EECS2200

Lab #4

RLC Circuits

Introduction

Consider the circuit in Fig. L4.1.

Assume the initial conditions of the circuit are $i(0^+)=I_0$ and $v_c(0^-)=V_0$ .For $Vs=0$, applying KVL gives:

$$v_R + v_L + v_c = 0 \qquad \Rightarrow \qquad Ri + L\frac{di}{dt} + \frac{1}{C}\int_0^t i d\tau + V_0 = 0$$

Differentiating the two sides of the above equation with respect to t yields:

$$L\frac{d^2i}{dt^2} + R\frac{di}{dt} + \frac{1}{C}i = 0 \qquad \Rightarrow \qquad \frac{d^2i}{dt^2} + \frac{R}{L}\frac{di}{dt} + \frac{1}{LC}i = 0$$

In the above differential equation, assume:

$$\alpha = \frac{R}{2L} \text{ neper/s}, \quad \omega_0 = \frac{1}{\sqrt{LC}} \text{ rad/s}$$

The solution to this differential equation, which is the response of the series RLC circuit, depends on α and ω_0. The circuit can have one of the following conditions.

Case I: $\alpha > \omega_0$

The response is called overdamped, and the solution to the differential equation is

$$i(t) = A_1 e^{s_1 t} + A_2 e^{s_2 t}$$

in which $s_{1,2} = -\alpha \pm \sqrt{\alpha^2 + \omega_0{}^2}$

Case I: $\alpha = \omega_0$

The response is called critically damped, and the solution to the differential equation is

$$i(t) = A_1 e^{\alpha t} + A_2 t e^{\alpha t}$$

Case I: $\alpha < \omega_0$

The response is called underdamped, and the solution to the differential equation is

$$i(t) = e^{-\alpha t}\left(A_1 \cos(\omega_d t) + A_2 \sin(\omega_d t)\right)$$

in which $\omega_d = \sqrt{\omega_0{}^2 - \alpha_0{}^2}$

To find the constants A_1 and A_2 in the above relations for *i(t)*, the initial conditions of the circuit can be used. Keep in mind that the voltage across a capacitor and the current through an inductor cannot change suddenly, that is

$$v_C(0^-) = v_C(0^+) \quad , \quad i(0^-) = i(0^+)$$

$i(0^+)=I_0$ gives one equation for finding A_1 and A_2. $v_c(0^+)=V_0$ can be used to find $di/dt(0^+)$. Finding the derivative of *i(t)* given by the above relations and equating it with $di/dt(0^+)$ gives another equation for A_1 and A_2.

Once *i(t)* is known, the voltages across different elements of the circuit can be calculated:

$$v_R = i(t)R \ , \ v_L = L\frac{di}{dt} \ , \ v_C = -v_R - v_L \ \text{or} \ v_C = -\frac{1}{C}\int_0^t i(\tau)d\tau + V_0$$

In the presence of a non-zero *Vs*, the final value of each voltage or current (at time infinity) must be added to the time relations obtained for that voltage or current in the absence of *Vs*. For example, the capacitor voltage will be

$$v_c(t) = \begin{cases} V_s + B_1 e^{s_1 t} + B_2 e^{s_2 t} & \text{Overdamped} \\ V_s + e^{-\alpha t}\left(B_1\cos(\omega_d t) + B_2\sin(\omega_d t)\right) & \text{Underdamped} \\ V_s + B_1 e^{-\alpha t} + B_2 t e^{-\alpha t} & \text{Critically damped} \end{cases}$$

PreLab

Consider the circuit shown in Fig L4.1 with R=2 kΩ, L=100 mH, and C=0.1 μF.

