Noise Factor of Active Reflector Antennas

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***Abstract*— A brief discussion on the noise performance of active reflector antennas with the goal of deriving an expression for the reflector noise factor.**

I. Background

We want to find an expression for the noise factor of an active reflector antenna. To study the noise behavior of active reflector antennas (essentially negative-resistance) amplifiers. We turn to Paul Penfield's report from 1960 [1]. Penfield employed a scattering matrix approach in his analysis (still rather new for the time). Conforming to this approach leads to a description of our circuit as illustrated in Fig. 1 where a lossless "imbedding network" has been placed between the negative-resistance amplifier and the antenna (not explicitly shown) to help with the formal development of our intended result.

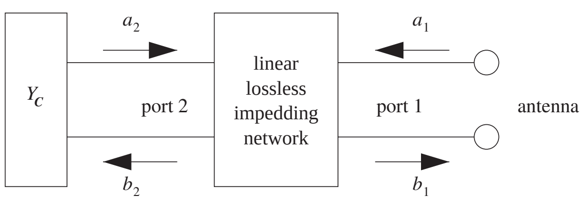

Fig. 1. A sketch of the reflector from the power wave formalism.

The variables a_1, a_2, b_1, b_2 are so called "power waves" (although they are in units of $\sqrt{\text{Watt}}$). Kurokawa [2] notes that Penfield was among the first to utilize the power wave concept in circuit theory. Kurokawa [2] also notes that Penfield's description of these variables was very brief (as his primary goal was deriving noise characteristics). Today, microwave engineers are much more familiar with the power wave concept, nonetheless a good discussion of these is available in the Kurokawa paper [2].

For the purposes of this paper we do not need to be intimately familiar with the details of the power wave concept. It is sufficient to realize that

$$\mathbf{b} = \mathbf{S}\mathbf{a} \tag{1}$$

and that the power absorbed by port 1 of the imbedding network is

$$\frac{1}{2}\text{Re}\{V_1 I_1^*\} = |a_1|^2 - |b_1|^2 \tag{2}$$

and that the power absorbed by port 2 of the imbedding network is

$$\frac{1}{2}\text{Re}\{V_2 I_2^*\} = |b_2|^2 - |a_2|^2. \tag{3}$$

The reason for the re-arrangement of the b and a in the last equation is because our circuit termination Y_C has a negative real component. That is, the power wave symbols, a and b, are defined with respect to a source with a positive real impedance component (i.e. the source driving the n-port under consideration), however in this case the source "driving" the two-port from the left has a negative impedance so we flip the order of a and b in describing the power absorbed at port 2.

The exchangeable power [2], P_e, (another microwave engineering term, but of a meaning similar to available power) of the circuit noise is a negative value (the exchangeable power concept allows this) related to our power waves by

$$P_e = -|a_2|^2. \tag{4}$$

II. Reflector Noise Factor

Since the imbedding network is lossless we have no net power going into it, that is

$$|a_1|^2 - |b_1|^2 = |b_2|^2 - |a_2|^2 \tag{5}$$

$$|a_1|^2 + |a_2|^2 - |b_1|^2 + |b_2|^2 = 0. \tag{6}$$

Employing the matrix

$$\mathbf{P} = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

we can express this power balance as

$$\mathbf{a}^\dagger \mathbf{P} \mathbf{a} - \mathbf{b}^\dagger \mathbf{P} \mathbf{b} = 0 \tag{7}$$

where the superscript † denotes the hermitian. Invoking the relationship between **a** and **b** (mitigated by **S**) cited above we have

$$\mathbf{a}^\dagger \mathbf{P} \mathbf{a} - \mathbf{S}^\dagger \mathbf{a}^\dagger \mathbf{P} \mathbf{S} \mathbf{a} = 0 \tag{8}$$

$$\mathbf{a}^\dagger (\mathbf{P} - \mathbf{S}^\dagger \mathbf{P} \mathbf{S}) \mathbf{a} = 0. \tag{9}$$

If this expression is to hold for any input then

$$\mathbf{S}^\dagger \mathbf{P} \mathbf{S} = \mathbf{P} \tag{10}$$

. Following Penfield, if we pre-multiply the above by $\mathbf{SP^{-1}}$ and post-multiply by $\mathbf{S^{-1}P^{-1}}$ we obtain

$$\mathbf{SP^{-1}S}^\dagger = \mathbf{P^{-1}}. \tag{11}$$

The 11 element of this states that

$$|S_{12}|^2 = |S_{11}|^2 - 1. \tag{12}$$

At last we are ready to generate an equation for the reflector noise factor which is defined as

$$F = 1 + \frac{\text{Power at output due to amplifier noise}}{\text{Power at output due to generator noise at input}}. \tag{13}$$

Symbolically we have

$$F = 1 + \frac{|S_{12}|^2 \cdot \overline{|a_2|^2}}{|S_{11}|^2 \cdot \overline{|a_1|^2}}. \tag{14}$$

Many thanks to the friends of FishLab.

With the help of Eq. (12) we can rewrite this as

$$F = 1 + \left(1 - \frac{1}{|S_{11}|^2}\right) \frac{\overline{|a_2|^2}}{\overline{|a_1|^2}} \tag{15}$$

Invoking Eq. (4) and assuming only thermal input noise such that

$$|a_1|^2 = kT\Delta f \tag{16}$$

we at last have

$$F = 1 + \left(1 - \frac{1}{|S_{11}|^2}\right) \frac{-P_e}{kT\Delta f}. \tag{17}$$

The S_{11} is simply the reflection coefficient in this circuit. That is, it is the reflector's (available/exchangeable) power gain

$$S_{11} = G = \Gamma_c = \frac{(1 + K_G) - j(1 + K_B)Q_A}{(1 - K_G) + j(1 + K_B)Q_A}. \tag{18}$$

where $Q_A = B_A/G_A$, $K_G = -G_C/G_A$, $K_B = B_C/B_A$ ($Y_C = G_C + jB_C$ is the admittance of the circuit attached to the antenna and $Y_A = G_A + jB_A$ is the admittance of the antenna itself).

References

[1] P. Penfield, "Noise in negative-resistance amplifiers," *IRE Trans. on Circuit Theory*, vol. 7, no. 6, pp. 166–170, June 1960.

[2] K. Kurokawa, "Power waves and the scattering matrix," *IEEE Trans. on Microwave Theory and Techniques*, vol. 13, no. 3, pp. 194–202, March 1965.