# An Active Reflector Circuit for PSTD

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***Abstract*— Some power gain and consumption numbers are considered for an amplify-and-forward reflection amplifier to be used in a communication system employing phase sweep transmit diversity.**

## I. Introduction

A description of the phase sweep transmit diversity system (PSTD) is given in [1]. Simply put, in these systems an amplify-and-forward repeater re-transmits the original transmitted signal with a time-varying phase. When received in combination with the original signal at the target receiver, this system reduces the effect of flat Rayleigh fading.

Simulations on the PSTD system [1] indicate that BERs of $\sim 10^{-3}$ can be obtained for SNRs $\sim 17$ dB at the target receiver. This is a suitable starting point for determining the necessary circuit characteristics of the repeater. We start by reviewing the noise present at the input of the target receiver

## II. Receiver Noise

The thermal noise available from an antenna is

$$P_{an} = kT_aB \tag{1}$$

where $T_a$ is the antenna temperature and $B$ is the bandwidth of the signal. The receiver also adds noise of its own which is characterized with the noise factor

$$F \triangleq \frac{\text{total output noise}}{\text{total output noise due to source}}. \tag{2}$$

That is

$$F = \frac{G_aP_{an} + G_aP_{ampn}}{G_aP_{an}} \tag{3}$$

where $G_a$ is the receiver's available gain and $P_{ampn}$ is the input equivalent of the noise contributed strictly by the amplifier alone. Thus, accounting for the noise characteristic of the amplifier in input (available) noise power is

$$P_n = FP_{an} = FkT_aB. \tag{4}$$

For a given minimum SNR, $SNR_{min}$, (17-dB in the PSTR system under consideration), the minimum detectable signal becomes

$$P_{min} = SNR_{min} \cdot FkT_aB. \tag{5}$$

Assuming $T_a$ equals the ambient temperature of 290 K and that we consider a signal bandwidth of 100-kHz and that we the target receiver has a net noise figure of 5-dB (achievable in production-level integrated technologies) the minimum detectable signal becomes only 63.4 fW or -102 dBm.

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For a transmitter and receiver with antenna gains of unity (i.e. $G_T = G_R = 0$ dB) and a signalling frequency at 5.5 GHz the large-scale path lose in a line-of-sight environment per meter of separation is

$$L \text{ [dB]} = -47.25 - 20\log r \tag{6}$$

where $r$ is the transmitter-receiver separation in meters and the -47.25 dB is computed from $\lambda^2 G_T G_R/(4\pi)^2$. Thus the maximum range for a given transmit power $P_{tx}$ is

$$r_{max} = 10^{(P_{tx} - 47.25 + 102)/20}. \tag{7}$$

For a 0-dBm source the maximum separation distance is 546 m. This is orders of magnitude greater than the distances encountered by indoor networks, the kinds of concern to us. For a 10-m separation the required transmit power is

$$P_{tx} = 47.25 - 102 + 20\log 10 = -34.75 \text{ [dBm]} = 0.335 \text{ [}\mu\text{W]}. \tag{8}$$

Clearly, rather low transmit powers are possible under the stated assumptions.

## III. Circuit Discussion

This technical note was prompted by the worry that the need for a large transmit power could significantly complicate the design of the repeater node. The above analysis, although simplistic, eases some of this worry. Granted other effects such as shadowing could be present. Assuming a worst case shadowing loss of 10 dB increases the necessary transmit power to 3.4 $\mu$W. More generally we can speak of a spectral density and consider $3.4 \times 10^{-6}/10^5 = 34$ pW/Hz.

If the repeater is stationed 1 m away from the source transmitter, it will need to provide 47-dB of power gain to the signal it forwards in order to reach the destination receiver at a power comparable to that forwarded by the transmitter. What kind of repeater is needed that can amplify the signal by 47-dB to about 3 $\mu$W (30 pW/Hz)?

### A. Antenna

A critical part of the answer to this question hinges on the repeater's antenna. How must we drive the antenna in order to radiate out approximately 3 $\mu$W in the first place? A recent technical note [2] discussed the simulated results of a small loop antenna (6 mm on a side) that may serve as a fair illustrative example at this point. At 5.5 GHz and resting on glass the antenna has a net series resistance of about 10.8-$\Omega$, a (maximum) gain of 0.66-dB, and an efficiency of 65%. With a series equivalent inductance of 11.2-nH the parallel equivalent resistance of this antenna is

$$R_p = 10.8 \cdot \left( \left( \frac{2\pi 5.5 \times 10^9 \cdot 11.2 \times 10^{-9}}{10.8} \right)^2 + 1 \right) \tag{9}$$

