## FDM and TDM Parallel MRI

Sebastian Magierowski

*Abstract*—A brief discussion of TDM and FDM in the parallel MRI technique for the circuit oriented systems designer.

## I. BASIC MRI

An illustration of a coil array (imaging some oval sample) with reference directions is shown in Fig. 1. Using the singleecho acquisition (SEA) technique this type of pick-up coil structure is capable of imaging a slice in the xz-plane very quickly. Before diving into a discussion of how a coil array facilitates this however, we first consider how an image would be taken in a basic MRI machine.

To facilitate this a magnetic field gradient must be established in the y-direction. This is accomplished by a so-called "gradient pulse". Depending on the fields in question, this gradient pulse establishes a Larmor frequency of  $\omega_0 + \omega_{xz}(y)$ for nuclei in the xz-plane located at y.

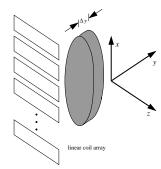


Fig. 1. A sketch of parallel coil array with laboratory frame reference coordinates.

At the same time that the gradient pulse is applied (the gradient pulse is applied in the y direction) an RF pulse of magnetic flux also varying around  $\omega_0 + \omega_{xz}(y)$  with some bandwidth  $\Delta \omega$  excites the system from some orthogonal direction (i.e. x or z direction in this case). Since this pulse has a bandwidth  $\Delta \omega$  it causes the precession of nuclei spins in a slice of finite thickness  $\Delta y$ . This precession generates a magnetic flux which is sensed by the coils.

Given the sample perturbations described so far (i.e. gradient pulse and RF pulse), the RF engineer can imagine that the  $\Delta y$  thick slice is generating a voltage signal across the coil with a bandwidth of  $\Delta \omega$  at a center (Larmor frequency) of  $\omega_0$ .

Now in a (rudimentary) standard MRI machine a special signalling scheme must be employed in order to construct a coherent image of the signal. Specifically, we must find some way of isolating volume "pixels" (i.e. voxel) sections of the image such that the signals from the slice can be constructed into a complete picture.

Many thanks to the friends of FishLab

First, with all nuclei spins precessing due to the original RF pulse, a second magnetic field gradient is introduced in the the z-direction. This causes a shift in the frequency of the magnetic flux (from the spinning nuclei) in the z-direction. That is, the nuclei at (x, z) produce a slightly different magnetic flux frequency then the nuclei at  $(x, z+\delta z)$ .

There is something very interesting here. First, the total voltage sensed by the coil (in standard MRI we only have one pick-up coil)

$$v(t) \propto \sum_{n}^{N} \frac{\mathrm{d}}{\mathrm{d}t} \phi(x_0, y_0, z_0 + n \cdot \delta z, t).$$
 (1)

In a moment we will return to the technique by which a specific  $x_0$  is isolated for the magnetic flux  $\phi$  (we already explained how a specific slice of thickness  $\Delta y$  centered around some  $y_0$  can be established).

At this point, it must seem that the net signal registered by the coil from the nuclei located in the line between  $(x_0, y_0, z_0)$ and  $(x_0, y_0, z_0 + N \cdot \delta z)$  is an irreversible jumbled mess. However this is not the case, since the signal from each distinct z-location has a different frequency of precession (due to the z-directed gradient signal). Thus, (and I won't justify it any more deeply than that) we can recover the "image" signal, I, from each nuclei via the inverse Fourier transform

$$I(x_0, y_0, z_0 + n \cdot \delta z) = \int v(t) e^{-j2\pi f(z_0 + n \cdot \delta z)t} dt.$$
 (2)

The better that we sample the net signal v(t) in time, the better resolution we can get for I.

The selection of a specific  $x_0$  is the remaining loose end. This is easy to address. To select a specific  $x_0$ , a third magnetic field gradient must be applied, this time in the x-direction. The receiving coil is then tuned not just to the frequency  $f(z_0 + n \cdot \delta z)$ , but rather  $f(x_0, y_0, z_0 + n \cdot \delta z)$ . As with the y-direction gradient, which tunes the  $y_0$  location of the sample to the receiving coil (or possibly vice-versa), the x-direction gradient essentially selects only a row of the sample along in the zdirection at some  $x_0$ .

The need to form a slice image one *z*-line at a time makes standard MRI a very time consuming process. But this is one problem that parallel MRI (pMRI) systems can address (another is the ability to improve SNR without a time savings). Rather than stepping through one *z*-line at a time, we can use multiple coils to capture multiple *z*-lines simultaneously.

## II. FDM PMRI

One way of imaging the z-lines of an xz-plane simultaneously is via the frequency-division multiplexing (FDM) approach. In hindsight, the concept is simple, rather than picking up one z-line at a time (with one receive coil), we place a coil right above the z-line of interest, tune it to the frequency of that z-line and sense the magnetic flux of that line. The same idea goes for the remainder of the coils in the array. A picture of this is illustrated in Fig. 2.

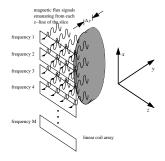


Fig. 2. A coil array picking up sample flux lines in an FDM pMRI.

The row of signals going through each coil represents the flux from each z-line (or z-row). The sums of these signals (i.e. just summing signals in each row) are centered around some frequency "frequency m" (m = 1...M) as indicated in Fig. 2. These center frequencies are mainly determined by the gradient pulse in the x-direction, but are also slightly influenced by the frequency encoding gradient (assumed in the z-direction in this note) without which we obviously would not be able to discern the signals in each row from one another<sup>1</sup>. The bandwidth of the signals around each center frequency is determined by the RF slice (thickness) pulse described above.

This is perhaps the most obvious approach in the parallel imaging technique as it does not fundamentally alter the imaging sample perturbation scheme (i.e. application of RF pulses and gradients) other than reducing the number of times this scheme needs to be applied (i.e. the major benefit). It should be noted that this is not exactly how a SEA MRI is done (different field directions are employed and the set-up is a little less complicated, but the end result in terms of signals going into the coils is the same).

## III. TDM PMRI

The TDM pMRI system is even simpler than the FDM technique described above. In a nutshell, it is FDM with the *x*-direction gradient removed. In this case the spins in all*z*-lines (on average) precess at the same frequency. That is, although frequency encoding is still present (hence the not to average frequency in the sentence above) any phase encoding has been completely removed.

In theory this is not a problem because each coil is position and designed such that it picks up only the z-line immediately above it (i.e. along the y-axis). Hence, even though all z-lines are the same (average) frequency they get "funnelled" into different receiver channels.

<sup>&</sup>lt;sup>1</sup>This is the all important frequency encoding, it causes slight frequency shifts in the flux signals coming from different z locations, thus orthogonalizing them and allowing us to eventually reconstruct each from the summed signal at the coil. Therefore, because of frequency encoding, each of the signals in a specific row is at a slightly different frequency.