Small Loop Antenna Calculations

Sebastian Magierowski

***Abstract*— A somewhat prolonged discussion of the performance of small loop antennas operating in free-space.**

I. Introduction

A previous technical note [1] presented a number of closed form expressions for a loop antenna in free-space. Here we generate some numbers to get an impression of this antenna's performance. Specifically, we are interested in very small antennas on the order of several millimeters. The kind that in one form or another can be wrapped around (or deposited on) a chip as illustrated in Fig. 1

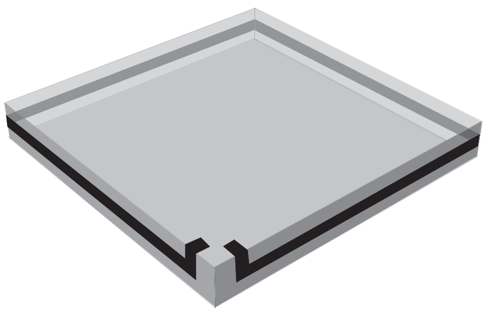


Fig. 1. A rectangular loop antenna wound around the periphery of a chip.

II. Calculation

Matlab is used to calculate all the key antenna performance parameters. The first order of business is to calculate the radiation resistance R_r. For a loop antenna the radiation intensity (in Watts per steradian) is [1]

$$
\begin{aligned}
U(\theta,\phi) &= \frac{1}{2}(\mathbf{E}(\theta,\phi)\times\mathbf{H}^*(\theta,\phi)) \qquad (1)\\
&= \frac{2I_0^2}{\pi^2}\sqrt{\frac{\mu_0}{\epsilon_0}}\frac{\sin^2(\frac{\pi a}{\lambda}\sin\theta\cos\phi)\sin^2(\frac{\pi a}{\lambda}\sin\theta\sin\phi)}{\sin^2\theta\sin^2 2\phi}
\end{aligned}
$$

for a square loop a meters on a side is considered.

The radiation resistance can be calculated with

$$
\begin{aligned}
R_r &= \frac{2}{I_0^2}\int_\Omega U(\theta,\phi)\mathrm{d}\Omega \qquad (2)\\
&= \frac{2}{I_0^2}\int_0^{2\pi}\mathrm{d}\phi\int_0^{\pi}U(\theta,\phi)\sin\theta\mathrm{d}\theta.
\end{aligned}
$$

The strip of Matlab code used to actually calculate R_r is

```
row=1;col=1;
for row=1:1:(lstop-lstart)/lstep+1
len=l1(row);
 for col=1:1:(fstop-fstart)/fstep+1
 lam=lambda(col);
 Rtemp = dblquad(@Rrdiff2,1e-10,...
 2*pi-1e-10,1e-10,pi,1e-10,[],len,lam);
 Rr(row,col)=Rtemp;
 end
end
Rr = 4/pi^2*sqrt(mu/eps)*Rr;
```

where the `Rrdiff2` function called by the `dblquad` integration command is

```
function z=Rrdiff2(phi,theta,lenx,lamx);
z=
(sin(pi*lenx/lamx*sin(theta)*cos(phi))).^2.*
(sin(pi*lenx/lamx*sin(theta)*sin(phi))).^2.*
(sin(theta)).^(-1).*
(sin(2*phi)).^(-2);
```

As per the discussion in [1] and [2] we also calculate the (maximum) gain, G, and the antenna efficiency, η, by calculating the resistive (thermal) loss, R_l. For the purpose of receiver and transmitter design, it is also important to know the self-inductance of the antenna. The equation for this was presented in [1].

III. Results

A natural, lumped element model of the antenna consists of an inductor in series with a resistor as shown shown in Fig. 2. The series inductance can be substantial even for the chip-scale dimensions under consideration (e.g. $\sim$ 10 nH). Compact expression for the loop inductance, L, and ohmic losses, R_l are given in [1].

