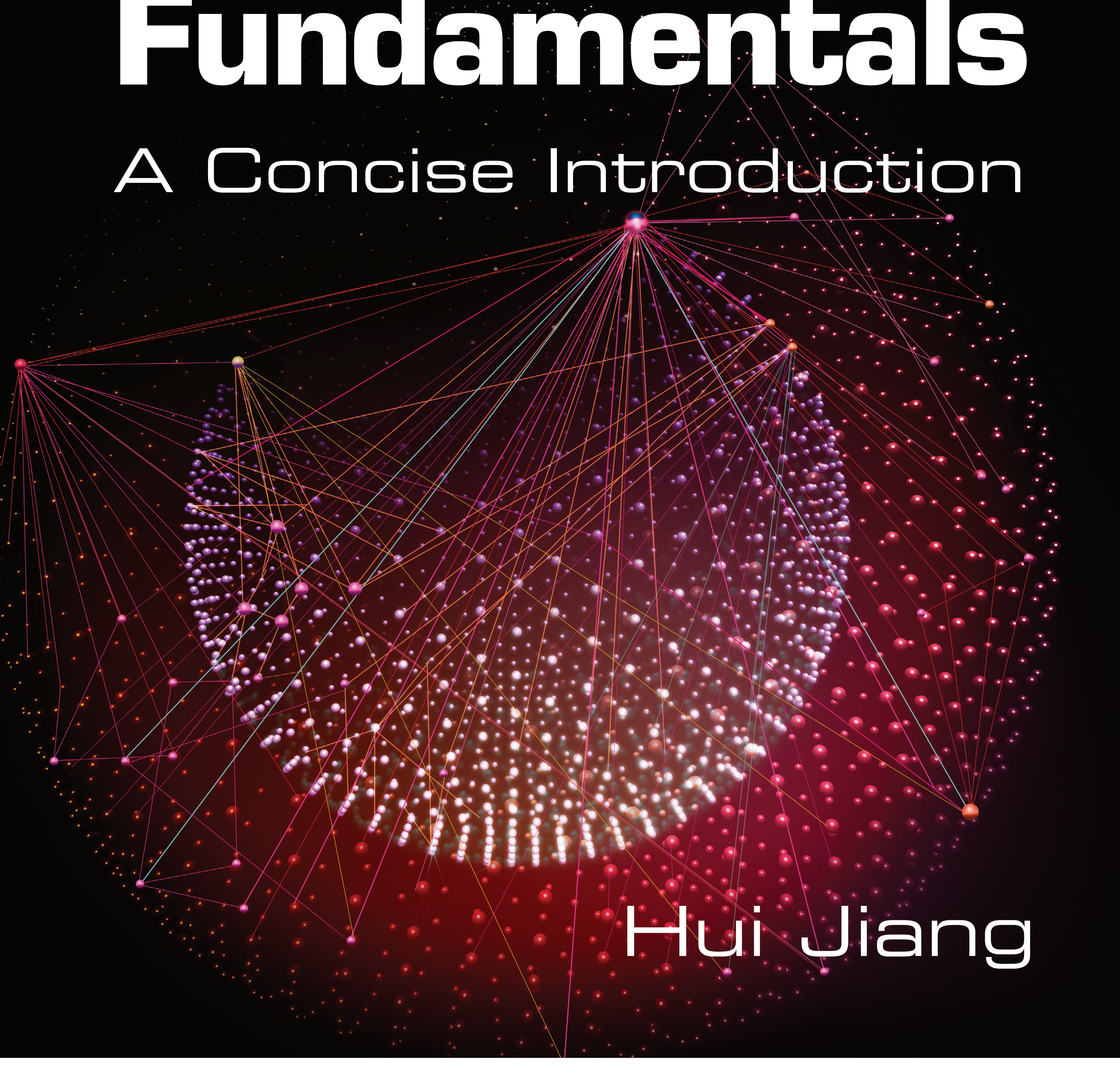
Machine Learning Fundamentals

A Concise Introduction



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