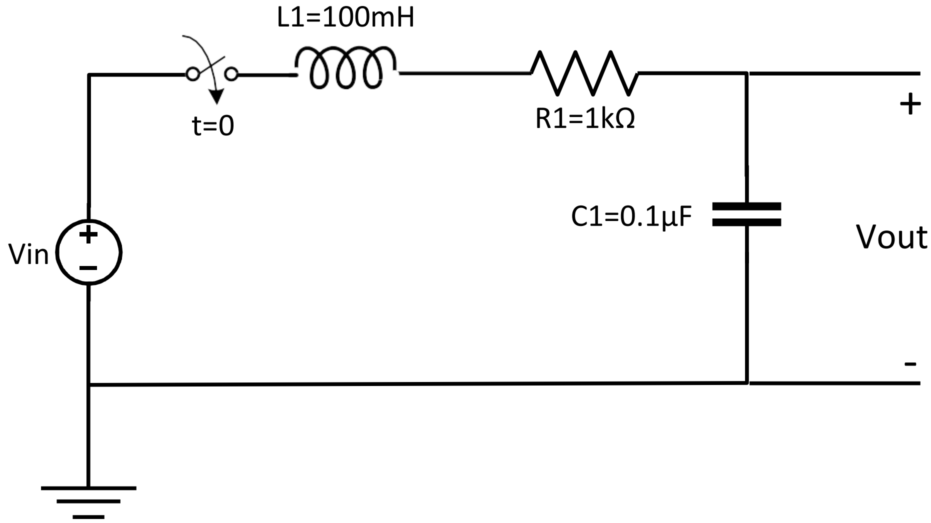

Figure L4.1

1) Solve the circuit to determine the voltage across the capacitor for t>0 assuming the capacitor's voltage and current to be 0 at *t*=0 and *Vin*=5V, and identify the type of damping.
2) Using MATLAB, plot the capacitor's voltage as a function of time.
3) What is the relation between R, L and C in critically damp situation? Also explain how will the circuit response change in case of increasing or decreasing R or L or C values. (Hint: Assume initial state of critically damped and fill the table with underdamped or overdamped.)

	R	L	C
Decreasing			
Increasing			

Lab experiment

Part I: RLC Circuit

Implement the RLC circuit as shown in Fig. L4.2 using L=100 mH, and C=0.1 μF. The value of the resistor is unknown and you have to calculate it for different situations.

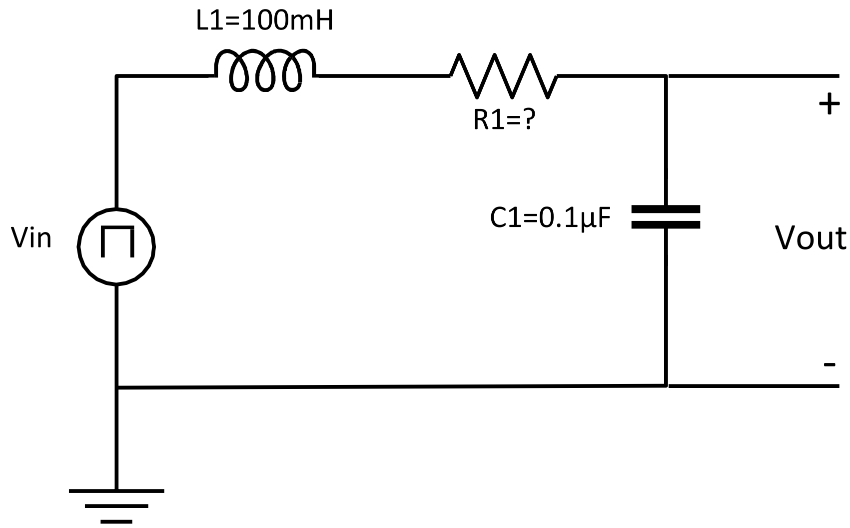

Figure L4.2

1) Calculate the value of ω_0 for this circuit.
2) Find the resistor value to generate the critically damped response.
3) Apply a square wave as the input voltage. (f=1kHz, amp=2V)
4) Measure capacitor voltage.
5) What would be the circuit response for the resistor values of 0.5 and 2 times of the value you found in question 2.
6) Measure the overshoot value for the underdamped situation.