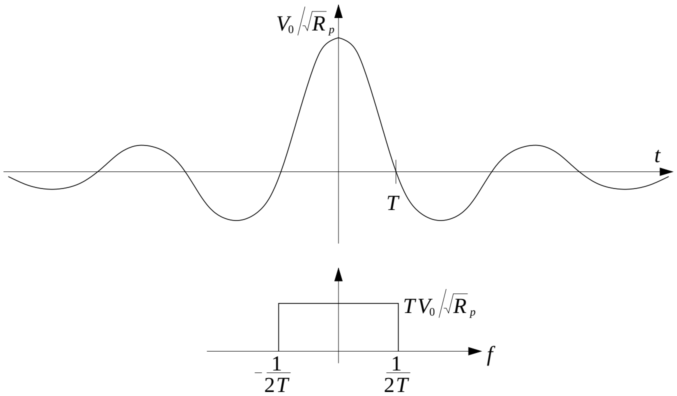


Fig. 1. The data pulse assumed for the system, a sinc function along with its fourier transform.

which results in $R_p = 13.9$ kΩ.

How big a voltage must the circuit apply across this antenna in order to get the necessary output power? The answer to this question will refine our idea of the type of voltage supply that we must provide for our circuit. The size of the voltage signal will also be a critical clue to the extent of circuit nonlinearities that we may have to deal with.

Let us assume that the information signal (separate from any coding signal) to be amplified consists of a series of sinc pulses with peak root-power of $V_0/\sqrt{R_p}$ (i.e. a peak voltage of $V_0$ is dropped across the parallel equivalent antenna resistance of $R_p$) and main-lobe width of $2T$ seconds (where $T = 1/B$ for an uncoded system) indicating a symbol period of $T_s = T$ as shown in Fig. 1 (along with its fourier transform).

Under the sinc-pulse assumption, the amplifier forwards a brickwall signal with power spectral density

$$S_{tx} = \frac{|F_{tx}(f)|^2}{T_o} \tag{10}$$

where $F_{tx}$ is the fourier transform of the root-power transmitted sinc pulse discussed above and $T_o$ is the observation time (i.e. the separation between pulses). To avoid ISI, $T_o = T$.

From Fig. 1 we then have

$$S_{tx} = \left(\frac{TV_0}{\sqrt{R_p}}\right)^2 \frac{1}{T} = \frac{TV_0^2}{R_p}. \tag{11}$$

We can refine this expression further by accounting for the transmit gain of the antenna,$G_T$ (which accounts for the antenna's efficiency)

$$S_{tx} = \frac{G_T T V_0^2}{R_p} = \frac{G_T V_0^2}{R_p B}. \tag{12}$$

Thus the peak voltage that needs to be dropped across the antenna is

$$V_0 = \sqrt{\frac{S_{tx} R_p B}{G_T}}. \tag{13}$$

If we allow for the possibility of spread-spectrum type signalling with a processing gain $P_G$, then we can drop the maximum necessary voltage (because the power spectral density is decreased at the expense of a wider bandwidth) according to

$$V_0 = \sqrt{\frac{S_{tx} R_p B}{G_T P_G}}. \tag{14}$$

A summary of calculations using Eq. (14) is shown in Table I for the loop antenna referred to above (6-mm, copper loop on glass at 5.5 GHz). The results are encouraging. Even a broad, uncoded bandwidth (top row) requires only a peak 189-mV signal across the resistor. Although this should be easily accommodated under standard supply voltage levels (∼ 1 V), it is still undesirable for the nonlinearities it may arouse in the amplifier circuit. For example, a cross-coupled differential MOS pair can be fully switched with less than 300-mV. Thus, there is the very real danger that signals around 200-mV may introduce gross nonlinear behavior.

TABLE I

THE MAXIMUM VOLTAGE NEEDED TO DRIVE A 6-MM LOOP ANTENNA

| $V_0$ [mV] | $B$ [kHz] | $P_G$ | $S_{tx}$ [pW/Hz] | $R_p$ [kΩ] | $G_T$ [dB] |
|---|---|---|---|---|---|
| 189 | 100 | 1 | 30 | 13.9 | 0.66 |
| 59.9 | 10 | 1 | 30 | 13.9 | 0.66 |
| 18.9 | 1 | 1 | 30 | 13.9 | 0.66 |
| 10.6 | 10 | 32 | 30 | 13.9 | 0.66 |

Luckily we have options as detailed in Table I to reduce the necessary maximum driving voltage. For instance with processing gain of 32 and a 10-kHz signal spectrum (i.e. 320 kHz spread signal) the maximum voltage drops to 10.6 mV, leaving an amplifier working in essentially the small-signal (i.e. linear) regime.

## B. *Transmitter-Repeater Separation*

By considering the antenna and the signaling (in the above sub-section) we came to the conclusion that under the right conditions it may be able to treat the amplifier as a linear element without distancing ourselves from practical communications scenarios. With this we can explore further corners of the amplifier design space.

A key concern is the power gain — 47 dB — that the repeater would need should it be placed 1-m away from the source transmitter. A high-level circuit analysis shows that this is a difficult figure to reach.