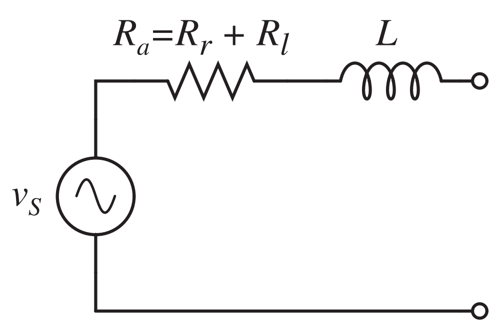


Fig. 2. Antenna lumped equivalent circuit.

The width, w, of the metal antenna trace possibly ranges from 2 μm to 500 μm (which would be rather wide considering typical chip dimensions). The conductivity, σ, of the trace obviously depends on the metal that it is made out of (3.77×10^7 $[\Omega\cdot\mathrm{m}]^{-1}$ for Al and 5.96×10^7 $[\Omega\cdot\mathrm{m}]^{-1}$ for Cu). We can expected the thickness of the loop's metal trace, t, to vary between 1 μm and 5 μm.

Many thanks to the friends of FishLab.

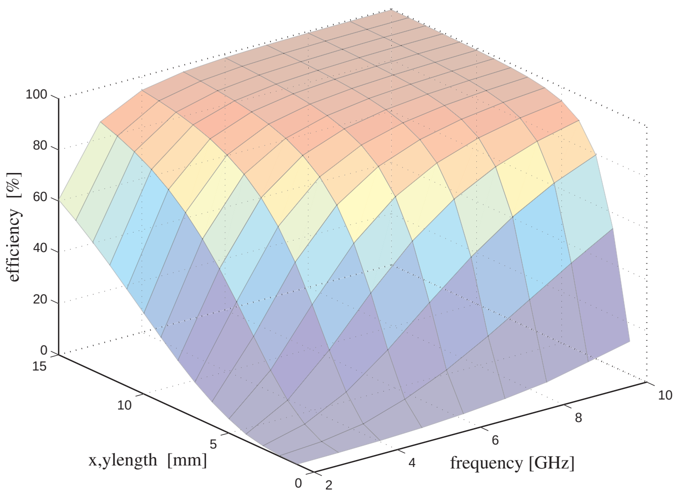


Fig. 3. Efficiency of square copper loop antenna ($w = 400$ μm and $t = 3$ μm) as a function of its dimensions and frequency of operation.

An earlier technical report [1] highlighted the proportionalities $R_r \propto \omega^4$ and $R_l \propto \sqrt{\omega}$. This is conveyed in the 3D efficiency plot shown in Fig. 3 as a function of the antenna's dimensions (a single square loop is simulated) and operational frequency. This result assumes a copper antenna trace that is 400-μm wide and 3-μm thick.

On the whole the results are quite promising, even at a 2-GHz carrier we can achieve a 9.1% efficiency for a 6-mm square loop antenna. At 6 GHz this improves to 83%. Even a compact 3-mm on-a-side loop operates with an efficiency of 39% at this frequency.

To gain some perspective on the trade-offs, a plot of the antenna dimensions (assuming copper traces) needed to achieve 50% and 10% efficiencies at various frequencies of operation are shown in Fig. 4.

As indicated by these plots a 50% requires a rather manageable footprint with a 4 mm $\times$ 4 mm 400-μm wide copper structure excited at 5 GHz. If we can settle for a 10% efficiency the antenna area is quartered to a 2 mm $\times$ 2 mm (still 400-μm wide) running at 5 GHz.

For the purpose of circuit design it is worthwhile to examine the total resistance of the antenna. This indicates the driving point impedance that the receiver has to present for optimum power transfer. Fig. 5 shows the total series resistance ($R_t = R_r + R_w$) of the copper antenna under consideration ($w = 400$ μm, $t = 3$ μm) at various sizes and frequencies of operation. Beside each resistance data point is indicated the efficiency of the antenna. In the options shown, the antenna resistance ranges from 0.3 Ω to 50 Ω. Within the rectangular region denoting an acceptable range of frequency operation and antenna size the antenna resistance varies from 0.35 Ω to 10 Ω. Practically speaking, it is hard to imagine that any structure with $R_t < 1$ Ω is worth considering, as the losses incurred in the interface to the receiver should far outweigh this therefore completely blotting out the ideal efficiency values quoted.

