot G_T(\theta, \phi). \tag{4}$$

Many thanks to the friends of FishLab.

In this case we consider the injected power distributed over a unit sphere (the second term in W/Ω_A) multiplied by the transmit gain $G_T(\theta, \phi)$ which gives the amount of power propagating in the (θ, ϕ) direction relative to the injected power distributed over a unit sphere. The maximum value of gain is typically denoted by

$$G_T = \max\{G_T(\theta, \phi)\}. \tag{5}$$

The gain of any particular antenna is the same wether in transmit or receive mode and is typically denoted by G, the subscripts are used only to identify any potential differences between transmit and receive antennas in communication system (where at any one time the transmit and receive antennas are physically different structures).

Now, the total power absorbed by a receiving antenna and available to a receive circuit (assuming conjugate match) is given by

$$P_R = S_T(\theta, \phi, r) \cdot A_R(\theta, \phi) \tag{6}$$

where $A_R(\theta, \phi)$ is the effective area of the receive antenna (roughly speaking λ away from the antenna terminals) looking in the (θ, ϕ) direction (now assumed relative to the receive antenna). This particular equation assumes only one path from transmitter to receiver (note that the arguments in S_T and A_R may be different), if more paths need to be accounted for they can be added to the above expression (including any potential phase offsets in the electric and magnetic fields registered by the receive antenna since the sum of the signals arriving from different paths is not necessarily constructive).

Another way to express this is with

$$P_R = [4\pi\lambda^2 S_T(\theta, \phi, r)] \cdot \frac{1}{4\pi} \cdot G_R(\theta, \phi) \cdot \frac{1}{4\pi}. \tag{7}$$

This expression is purposely expanded into four terms to help highlight some of the physics at work. With a nod to the fact that and antenna can "sense" electromagnetic radiation λ away from its terminals (again, no farther and no closer) the first takes the power, $S_T(\theta, \phi, r)$ flowing in from direction (θ, ϕ) and pretends as if all this power were flowing through the surface of a sphere with radius λ. This term expresses the total EM power λ away from the antenna.

However, the power is not actually coming in all around the antenna, as mentioned, it is only coming into one steradian in direction (θ, ϕ) (from the perspective of the receiving antenna), so the second term accounts for this by dividing by the solid angle of a whole sphere.

Next we multiply by the gain of the receiving antenna, $G_R(\theta, \phi)$ which describes how much of the power coming into the antenna from direction (θ, ϕ) is converted to available power at the antenna terminals (connected to the receiving circuit).

However it must be noted that the gain term, $G_R(\theta, \phi)$, relates the available power to an incoming power spread uniformly along the surface of the sphere, hence in the fourth term we divide by 4π again. Physically this means that we take the power coming in at (θ, ϕ) (i.e. $\lambda^2 S_T(\theta, \phi, r)$ left after multiplying the first term in Eq. (7) by the second term) and spread it over the surface of a unit sphere (again, to satisfy the meaning of the antenna gain coefficient).

Similar to the terminology used above

$$G_R = \max\{G_R(\theta, \phi)\}. \tag{8}$$

Summing up the above, assuming that the radiation patterns of the transmit and receive antennas are optimally aligned then the maximum (because they are optimally aligned) available power at the receive antenna is given by

$$P_R = \frac{\lambda^2 G_T G_R P_{inj}}{(4\pi r)^2} \tag{9}$$

for a single unobstructed EM path. This relation is better known as Friis' formula and highlights the importance of the antenna gain parameter.