In an earlier technical note [3] we examined the reflecting amplifier from a high-level circuit perspective. For one amplifier configuration we obtained the results shown again in Fig. 2. For a rather aggressive $K_G$ setting of 0.95 (when the compensation factor, $K_G$, exceeds unity the reflector begins to oscillate) the maximum reflected gain is 30 dB. However, the reflection amplifier must modulate the phase of the signal it is forwarding. For a phase modulation of $\pm 75°$ a gain of 20-dB is achieved.

This admittedly brief glance at the reflection amplifier indicated that a 47-dB gain is a challenging goal. Of course, the introduction of a novel means of efficiently controlling $K_G$ from exceeding unity, could open the door to substantial gain improvements. For the moment however, it seems that a 20-dB gain is close to the upper achievable performance leading

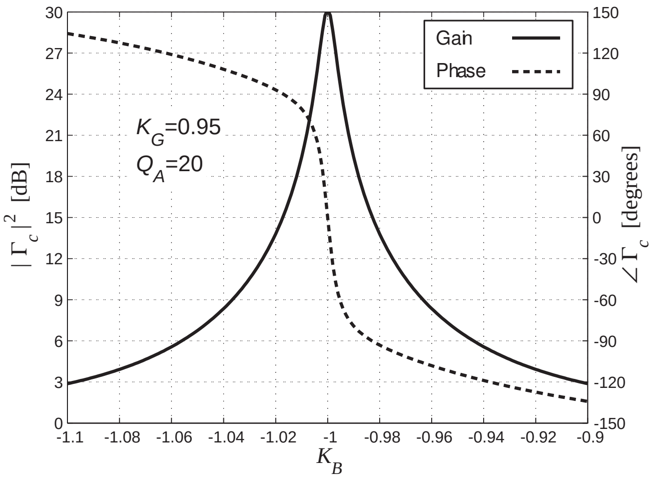


Fig. 2. Reflection coefficient of reflector antenna.

to a disappointing 4.3-cm separation between transmitter and repeater. If the system can adequately sustain a phase swept signal 10-dB below the source-transmitted information (at the destination receiver) than the transmitter-repeater distance increases to a lackluster 13.7 cm. Alternatively, the power of the source transmitter may need to be boosted.

### C. Amplifier Efficiency

In another technical note [4] a cross-coupled NMOS pair with (ideal) tail current source was proposed as a possible means of realizing the negative resistance needed to implement the reflection amplifier.

A simplified expression (valid for long-channel devices operating "sufficiently" below the unity power gain frequency $f_{max}$) for the compensation factor of this circuit was derived

$$K_G = \frac{G_R}{G_A} = \frac{P_{dc}}{2G_A V_{DD}(V_{DD} - V_T - V_B)}. \tag{15}$$

where $G_A$ is the parallel equivalent conductance of the antenna attached to the reflector, $P_{dc}$ is the dc power drawn by the reflector, $V_{DD}$ is the dc power supply voltage, $V_T$ is the transistor threshold voltage, and $V_B$ is the voltage needed across the current source (to sustain its behavior as a high-impedance current source).

For $K_G = 0.95$ (and hence an off-center gain of 20-dB), $V_{DD} = 1$ V, $V_T = 0.5$ V, $V_B = 0.2$ V (these values are reasonable starting points for the circuit design and were also explored in [4]) we have from Eq. (15).

$$P_{dc} \cdot R_p = 0.57 \tag{16}$$

where $P_{dc}$ is in watts and $R_p = 1/G_A$ is in ohms. Given the antenna characteristics (and of course the circuit properties defined above) we can use this equation to predict the amount of power needed to We must keep in mind that an antenna is not 100% efficient and that we can point it in an optimal direction (where the gain is maximum) hence, therefore the above expression should be augmented to

$$P_{dc} = \frac{0.57}{R_p G_T}. \tag{17}$$

For the antenna under consideration ($R_p = 13.9$ k$\Omega$, $G_T = 0.66$ dB) we have $P_{dc}$ = 35.4 $\mu$W.

Thus, for the amplifier to sustain a $K_G = 0.95$ requires 35.4 $\mu$W from the dc supply. In addition, the supply must also donate the actual power that must be radiated, about 30 pW/Hz·10 kHz = 0.3 $\mu$W (assuming that the repeater's 20-dB gain overcomes any loss between it and the source transmitter). Clearly this is negligible compared to the power drawn simply to make the circuit a suitable reflector making for a poor efficiency of only 0.85%. With a suitable supply (a solar cell say) this may be forgivable.

## References

[1] G. G. Messier, S. Magierowski, and J-F. Bousquet, "Co-operative phase sweep transmit diversity networks," *IEEE Communications Letters*, pp. 1–3, May 2007, submitted.
[2] S. Magierowski, "Small loop antenna simulations," Tech. Rep., June 20 2007.
[3] S. Magierowski, "Active analog reflector antennas: Scattered power," Tech. Rep., May 22 2007.
[4] S. Magierowski, "An active reflector circuit," Tech. Rep., May 23 2007.