Even in the 1 Ω to 10 Ω range the designer will have to exact considerable care in designing the interface. Still, when

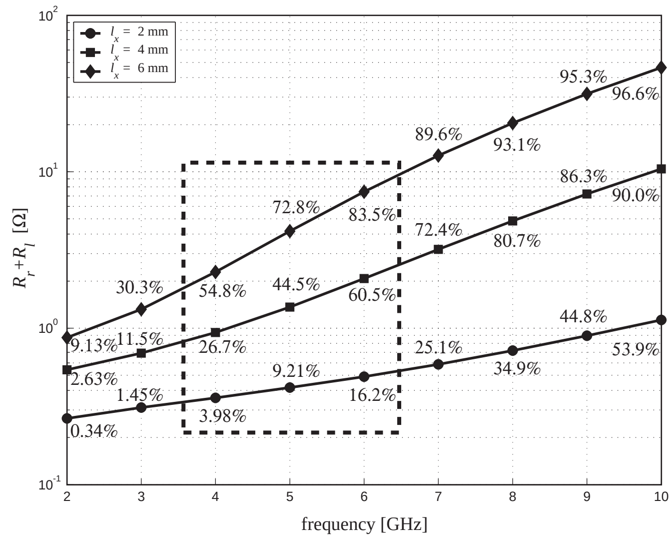


Fig. 5. Net resistance of square copper loop antenna ($w = 400$ μm and $t = 3$ μm) as a function of its dimensions and frequency of operation.

TABLE I

COPPER LOOP ANTENNAS ($w = 400$ μm, $t = 3$ μm) WITH $f_0 \leq 10$ GHz

$l_x = l_y$ [mm]	f_0 [GHz]	η [%]	Q	BW [MHz]	L [nH]	$R_r + R_l$ [Ω]	G [dB]
2	2.50	0.76	300	8.32	5.51	0.29	-19.0
2	5.50	12.5	422	13.0	5.51	0.45	-7.29
2	10.0	53.9	307	32.6	5.51	1.13	-0.93
4	2.50	6.29	341	7.33	13.2	0.61	-10.3
4	5.50	53.0	273	20.2	13.2	1.68	-1.02
4	10.0	90.0	80.0	125	13.2	10.4	+1.25
6	2.50	18.4	326	7.67	21.8	1.05	-5.60
6	5.50	78.9	135	40.9	21.8	5.60	+0.70
6	10.0	96.6	29.5	338	21.8	46.4	+1.50

