## Active Analog Reflector Antennas: Scattered Power

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*Abstract*—A brief discussion on active reflector antennas. Revised May 22, 2007. Revised May 26, 2007. Revised June 14, 2007.

## I. BASICS

Antennas are special arrangements of materials designed to be "good" at radiating and absorbing electromagnetic energy. These elements are difficult to analyze in detail, but their key characteristics for communications systems can often be treated from a simple lumped-circuit-element perspective.

One simple picture is the parallel equivalent shown in Fig. 1.  $G_A$  denotes the antenna's radiation conductance,  $B_A$ 



Fig. 1. Antenna lumped equivalent circuit.

is the antenna's susceptance  $(Y_A = G_A + jB_A)$  and  $I_i$  is the input signal current amplitude. The power that this structure launches towards any circuit attached to its a - a' terminal is

$$P_{in} = \frac{I_i^2}{8G_A}.$$
(1)

Assuming free-space signal exchange,  $P_{in}$  can be related to the power,  $P_{txin}$  driving a transmitter antenna r meters away (from our receiving antenna) with

$$P_{in} = \frac{\lambda^2 G_t G_r}{(4\pi r)^2} P_{txin}.$$
 (2)

The power reflected and radiated by the antenna is given by

$$P_{ref} = \Gamma_c \Gamma_c^* P_{in} \tag{3}$$

where  $\Gamma_c$  is the reflection coefficient of the circuit attached (with admittance  $Y_c$ ) to the antenna given by

$$\Gamma_c = \frac{Y_A^* - Y_C}{Y_A + Y_C}.\tag{4}$$

For our distributed MIMO system employing doppler transmit diversity the idea is to control  $\Gamma_c$  such that  $P_{ref}$  has suitable power and (time-varying) phase.

It is worthwhile to pause for a moment in consideration of Eq. (3). Indeed, the power that is re-radiated (reflected) by the antenna is  $P_{ref} = |\Gamma_c|^2 P_{in}$ . But an important part of our

Many thanks to the friends of FishLab.

system concerns the our ability to manipulate the phase of the reflected signals. The use of the word "signals" (in place of power) is intentional. The amount of power that an antenna extracts is dependent on the relationship of the electric and magnetic fields (**E** and **H**) of the signals impinging upon it. Yes, the power of each signal impinging upon the antenna is **related** to  $|\Gamma_c|^2 P_{in}$ , but the manner in which these signals are "processed" by the antenna, must be analyzed by the net combination of all the different **E** and **H** fields on that antenna. The phase differences between these E and H fields (between different signals, not within one EM signal) is dictated by the differences.

Thus, assuming there is a signal  $P_{inj}$  injected into a transmit antenna and relayed (i.e. reflected) signal  $P_{ref}$  coming into a particular node the total power into that node is given by

$$P_{rx} = \left| \sqrt{\frac{\lambda^2 G_T G_R P_{inj}}{(4\pi r_D)^2}} e^{j2\pi r_D/\lambda} \right|$$
(5)

$$+\sqrt{|\Gamma_c|^2 \frac{\lambda^2 G_T G_R P_{inj}}{(4\pi r_R)^2}} e^{j(\angle \Gamma_c + 2\pi (r_R + r'_D)/\lambda)} \bigg|^2 \qquad (6)$$

where (to facilitate a simple illustration) a free-space model is assumed, all antennas are assumed to have transmit and receive gains of  $G_T$  and  $G_R$ , respectively, the distance from the source to destination is  $r_D$  the distance from source to relay is  $r_R$  and the distance from relay to source is  $r'_D$ , and the signal carrier wavelength is  $\lambda$ .

## **II. ACTIVE REFLECTED POWER**

In general we can express the reflection coefficient with

$$\Gamma_c = \frac{G_A - G_C - j(B_A + B_C)}{G_A + G_C + j(B_A + B_C)}.$$
(7)

For passive loads where  $G_C \ge 0$   $P_{ref}$  cannot exceed  $P_{in}$ . The only way that  $P_{ref}$  can exceed  $P_{in}$  is if the load  $G_C$  is implemented in such a way that

$$G_C = -G_R \qquad G_R > 0. \tag{8}$$

In order for such a reflection system to remain stable it is necessary that  $G_R < G_A$ . Defining  $K_G = G_R/G_A$  and  $K_B = B_C/B_A$  we have

$$\Gamma_c = \frac{(1+K_G)G_A - j(1+K_B)B_A}{(1-K_G)G_A + j(1+K_B)B_A}.$$
(9)

Recalling that antenna quality factor can be defined with

$$Q_A = B_A/G_A \tag{10}$$

we have

$$\Gamma_c = \frac{(1+K_G) - j(1+K_B)Q_A}{(1-K_G) + j(1+K_B)Q_A}.$$
(11)



Fig. 2. The gain of the active reflection antenna vs.  $K_G$ .

What do we want out of our active reflector antenna?

- Adequate Gain.
- Adequate Phase Control.

There is only one way by which the gain can be increased:

1)  $K_G \longrightarrow 1$ .

The first point is obvious, increasing  $K_G$  increases the power supplied to the active reflecting circuit that drives the antenna. The gain of the reflection antenna as a function of  $K_G$  is shown in Fig. 2. For "practical" settings of  $K_G = 0.9$ , the reflected maximum power gain is only about 25-dB. Obviously very high gains are achievable as  $K_G$  is increased. However this also makes the system more susceptible to instability (i.e. oscillations) given fluctuations in antenna parameters (which can be both manufacturing and environmentally induced). Obviously the problem of instability needs to be more rigorously studied (and perhaps a gain control studied).

Increasing  $Q_A$  is mainly of help in reducing the variation of  $K_B$  needed to get any particular phase modulation of the reflected signal. It has no effect on the gain of the reflector antenna. Examples of the reflected wave characteristics for two different values of  $Q_A$  are shown in Figs. 3 and 4. As already mentioned, no gain advantage is evident in these results. Increasing  $K_G$  to 0.95 in a  $Q_A = 20$  systems results in the reflected performance summarized in Fig. 5.

Varying  $K_B$  obviously varies the phase of the output signal. It also results in a non-optimal match that lowers the net power reflected by the antenna. For instance, in Fig. 4, we see that when  $K_G = 0.8$  and  $Q_A = 20$  we can vary  $K_B$  by  $\pm 1.8\%$  to affect a phase variation of  $\pm 70^\circ$  at a reflected gain of about 14-dB.



Fig. 3. Reflection coefficient of reflector antenna.



Fig. 4. Reflection coefficient of reflector antenna.



Fig. 5. Reflection coefficient of reflector antenna.