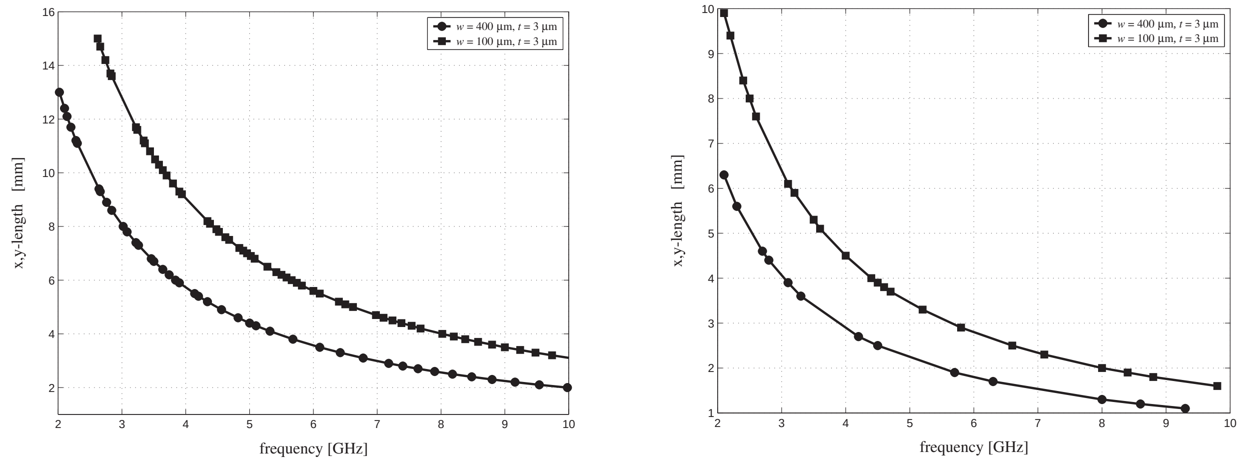
approaching this problem from a purely integrated viewpoint this challenge becomes somewhat less daunting. If confined to the 50-Ω RF system paradigm, the designer is indeed challenged with the implementation of a matching network that converts the low series resistance (on the order of 5 Ω as seen in Fig. 5) of the loop without introducing so much loss that the already strained antenna efficiency is even further compromised. Luckily, when considering a custom integrated circuit design, we have a good amount of freedom to determine our terminating impedances, thus giving hope to the usefulness of low-resistance antenna designs.

Obviously, a great many options are available to the prospective designer. Table I summarizes some possibilities with acceptable sizes and operating frequencies. The table includes measures of the antenna quality factor

$$Q = \frac{\omega_0 L}{R_a} \tag{3}$$

and the antenna's bandwidth in Hz

$$BW = \frac{f_0}{Q}. \tag{4}$$



(a) 50% efficiency (b) 10% efficiency

Fig. 4. The relationship between loop antenna dimensions (copper trace with characteristics given in plot) and frequency of operation for efficiencies of a.) 50% and b.) 10%.

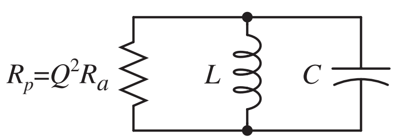


Fig. 6. Circuit equivalent of the loop antenna as seen by a transmitter.

IV. Power In/Out

As already mentioned, any practical matching network placed between the antenna and driving circuit is bound to introduce more ohmic losses, thus further lowering the efficiency. This problem can be sidestepped by designing the transceiver circuit to interface directly to the antenna, however in this case, it will be difficult to couple a significant amount of power into the antenna — not an unreasonable limitation in purported low-power systems.

To illustrate, we refer to the equivalent antenna circuit in Fig. 6 where we reconsider the general model of Fig. 2 in transmit mode (therefore dropping v_S) as a parallel equivalent circuit with C acting as some tuning capacitance. As shown

$$R_p \approx Q^2 R_a \tag{5}$$

where Q is the antenna's quality factor.

Assuming a sinusoidal voltage excitation with amplitude V_0 across this circuit the radiated power is

$$P_{rad} = \frac{\eta V_0^2}{2R_p}. \tag{6}$$

Now, we come across the limitation that without a matching network, V_0 is roughly limited to the DC supply level, V_{DD}. Assuming a $V_{DD} = 2$ V the possible power radiated from the antennas (only 5.5-GHz frequencies are considered) is given in Table II. Also shown is the power that must be injected into the antenna (presumably from the DC supply) in order to sustain the radiation. If we imagine that a DC current I_{dc} is (sinusoidally) switched back and forth across the resistor then

TABLE II

Copper Loop Antennas Driven by 5.5-GHz Sinusoid with 2-V Amplitude

l [mm]	R_p [kΩ]	I_{dc} [μA]	P_{rad} [dBm]	P_{rad} [μW]	P_{inj} [μW]
2	80	25	-25.1	3.10	24.8
4	125	16	-20.7	8.56	16.1
6	102	20	-18.1	15.6	19.7

the relationship between I_{dc} and V_0 is simply

$$I_{dc} = \frac{V_0}{R_p}. \tag{7}$$

The limited supply and relatively poor radiative characteristics of the antenna combine to necessitate the very low dc currents listed in Table II.

A drawing of an antenna driver in the form of an active antenna connection is shown in Fig. 7. With ideal switches, the efficiency of this amplifier is 100%. Unfortunately, this property is deceiving as even an extremely good (but lossy) switch in the circuit will immediately reduce the efficiency of this circuit to 50%. Thus, no currently conceivable technology can hope to improve the efficiency of this circuit beyond 50%.

As shown in Table I the maximum gain of the antennas under consideration falls to the 6 mm $\times$ 6 mm model and is only 0.7 dB. The distribution of this antenna's gain parameter in space is shown in Fig. 8.

Assuming maximum practical injected power (Table II) the available power of the receiving antenna as well as the peak voltage and current values from the Thevenin and Norton antenna equivalent models (see Fig. 9) are tabulated in Table III as a function of the separation distance, r, (between transmitter and receiver) where the peak voltage and current values are calculated according to

$$V_{pk} = \sqrt{8 \cdot P_R \cdot R_a} \tag{8}$$

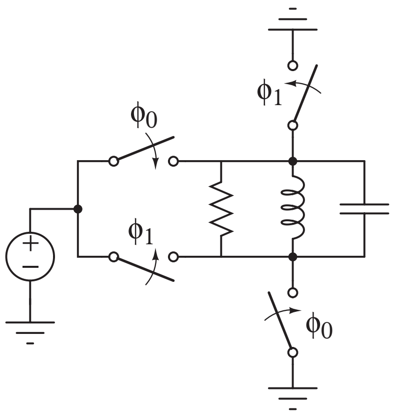


Fig. 7. An idealized schematic of an active antenna.

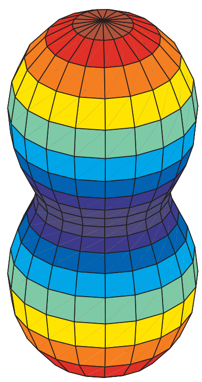


Fig. 8. The gain distribution (maximum of 0.7 dB) of the 6 mm loop antenna at 5.5 GHz.

and

$$I_{pk} = \sqrt{8 \cdot P_R / R_p} \tag{9}$$

where P_R is the received radiation power that can appear as useful electrical energy at the antenna terminals (i.e. the part of the received power that is not dissipated as heat through the ohmic losses of the antenna). Using Friis' formula [2]

$$P_R = \frac{\lambda^2 G_T G_R P_{inj}}{(4\pi r)^2} \tag{10}$$

we can express, I_{pk} for instance, as

$$I_{pk} = \sqrt{8 \cdot \frac{\lambda^2 G_T G_R}{(4\pi r)^2} \cdot \frac{I_{dc}^2 R_p}{2} \cdot \frac{1}{R_p}}. \tag{11}$$

TABLE III

6-mm RECEIVE ANTENNA SIGNAL STRENGTHS

r [m]	P_R [pW]	P_R [dBm]	V_{pk} [μV]	I_{pk} [nA]
0.1	51200	-42.9	1520	2010
1	512	-62.9	152	201
5	20.5	-76.9	30.1	40.2
10	5.12	-83.0	15.2	20.1

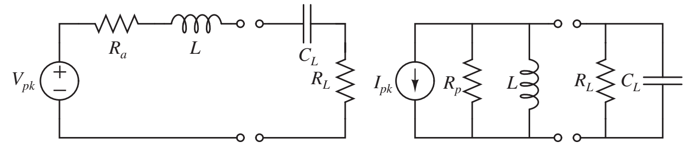


Fig. 9. Thevenin and Norton equivalent models of the loop antenna.

REFERENCES

[1] S. Magierowski, "Loop antenna equations," Tech. Rep., June 5 2007.
[2] S. Magierowski, "Basics of antenna power transfer," Tech. Rep., June 2 2007.